

Module-1

When we say 'materials' we think of nearly all materials known to science and in all states of matter like solid, liquid & gaseous.

But material science concern itself basically with the nature & behaviour of only solid materials.

'Solid engineering materials' are those which helps engineers to build machines, structures, automobiles, and air craft.

Classifications of engineering materials:

- (i) Metals & alloys: Ex: cast iron, steels, aluminium, copper, silver, gold, brass & bronze
- (ii) Ceramics & glasses : Ex: MgO, ZnO, SiC, concrete & cement
- (iii) Polymers: plastics, polyethylene, PVC, nylon, cotton & rubber
- (iv) Composites: metal-matrix composites

Each of above group of materials has their own set of properties. Some of the most engineering materials as follow:

- ✓ Mechanical: strength, hardness, ductility, malleability, toughness, resilience & fatigue
- ✓ Physical: shape, size, density, porosity & colour
- ✓ Chemical: acidity, alkalinity, composition, corrosion resistance, atomic number & molecular weight
- ✓ Electrical: conductivity, resistivity, dielectric constant, dielectric strength & power factor
- ✓ Thermal: Specific heat, refractoriness & conductivity
- ✓ Aesthetic: feel, texture, appearance, lustre

The above properties of the materials which guide us in the selection of the materials for specific operations. Ex: (i) an aircraft structure has to be built with materials having low density but high strength, (ii) a steel melting furnace

has to be lined with refractory materials to with stand high temperature, (III) buildings & structures have to be built with materials having high compressive strength to with stand heavy loads.

Structure of crystalline solids:

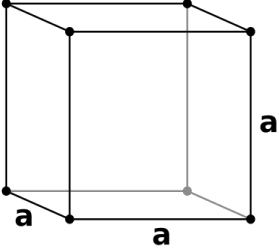
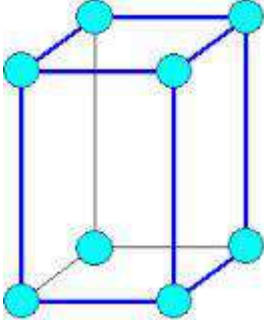
1. Crystalline solids,
2. Non- Crystalline solids

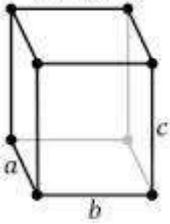
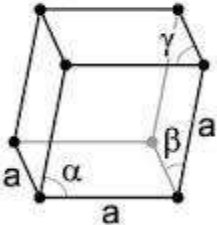
Crystalline solids	Non- Crystalline solids/ amorphous solids
1. The basic structural unit is a crystal [a solid whose constituent molecules or atoms are arranged in a systematic geometric pattern.	The basic structural unit is a molecule & chains of these molecule come together to form an amorphous solids
2. Each crystal [also called as a grain] is made up of a number of respective blocks called unit cells[the smallest group of atoms possessing the symmetry of the crystal] which are arranged neatly in relation to each other	The chains of molecules are random within the solid & occur in no particular relation to each other. They are irregular & lack symmetry
3. Compare crystalline solid with a military parade where all soldiers are arranged in order with respect to each other.	In this, crowd where people are random & not arranged in order with respect to each other.

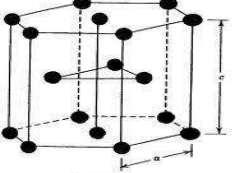
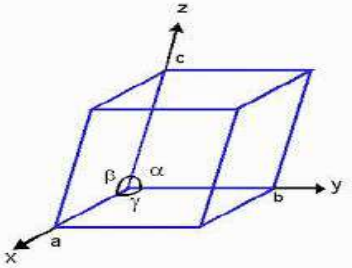
4. A crystalline solid therefore is made up of millions of unit cells orderly arranged. Each unit cell is itself made up of atoms & the number of atoms depends on the type of unit cell.	In this it is made up of millions of molecules disorderly arranged
5. Metals, alloys, some salts like NaCl, KCl, many oxides & ceramics, non metals like diamond, Gem stones	Glass, polymers, rubber & plastics
6. Density of crystalline solids is generally high. They have higher melting point & strength	Generally low because molecules cannot be compacted. They have lower melting point & strength
7. Structures are stable & materials are stronger	Structures are unstable & materials are less stronger

Aggregates: some materials are obtained both in crystal as well as amorphous. Ex: silicate can occur as crystalline solid [quartz] or a Non- Crystalline solids/ amorphous solids [silicate glass]. Aggregates type of materials which have short range order but no long range order. Ex: concrete, rocks & minerals.

