BEARYS INSTITUTE OF TECHNOLOGY

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INTERNAL ASSESSMENT ASSIGNMENT BOOK

NAME OF THE STUDENT : ARSHEEN ARA

YEAR : 2nd year 2022-2023 BRANCH : ECE SECTION : —

ROLL/UNIVERSITY SEAT NUMBER : 4BP21EC004

SUBJECT : DIGITAL SIGNAL PROCESSING [21EC42]

	DATE	MAX. MARKS	MARKS OBTAINED	TEACHER'S INITIAL	REMARKS
FIRST ASSIGNMENT	30/08/23	~~5~~ 10	10	Akbay	- very good -
SECOND ASSIGNMENT	22/09/23	~~5~~ 10	10	Akbay	- Very good -
THIRD ASSIGNMENT		5			
FINAL ASSIGNMENT		5	10/10		

FINAL ASSIGNMENT MARKS IN WORDS : Ten only.

Signature Staff HOD Principal

Assignment - 01

1. Compute N point DFT of the following signals

i) $x(n) = a^n$; $0 \le n \le n-1$

ii) $x(n) = \cos(2\pi k_0 n/N)$; $0 \le n \le N-1$

ans i) $x(n) = a^n$; $0 \le n \le n-1$

WKT DFT formula $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(kn)}$

$$X(k) = \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = \sum_{n=0}^{N-1} \left[a e^{-j\frac{2\pi}{N}L}\right]^n$$

W.K.T $\boxed{\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & \forall \alpha \ne 1 \\ N & \forall \alpha = 1 \end{cases}}$

$$X(k) = \frac{1-\left(a e^{-j\frac{2\pi}{N}L}\right)^N}{1-ae^{-j\frac{2\pi}{N}k}}$$

$$X(k) = \frac{1-a^N e^{-j\frac{2\pi}{\cancel{N}}k\cancel{N}}}{1-ae^{-j\frac{2\pi}{N}k}}$$

$$X(k) = \frac{1-a^N e^{-j2\pi k}}{1-ae^{-j\frac{2\pi}{N}k}} \longrightarrow (1)$$

W.K.T $e^{-j2\pi k} = \cos k2\pi - j\sin 2\pi k$

$$e^{-j2\pi k} = 1-0$$

$$\boxed{\therefore\ e^{-j2\pi k} = 1}$$

substitute the above eqⁿ in ①

$$\therefore ① \Rightarrow \boxed{X(k) = \frac{1 - a^N(1)}{1 - ae^{-j\frac{2\pi}{N}k}}}$$

ii] $x(n) = \cos\left(\frac{2\pi}{N}k_0 n\right)$; $0 \le n \le N-1$

W.K.T DFT formula $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

$$\therefore X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} \quad ; \ 0 \le n \le N-1$$

$$X(k) = \sum_{n=0}^{N-1} \cos\left(\frac{2\pi}{N}k_0 n\right) e^{-j\frac{2\pi}{N}kn}$$

W.K.T $\boxed{\cos\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}}$

$$\cos\left(\frac{2\pi}{N}k_0 n\right) = \left[\frac{e^{\left(j\frac{2\pi}{N}k_0 n\right)} + e^{\left(-j\frac{2\pi}{N}k_0 n\right)}}{2}\right] \rightarrow ⓐ$$

2

$\therefore$ substitute above equation ⓐ in ①

$$① \Rightarrow X(k) = \sum_{n=0}^{N-1} \left[\frac{e^{j\frac{2\pi}{N}k_0 n} + e^{-j\frac{2\pi}{N}k_0 n}}{2}\right] e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = \frac{1}{2}\left[\sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-k_0)n} + \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k+k_0)n}\right] \longrightarrow ②$$