Unit cell: it is the smallest repeatable unit of a crystalline solid. In other words every unit cell is a crystalline solid consists of a group of atoms arranged in a definite order.

Unit cell [crystal system]	Space lattice [Atomic arrangement]	Abb.	Examples of materials
 <p>Fig: simple cubic</p> <p>1. Cubic $a=b=c$ $\alpha=\beta=\gamma=90^\circ$</p>	<p>1. Simple cubic Atoms only at the 8 corners of the unit cell</p> <p>2. Body centred cubic Atoms at the 8 corners plus one at the centre of the body</p> <p>3. Face centred cubic Atoms at the 8 corners plus one each at the centre of 6 faces</p>	<p>SC</p> <p>BCC</p> <p>FCC</p>	<p>Mn, Po, NaCl</p> <p>Na, V, Cr, Mo, W, Fe, CsCl, α-iron, δ-iron</p> <p>Ni, Cu, Ag, Al, Pb, Pt, γ-iron, Fe,</p>
<p>2. Tetragonal</p>  <p>$a=b \neq c$ $\alpha=\beta=\gamma=90^\circ$</p>	<p>4. Simple Tetragonal Atoms only at the 8 corners of the unit cell</p> <p>5. Body centred Tetragonal Atoms at the 8 corners & one atom at centre</p>	<p>ST</p> <p>BCT</p>	<p>Pa, In</p> <p>Sn, U, Martensite</p>

<p>3. Orthorhombic $a \neq b \neq c$</p>  <p>$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$</p>	<p>6] Simple Orthorhombic Atoms at the 8 corners</p> <p>7] End centred Orthorhombic Atom at 8 corners & at 2 face centres opp. To each other</p> <p>8] Body centred Orthorhombic Atoms at 8 corners & one at body centre</p> <p>9] Face centred Orthorhombic Atoms at 8 corners & at 6 face centres</p>	<p>SO</p> <p>ECO</p> <p>BCO</p> <p>FCO</p>	<p>As, Bi, Sb</p> <p>Mg, SO_4, KNO_3</p> <p>Cementite</p> <p>Ga, S</p>
<p>4. Rhombohedral $\alpha, \beta, \gamma \neq 90^\circ$</p> 	<p>10. Simple Rhombohedra Atoms at 8 corners</p>	<p>SR</p>	<p>B, CaCO_3, SiO_2</p>

$a=b=c$ $\alpha=\beta=\gamma \neq 90^\circ$			
<p>5. Hexagonal</p>  <p>HCP</p> $a=b \neq c$ $\alpha=\beta=90^\circ \quad \gamma=120^\circ$	<p>11. Hexagonal close packed Atoms each at 6 corners of top face, 6 corners of base, 2 hexagonal face centres & 3 within the body</p>	HCP	Mg, Be, Ti, Hf, Co, Cd, Zn
<p>6. Monoclinic</p>  $a \neq b \neq c$ $\alpha \neq \beta = 90^\circ \neq \gamma$	<p>12. Simple Monoclinic Atoms at 8 corners</p> <p>13. End centred Monoclinic Atoms at 8 corners & at 2 face centres opp. to each other</p>	SM ECM	FeSO ₄ , NaSO ₄ ,
7. Triclinic	14. Simple triclinic	STC	CuSO ₄ , K ₂ Cr ₂ O ₇



Space lattice in crystals:

- ✓ We know that a number of crystals come together to form a crystalline solid & each of these crystals in turn contain a number of respective blocks called Unit cells.
- ✓ In each unit cell itself, we have atoms which are regularly spaced along rows, columns & each in other directions.

'This three dimensional pattern where the atoms arrange themselves in an orderly manner along various directions called Space lattice'.

Bravais space lattice:

- ✓ Although every crystalline solid has a certain kind of space lattice, there are only 14 possible types, according to Bravais.
- ✓ He said that there are only 14 independent ways of arranging points in 3 dimensional spaces such that each arrangement confirms to the definition of a space lattice. These 14 lattices are called as Bravais space lattice, & they fall under seven basic crystals or crystal shapes.

Co-ordination number:

Each atom in a crystal structure is surrounded by a number of atoms. The surrounding atoms are located at different distances. The Co-ordination number of a crystal structure is defined as *the number of nearest & equidistant neighbouring atoms that each atom has in a space lattice*. Ex: Simple cubic-6, BCC-8, FCC-12, HCP-12.

Atomic packing factor:

It is the ratio of the volume of atoms contained in each unit cell to the volume of the unit cell itself OR the APF represents the percentage of space within each unit cell of a crystal structure which is packed with atoms.

- ✓ It also gives us information on the free space available within the unit cell.
- ✓ APF basically affects the density of the material.

$$\text{APF} = \frac{\text{Volume of atoms}}{\text{Volume of unit cell}}$$

CALCULATION OF APF FOR SOME UNIT CELLS:

1] SIMPLE CUBIC:

Let a = lattice constant, r = atomic radius

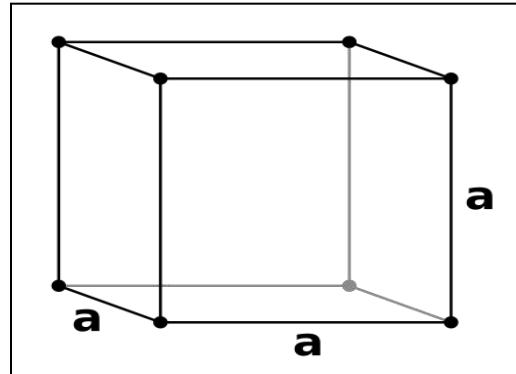


Fig: SCC

- ✓ The centre of each atom co-insides with the different corners of the cube & the atoms touch each other at their periphery. Hence let us take $a = 2r$.

- ✓ Each one of the atoms at the corners of the cube has effectively only $1/8^{\text{th}}$ of its volume present inside the cubic cell.
- ✓ The remaining volume of the atom is shared by its seven neighbouring unit cells.
- ✓ Therefore, no. of atoms in one simple cubic unit cell = $8 \times 1/8^{\text{th}}$ of an atom
= 1 atom.

$$\text{APF} = \frac{\text{Volume of atoms}}{\text{Volume of unit cell}} = \frac{\text{No. of atom} \times \text{Vol. of each atom}}{\text{Vol. of unit cell}} = 1 \times \frac{4\pi r^3/3}{a^3}$$
$$= 4\pi r^3/3 \{2r\}^3$$
$$= 0.52$$

[Since Vol. of sphere = $4\pi r^3/3$ & $a = 2r$]

APF = 0.52

Thus, it means that 52% of space inside the unit cell of a SIMPLE CUBIC structure is packed with atoms & the remaining 48% is empty space.

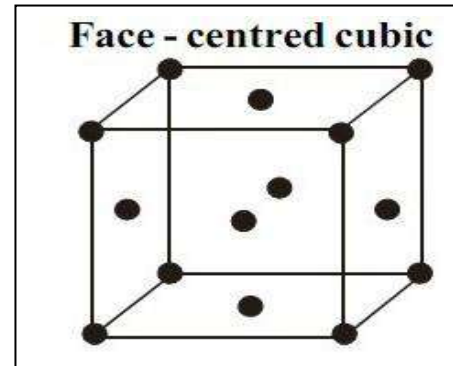
2] FACE CENTERED CUBIC:

One FCC unit cell has an atom at each corner of the cube as well as one atom at the centre of each of the six faces.

There is only $1/8^{\text{th}}$ of each corner atom effectively inside the cell where as $1/2$ of the volume of each atom at the centre of the faces are within the unit cell.

Therefore,

Effective no. of atoms in FCC one unit cell = $8 \times 1/8 + 6 \times 1/2 = 4$ atoms.