W.K.T $$\boxed{\begin{aligned} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-k_0)} &= N\delta(k-k_0) \\ \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-N+k_0)} &= N\delta(k-N+k_0) \end{aligned}} \longrightarrow (b)$$

substitute (b) in (2)

$$X(k) = \frac{1}{2}\left[N\delta(k-k_0) + N\delta(k-N+k_0)\right]$$

$$X(k) = \frac{N}{2}\left[\delta(k-k_0) + \delta(k-N+k_0)\right]$$

2. Consider a finite duration sequence $x(n) = \{0, 1, 2, 3, 4\}$ sketch the sequence $s(n)$ with 6 point DFT $S(k) = W_2^{*} X(k)$; $0 \le k \le 6$

ans given $x(n) = \{0, 1, 2, 3, 4\}$

$$S(k) = W_2^{*} X(k) \quad ; 0 \le k \le 6 \longrightarrow (1)$$

$$N = 6$$

W.K.T $$W_2^{*k} = e^{\frac{-j\pi}{2} \times k \times 2} \longrightarrow (2)$$

$\therefore x(n) = \{0, 1, 2, 3, 4\}$

substitute (2) in (1)

$$\therefore S(k) = e^{\frac{-j\pi k 2}{2}} X(k) \longrightarrow (3)$$

W.K.T $$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$\therefore (3) \Rightarrow S[k] = e^{\frac{-j2\pi}{2} kl} X(k) \qquad l = \frac{N}{2} = \frac{6}{2} = 3$$

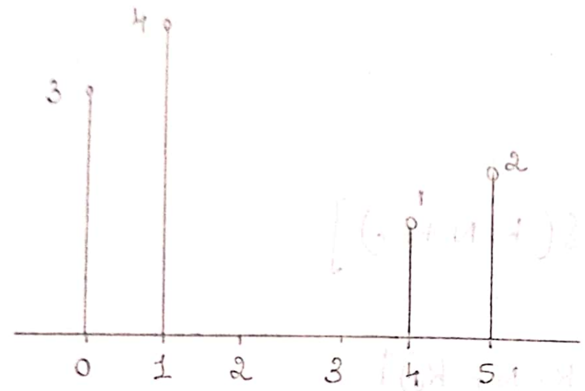
$$S[k] = e^{-j\frac{2\pi \times 3k}{2 \times 3}} X(k) \qquad l = 3$$

Take IDFT on both sides using circular time shift property

$l = 3$

$\therefore \ s(n) = x((n-3))_6$

$s(n) = \{3, 4, 0, 0, 1, 2\}$



3. Find 6 point DFT for $x(n) = \{1, 1, 0, 0, 0, 2\}$

ans given $N = 6$

$x(n) = \{1, 1, 0, 0, 0, 2\}$

W.K.T $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

$\therefore \ W_N = e^{-j\frac{2\pi}{N}}$

$W_6 = e^{-j\frac{\pi}{3}}$

$\therefore \ W_6^0 = e^0 = \underline{\underline{1}}$

$W_6^1 = e^{-j\pi/3} = \cos \pi/3 - j \sin \pi/3$

$W_6^1 = \underline{\underline{0.5 - 0.866j}}$

$W_6^2 = e^{-j2\pi/3} = \cos 2\pi/3 - j \sin 2\pi/3$

$W_6^2 = \underline{\underline{-0.5 - 0.866j}}$

$W_6^3 = e^{-j\frac{\pi 3}{3}} = e^{-j\pi} = \cos \pi - j \sin \pi$

$W_6^3 = \underline{\underline{1}}$

$$W_6^4 = e^{-j\pi 4/3} = \cos 4\pi/3 - j \sin 4\pi/3$$

$$W_6^4 = \underline{-1/2 + j\sqrt{3}/2}$$

$$W_6^5 = e^{-5j\pi/3} = \cos 5\pi/3 - j \sin 5\pi/3$$

$$W_6^5 = \underline{0.5 + 0.866j}$$

$$\therefore \text{WKT} \quad X[k] = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

For $k=0$

$$X(0) = \sum_{n=0}^{5} x(n) W_6^0$$

$$X(0) = x(0)W_6^0 + x(1)W_6^0 + x(2)W_6^0 + x(3)W_6^0 + x(4)W_6^0 + x[5]W_6^0$$

$$X(0) = 1(1) + 1(1) + 0 + 0 + 0 + 2(1)$$

$$X(0) = \underline{4}$$

For $k=1$

$$X(1) = \sum_{n=0}^{5} x(n) W_6^n$$

$$X(1) = x(0)W_6^0 + x(1)W_6^1 + x(2)W_6^2 + x(3)W_6^3 + x(4)W_6^4 + x(5)W_6^5$$

$$X(1) = 1(1) + (0.5 - 0.866j) + 0 + 0 + 0 + 2(0.5 + 0.866j)$$

$$X(1) = 1 + 0.5 - 0.866j + 1 + 1.732j$$

$$X(1) = \underline{2.5 + 0.866j}$$

For $k=2$

$$X(2) = \sum_{n=0}^{5} x(n) W_6^{2n}$$

$$X(2) = x(0)W_6^0 + x(1)W_6^2 + x(2)W_6^4 + x(3)W_6^6 + x(4)W_6^8 + x(5)W_6^{10}$$