To find 'a' in terms of 'r'

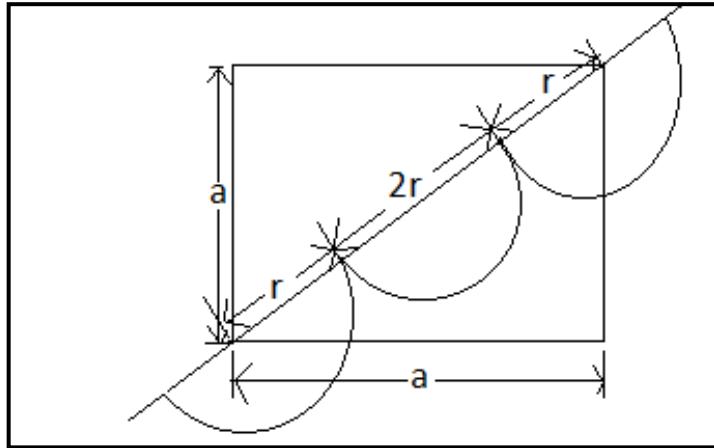


Fig: Front of FCC cell

From above fig, where r = radius of the atom

$$(4r)^2 = a^2 + a^2$$

$$4r = \sqrt{a^2 + a^2} = \sqrt{2} (a)$$

$$a = 4r/\sqrt{2}$$

$$\text{APF} = \frac{\text{No. of atom} \times \text{Vol. of each atom}}{\text{Vol. of unit cell}} = 4 \times \frac{4\pi r^3}{3} / 3a^3 = 4 \times \frac{4\pi r^3}{3} / 3 \times \{4r/\sqrt{2}\}^3$$

$$\text{APF} = 0.74$$

3] BODY CENTERED CUBIC:

There is only full atom at the centre & one atom at each of the eight corners.

Therefore,

$$\text{Effective no. of atoms inside each unit cell} = 1 + 8 \times 1/8 \\ = 2 \text{ atoms}$$

To find 'a' in terms of 'r'

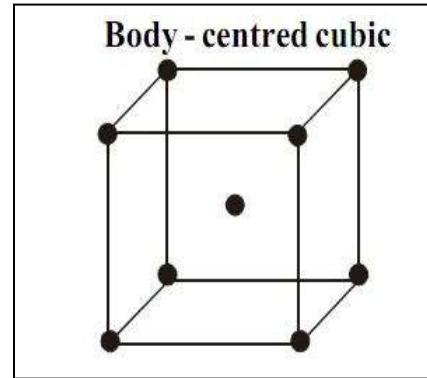
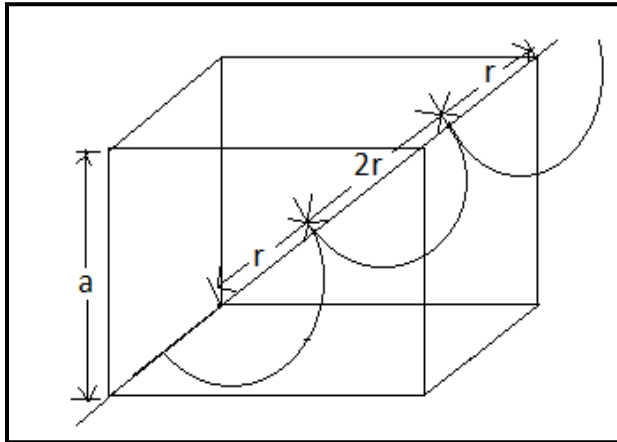


Fig: BCC Cell

$$\text{From above fig } xy = \sqrt{a^2 + a^2} = \sqrt{2} (a)$$

$$\text{Solid diagonal, } (xy)^2 = (4r)^2 = [\sqrt{2} (a)]^2$$

$$a = 4r/\sqrt{3}$$

$$\text{APF} = \frac{\text{No. of atom} \times \text{Vol. of each atom}}{\text{Vol. of unit cell}} = 2 \times 4\pi r^3 / 3a^3 = 0.68$$

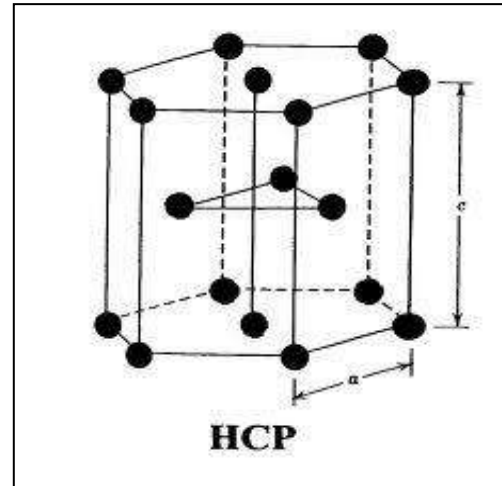
$$\text{APF} = 0.68$$

4. HEXAGONAL CLOSE PACKED (Given $c = 1.633a$)

A HCP unit cell has atoms at the centre as well as at the corner of the hexagonal top face & base.

Each of the 12 atoms at the corner of the top face & base are shared by 6 HCP unit cells whereas the atoms at the centre of the top face & base are each shared by only 2 unit cells.

There are also 3 full atoms within the volume of each unit cell.



Therefore, no. of atoms in one HCP unit cell = $12 \times 1/6 + 2 \times 1/2 + 3 = 6$ atoms.

Since, the corner atoms touch their neighbouring atoms at their periphery, we have $a = 2r$

The ratio of the height of the hexagonal prism to the side of the hexagonal faces is given as, $C = 1.633a$.

To find volume of hexagonal unit cell

Volume of HCP unit cell = area of the hexagonal face \times height of the hexagonal prism

From the above fig, area of the hexagonal face = area of each triangle $\times 6$

Area of each triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times a \times h = \frac{1}{2} \times a \times a\sqrt{3}/2$$

$$\text{Area of the hexagon} = a^2 \frac{\sqrt{3}}{4} \times 6$$

$$\text{Volume of HCP unit cell} = a^2 \frac{\sqrt{3}}{4} \times 6 \times C = a^2 \frac{\sqrt{3}}{4} \times 6 \times 1.633a$$

$$\text{APF} = \frac{\text{No. of atom} \times \text{Vol. of each atom}}{\text{Vol. of unit cell}} = \frac{6 \times 4\pi r^3/3a^3 \times \frac{\sqrt{3}}{4} \times 6 \times 1.633}{a^2 \frac{\sqrt{3}}{4} \times 6 \times 1.633a} = 0.74 \quad [a = 2r]$$

$$\text{APF} = 0.74$$

To show $c/a = 1.633$ for HCP unit cell

In a HCP unit cell if we consider 2 atoms at 2 of the base corners, one atom at the base centre & one atom above them. We may draw the following front view & top view of the arrangement of the 4 atoms.

- ✓ Fig a. shows that top & front view of the 4 atoms chosen & the arrangement between them.
- ✓ Fig b. shows that TV & FV of the tetrahedron formed when the centres of the 4 atoms chosen are considered. The side of the tetrahedron is 'a' (same as side of hexagonal unit cell) & the height is $c/2$ (half of the height of hexagonal unit cell).

- ✓ Fig c. shows that the right angled triangle formed by connecting base corner (p), apex & base of axis OO_1 .
- ✓ Fig d.shows that the right angled triangle formed on the base of tetrahedron joining base corner (p), base of axis O_1 & the centre of line pq.

$$\text{From fig c, } a^2 = b^2 + (c/2)^2 \dots\dots\dots (1)$$

$$\text{From fig d, } \cos 30^\circ = \frac{a/2}{b}$$

$$b = \frac{a}{2 \cos 30} \dots\dots\dots (2)$$

Substituting values of b in (1)

$$a^2 = (a/2 \cos 30)^2 + c^2/4$$

$$c^2/4 = a^2[1 - 1/4(\cos 30)^2]$$

$$c^2/a^2 = 4[1 - 1/4 \times (0.866)^2]$$

$$\mathbf{c/a = 1.633}$$

LIST OF FORMULAE

1] Volume of an atom of radius $r = \frac{4\pi r^3}{3}$

2] APF (SIMPLE CUBIC) = 0.52

3] [i] In FCC, $a = \frac{4r}{\sqrt{2}}$ where, r = radius of atom;
a = lattice constant [or lattice parameter or side of unit cell]

[ii] APF (FCC) = 0.74

4] [i] In BCC, $a = \frac{4r}{\sqrt{3}}$

[ii] APF (BCC) = 0.68

5] (i) Volume of HCP unit cell = $3\sqrt{3} \times a^2 c / 2$ where c = height of unit cell

(ii) If $C = 1.633a$, Volume of HCP cell = $4.24 a^3$

(iii) APF (HCP) = 0.74

6] Effective no. of atoms present in one

[i] Simple cubic unit cell = 1

[ii] Body centred cubic unit cell = 2

[iii] Face centred cubic unit cell = 4

[iv] Hexagonal closed packed unit cell = 6

7] [i] 1 Nanometer = 10^{-9} m

[ii] 1 Angstrom = (\AA) = 10^{-10} m

8] (i) Density of any material (ρ) = $nA / V_c N_a$

Where, n = No. of atoms in unit cell

A = Atomic weight of the metal (g/mol)