$$X(2) = 1(1) + 1(-0.5 - 0.866j) + 0 + 0 + 0 + 2(-0.5 + 0.866j)$$

$$X(2) = \underline{-0.5 + 0.866j}$$

For k = 3

$$X(3) = \sum_{n=0}^{5} x(n) W_6^{3n}$$

$$= x(0)W_6^0 + x(1)W_6^3 + x(2)W_6^{6} + x(3)W_6^{9} + x(4)W_6^{12} + x(5)W_6^{15}$$

$$= 1(1) + 1(-1) + 0 + 0 + 0 + 2(-1)$$

$$X(3) = \underline{\underline{-2}}.$$

For k = 4

$$X(4) = \sum_{n=0}^{5} x(n) W_6^{4n}$$

$$= x(0)W_6^0 + x(1)W_6^4 + x(2)W_6^8 + x(3)W_6^{12} + x(4)W_6^{16} + x(5)W_6^{20}$$

$$= 1(1) + 1(-0.5 + 0.866j) + 0 + 0 + 0 + 2(0.5 - 0.866j)$$

$$= 1 - 0.5 + 0.866j - 1 - 1.732j$$

$$X(4) = \underline{\underline{-0.5 - 0.866j}}$$

For k = 5

$$X(5) = \sum_{n=0}^{5} x(n) W_6^{5n}$$

$$= x(0)W_6^0 + x(1)W_6^5 + x(2)W_6^{10} + x(3)W_6^{15} + x(4)W_6^{20} + x(5)W_6^{25}$$

$$= 1(1) + 1(0.5 + 0.866j) + 0 + 0 + 0 + 2(0.5 - 0.866j).$$

$$= 1 + 0.5 + 0.866j + 1 - 1.732j$$

$$= \underline{\underline{2.5 - 0.866j}}$$

$$\therefore X(k) = \{4,\ 2.5 + 0.866j,\ -0.5 + 0.866j,\ -2,\ -0.5 - 0.866j,\ 2.5 - 0.866j\}$$

4. A designer has a number of 8 point FFT chips. Show explicitly how he should interconnect four such chips in order to compute a 32 point DFT.

ans W.K.T $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \longrightarrow ①$

$$X(k) = \left[\sum_{n=0,4,8,12,16,20,24}^{28} x(n) W_N^{kn} + \sum_{n=1,5,9,13,17,21,25} x(n) W_N^{kn} + \sum_{n=2,6,10,14,18,22,26} x(n) W_N^{kn} \right.$$

$$\left. + \sum_{n=3,7,11,15,19,23,27} x(n) W_N^{kn} \right] \longrightarrow ②$$

For eq ② Put $n=4i$ in first term, $n=4i+1$ in second term, $n=4i+2$ in 3rd term and $n=4i+3$ in fourth term

$$\therefore X(k) = \left[\sum_{i=0}^{7} x(4i) W_N^{k(4i)} + \sum_{i=0}^{7} x(4i+1) W_N^{k(4i+1)} + \sum_{i=0}^{7} x(4i+2) W_N^{k(4i+2)} + \right.$$

$$\left. \sum_{i=0}^{7} x(4i+3) W_N^{k(4i+3)} \right]$$

$$X(k) = \left[\sum_{i=0}^{7} x(4i) W_{N/4}^{ki} + \sum_{i=0}^{7} x(4i+1) W_N^{k} W_{N/4}^{ki} + W_N^{2k} \sum_{i=0}^{7} x(4i+2) W_{N/4}^{ki} + \right.$$