V_c = Volume of the unit cell

N_a = Avogadro's number = 6.023×10^{23} atoms/mol

CRYSTAL IMPERFECTIONS

All the different types of crystal structures with their arrangement of atoms are true only under ideal conditions. This means that all real crystals generally have imperfections among their unit cells. These are important because they affect many structure-sensitive properties of the material. Ex: tensile strength, shear strengths of materials.

Crystal imperfections can be broadly classified as

1] Point imperfections, 2] Line imperfections, 3] Surface imperfections, 4] Volume imperfections

1] Point imperfections: There are 4 types:

1] Vacancies, 2] Interstitialcies, 3] Substitutional impurities, 4] Electronic defects

1] Vacancy: A vacancy refers to an atomic site from where the atom is missing. This may be due to imperfect packing during original crystallization or from thermal vibration of the atom.

SCHOTTKY DEFECT: If two ions of opposite charges are missing but are found elsewhere in the same crystal, called as 'SCHOTTKY DEFECT'.

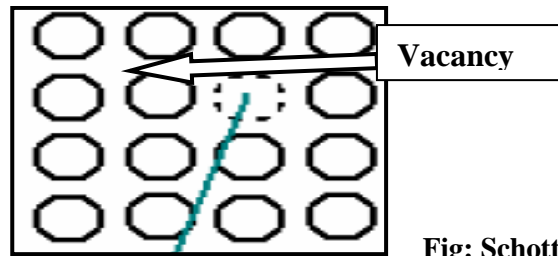


Fig: Schottky defect

2] Interstitialcy: Here, a small sized foreign atom occupies the space in between the atoms of a crystal without dislodging any of the parent atoms. This interstitial atom is usually of much smaller size than the atoms among which it is present. Ex: Carbon in iron.

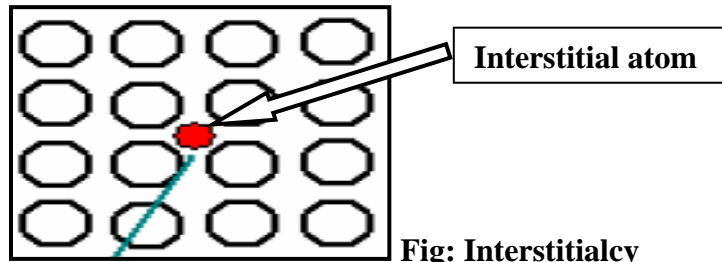


Fig: Interstitialcy

3] Frenkel defect: When an ion of the same crystal tries to occupy an interstitial position jumping from another site, then it is called 'Frenkel defect'.

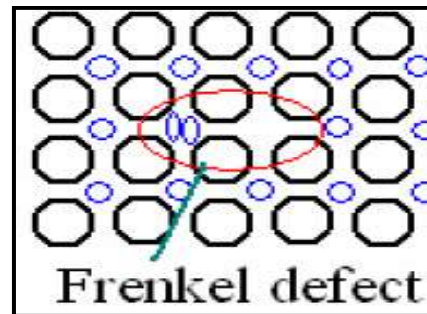


Fig: Frenkel defect

4] Substitutional impurity: This impurity is created when a foreign atom substitutes a parent atom in the lattice structure. Ex: a Zinc atom replaces a Copper atom in the FCC structure of Copper.

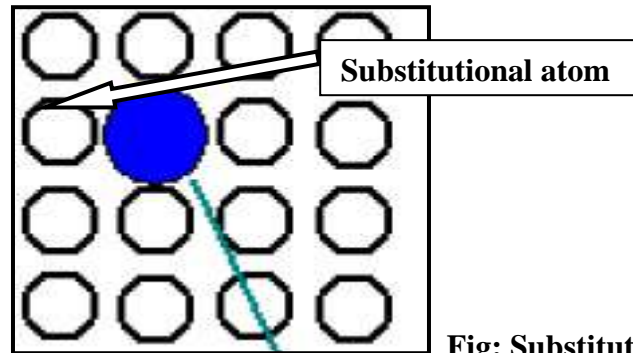


Fig: Substitutional impurity

5] Electronic defects: Electronic defects are the results of errors in the charge distribution in solid. For ex: An impurity atom whether interstitial or Substitutional may have a charge quite different from that of the host atoms & hence may produce local electronic disturbances. These are called electronic imperfections & are necessary to explain electrical conductivity & related phenomenon in solids.