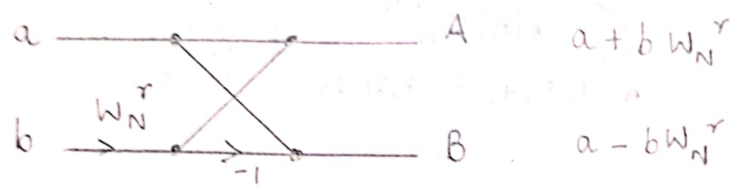
$$\left. W_N^{3k} \sum_{i=0}^{7} x(4i+3) W_{N/4}^{ki} \right]$$

$$\boxed{X(k) = X_1(k) + W_N^{k} X_2(k) + W_N^{2k} X_3(k) + W_N^{3k} X_4(k)}$$

where $X_1(k)$, $X_2(k)$, $X_3(k)$ and $X_4(k)$ are 8 point FFT chips

5. What is in place computation? what is the total number of complex addition, complex multiplication required for N = 512 point if DFT is computed directly and if FFT is used? Also find the number of stages

ans inplace computation



the FFT algorithm is performed in stages involves only N/2 butterflies and these butterflies operates on a pair of complex numbers and produces another pair of complex number. once the output pair is calculated we don't need the input pair anymore. Hence the output pair can be stored in the same location as input pair this is called as inplace computation.

* If N = 512
* The total number of complex multiplication if DFT is computed directly is $(N^2) = \underline{262144}$
* The total number of complex addition if DFT is computed directly is $N(N-1) = \underline{261632}$
* The total number of complex addition if FFT algorithm is used is $N \log_2 N = \underline{4608}$
* The total number of complex multiplication if FFT algorithm is used is $N/2 \log_2 N = \underline{2304}$
* The number of stages is $\log_2 N = \underline{9}$
* The total number of butterflies is $(N/2) = \underline{256}$

?

10/10

01/09/23

Assignment - 02

1. Convert the following decimal number into the ~~fee~~ floating point representation use 4 bits to represent exponent and 12 bits for mantissa.

i] 0.640492×2^{-2}

ans $0.640492 \times 2^{-2} = 0.1601230$

mantissa = 0.640492

Exponent = -2 [1110 in binary]

convert mantissa 0.640492 to binary [12 bits]

$0.640492 \longrightarrow 0.640492 \longrightarrow 0$

$0.640492 \times 2 \longrightarrow 1.280984 \longrightarrow 1$

$0.280984 \times 2 \longrightarrow 0.561968 \longrightarrow 0$

$0.561968 \times 2 \longrightarrow 1.123936 \longrightarrow 1$

$0.123936 \times 2 \longrightarrow 0.247872 \longrightarrow 0$

$0.247872 \times 2 \longrightarrow 0.445744 \longrightarrow 0$

$0.445744 \times 2 \longrightarrow 0.991488 \longrightarrow 1$

$0.991488 \times 2 \longrightarrow 1.982976 \longrightarrow 1$

$0.982976 \times 2 \longrightarrow 1.931904 \longrightarrow 1$

$0.931904 \times 2 \longrightarrow 1.863808 \longrightarrow 1$

$0.863808 \times 2 \longrightarrow 1.727616 \longrightarrow 1$

0.638454 = 010100011111

~~-0.638454 obtained by taking 2's compliment~~

~~101011100100~~

cascading exponent bits and mantissa bits : ~~01 0110 10 1110 0100~~
111 00101 000111 111

ii] -0.638454×2^5

$-0.638454 \times 2^5 = -20.430526$

mantissa = -0.6385454

Exponent = 5 [0101] in binary

Convert -0.638454 into binary (12 bits)

0.638454 →	0.638454 →	0
0.638454×2 →	1.276908 →	1
0.276908×2 →	0.553816 →	0
0.553816×2 →	1.107632 →	1
0.107632×2 →	0.215264 →	0
0.215264×2 →	0.430528 →	0
0.430528×2 →	0.861056 →	0
0.861056×2 →	1.722112 →	1
0.722112×2 →	1.444224 →	1
0.444224×2 →	0.888448 →	0
0.888448×2 →	1.776896 →	1
0.776896×2 →	1.553792 →	1

0.638454 = 0101 000 11011

-0.638454 obtained by taking 2's compliment

-0.638454 = 1010111 00100

Cascading exponent bits and mantissa bits : 0101101011100100

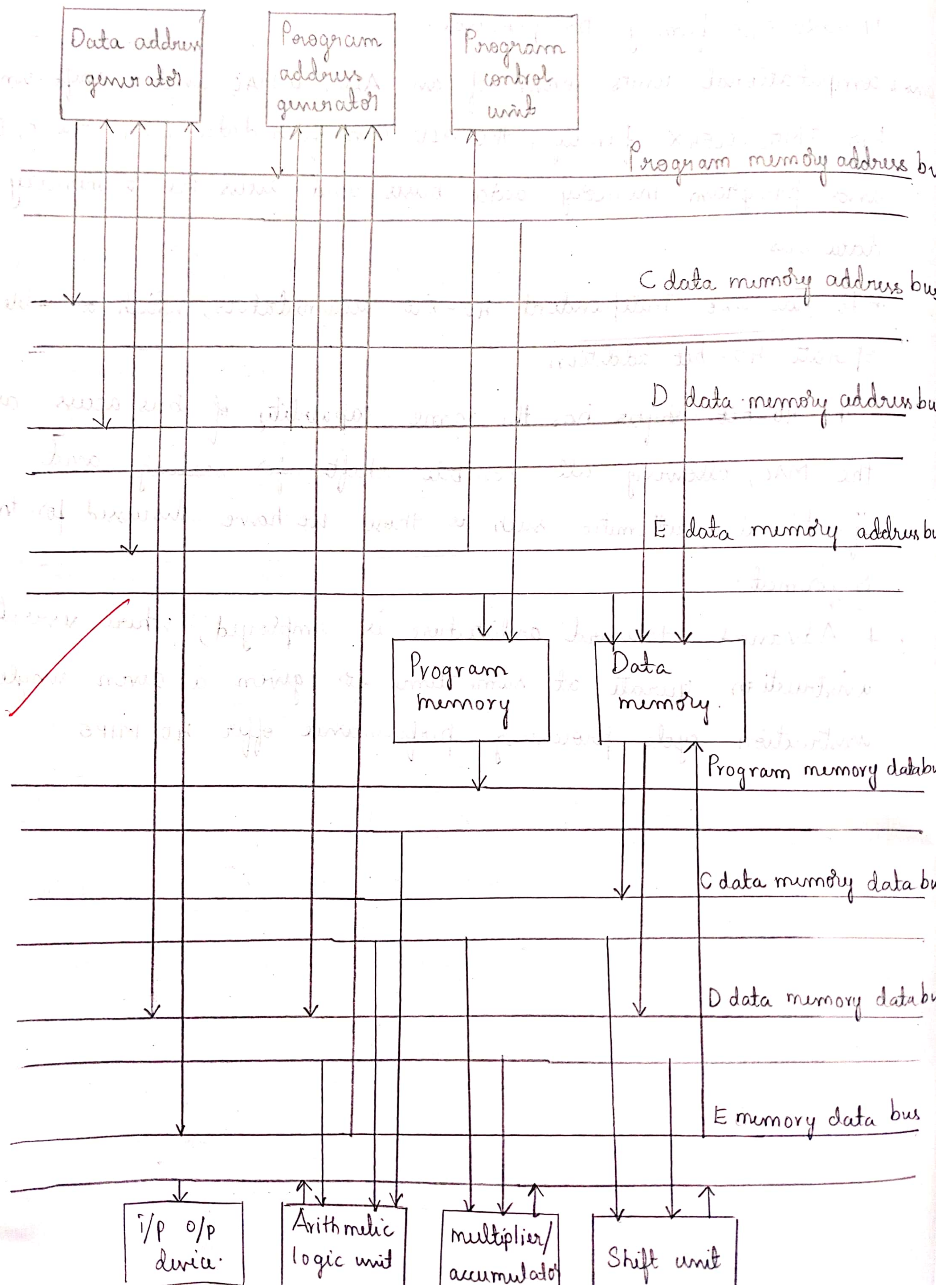
2. With a neat diagram explain the basic architecture of TMS320C54X family DS processor.

ans * computational units consist of an ALU, a MAC and a shift unit For TMS320C54X family, the ALU can fetch data from the C, D and program memory data buses and access the E memory data bus

* It has two independent 40-bit accumulators, which are able to operate 40-bit addition

* The 40-bit shifter has the same capability of bus access as the MAC, allowing all possible shifts for scaling and fractional arithmetic such as those we have discussed for the Q format.

* Advanced Harward architecture is employed, where several instruction operate at same-time for given a given single instruction cycle processing performance offer 40 MIPS



Data address generator
Program address generator
Program control unit
Program memory address bus
C data memory address bus
D data memory address bus
E data memory address bus
Program memory
Data memory
Program memory data bus
C data memory data bus
D data memory data bus
E memory data bus
i/p o/p device
Arithmetic logic unit
multiplier/ accumulator
Shift unit

4. What are the advantages and disadvantages of the window technique for designing FIR filters

ans Advantages:

* It is a simple method to implement to get the desired response

* There are minimum computations involved in executing this technique.

* No matter how much we increase the value of m, the maximum magnitude of ripple is fixed.

* There are many window functions, so various window functions can be used depending upon the applications

Disadvantages

* It does not have much flexibility as there are equal amount of Passband and stopband ripple present in the response that limits the ability of the design to make output more ideal.

* No trade off between stopband and transition band

* Lack of precious control of critical frequencies such as Ω_p and Ω_s

2

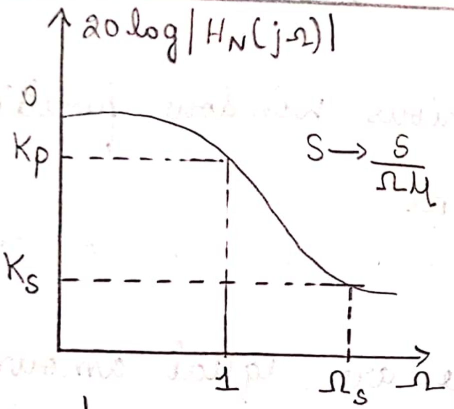
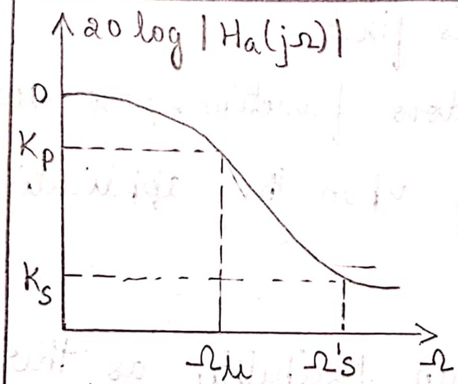
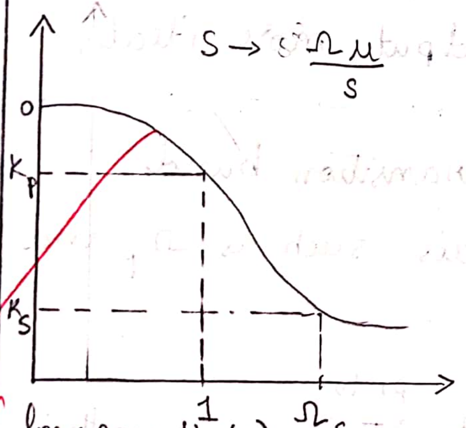
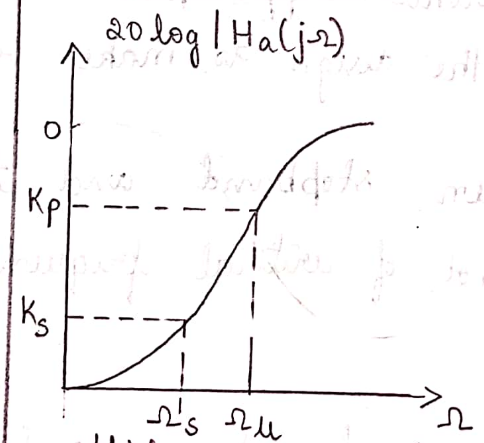
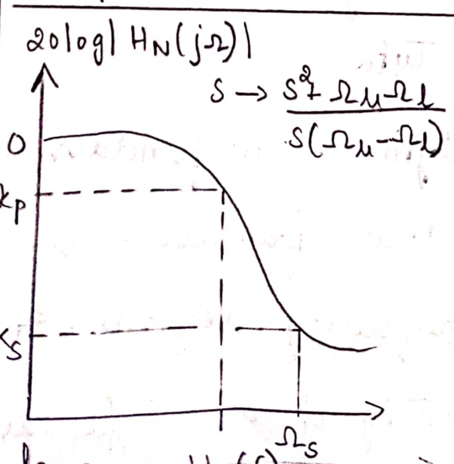
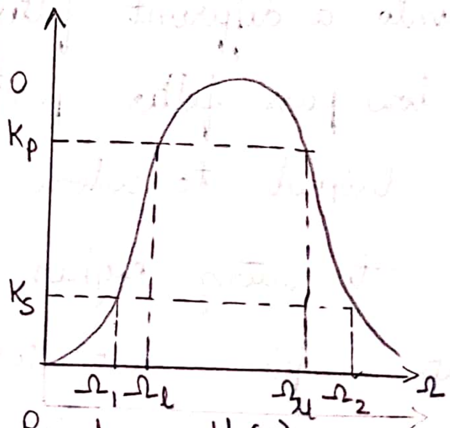
5. Define the first order analog low pass filter proto type. how this prototype transformed into a different filter type

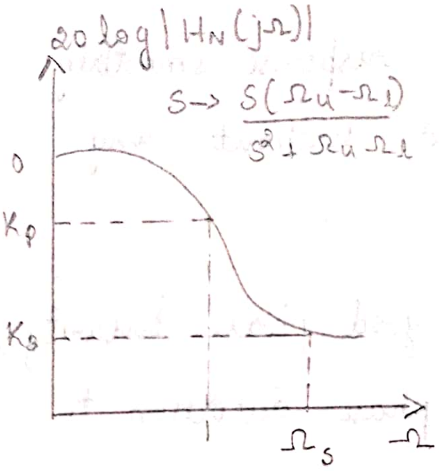
ans The first order analog low pass filter prototype is a fundamental electronic filter circuit designed to allow low frequency signals to pass through while attenuating higher frequency signals. It is characterized by its simple circuit consisting of a single resistor

(R) and a single capacitor C connected in series the transfer function H(s) for this filter is given by

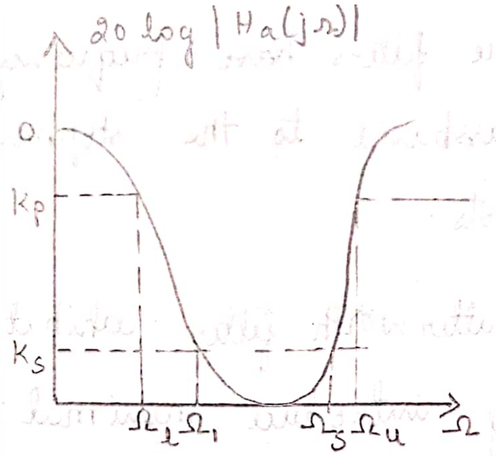
$$H(s) = \frac{1}{(1+sRC)}$$

where, s → represents the complex frequency variable
R → is the resistance
C → is the capacitance

Prototype frequency response	Transformed frequency response	Backward design equations.
 low pass, $H_N(s)$	 low pass $H_a(s)$	$\Omega_s = \frac{\Omega'_s}{\Omega_u}$
 low pass $H_N(s)$	 High pass ($H_a(s)$)	$\Omega_s = \frac{\Omega_u}{\Omega'_s}$
 lowpass $H_N(s)$	 Bandpass $H_a(s)$	$\Omega_s = \mathrm{Min}\{\lvert A\rvert, \lvert B\rvert\}$ $A = \frac{-\Omega_1^2 + \Omega_l\Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$ $B = \frac{-\Omega_2^2 - \Omega_l\Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$



low pass $H_N(s)$



Bandstop $H_a(s)$

$$\Omega_s = \text{Min}\{|A|, |B|\}$$

$$A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l\Omega_u}$$

$$B = \frac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_u\Omega_l}$$

3. Define the normalized low pass prototype function of butterworth filter and explain the butterworth filter characteristics

ans A filter is said to be normalized if the cut off frequency of the filter is rad/sec

first we need to design normalized low pass filter then low pass, high pass, band pass, band stop filters can be designed by applying a specific analogue to analogue transformation to a normalized low pass filter

$$\therefore |H(j\Omega)| = \frac{1}{\left[1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/N}}$$

Characteristics of butterworth filter :-

1) Maximally flat response : butterworth filters are designed to have a maximally flat frequency response in pass band

2) Slow roll off :- Butterworth filters have a gradual roll off in the stopband, which means that as you move away from the cut off frequency, the attenuation of the filter decreases slowly.

8. 3) Monotonic decay :- The filters have frequency response smoothly decreases from the passband to the stopband without any osillations or overshoots.

4) Phase linearity :- Butterworth filter exhibit good phase linearity, which means that they introduce minimal phase distortion to the filtered signal.

5) Transfer function shape :- The transfer function of a butterworth filter is characterized by its N^{th} order polynomial in the denominator, which determines the filters order and ultimately its roll-off rate.

10/10

Veeru
23/09/2023