(ii) Line imperfections: Line imperfections are called dislocations. A dislocation is a line defect where a uniform alignment of atoms is broken to form a discontinuity or a localised distortion in the crystal. The dislocations are responsible for the phenomenon of slip, by which most metals plastically deform.

There are two basic types of dislocations:

(1) Edge dislocation, (2) Screw dislocation or cross slip

1. Edge dislocation:

- Consider a perfect crystal fig :(a) to be made up of a number of vertical planes of atoms. If one of these vertical does not extend from to bottom.
- But ends at only a part of the way within the crystal as shown in fig: (b), an edge dislocation is present.

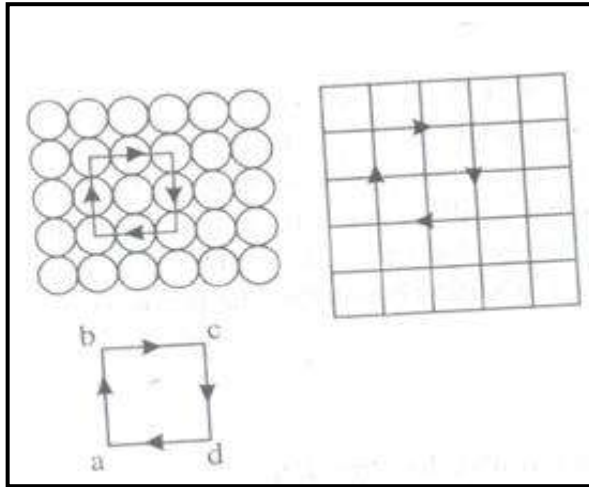


Fig (a): Ideal crystal

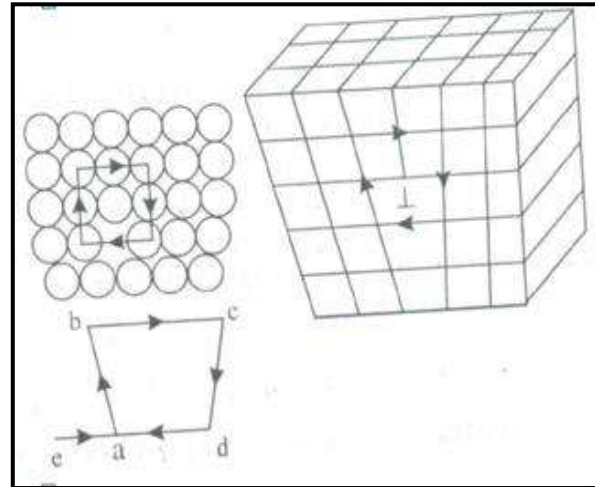


Fig (b): Crystal with edge dislocation

- In the perfect crystal, the atoms are in equilibrium positions & all the bond lengths are of equilibrium value.
- Where as in an imperfect crystal the atoms do not occupy equilibrium positions & the bond lengths are either compressed or pulled apart. Dislocations are denoted by the symbol \perp .

Burger's Vector

The magnitude & direction of the displacement of atoms in a dislocation is defined by a vector called the *Burger's Vector*.

From fig (b): Burger's Vector = $\overrightarrow{EA} = b$

The Burger's Vector is always *perpendicular* to the edge dislocation line.

2. Screw dislocation or cross slip:

Consider the shaded area AFED on the plane ABCD. Let top part of the crystal over the shaded area be placed by one inter atomic distance to the left with respect to the bottom part as shown in fig:(b), just like in a *Rubik's cube*.

Now there is said to be a screw dislocation about the line EF which is known as the screw dislocation line. The atomic bonds in this region immediately surrounding the dislocation line undergo a shear distortion. Also the forces required to form a screw dislocation is greater than that required for an edge dislocation.

From fig :(b), the Burger's Vector, $\vec{b} = xy$

The Burger's Vector is always *parallel* to the screw dislocation line.

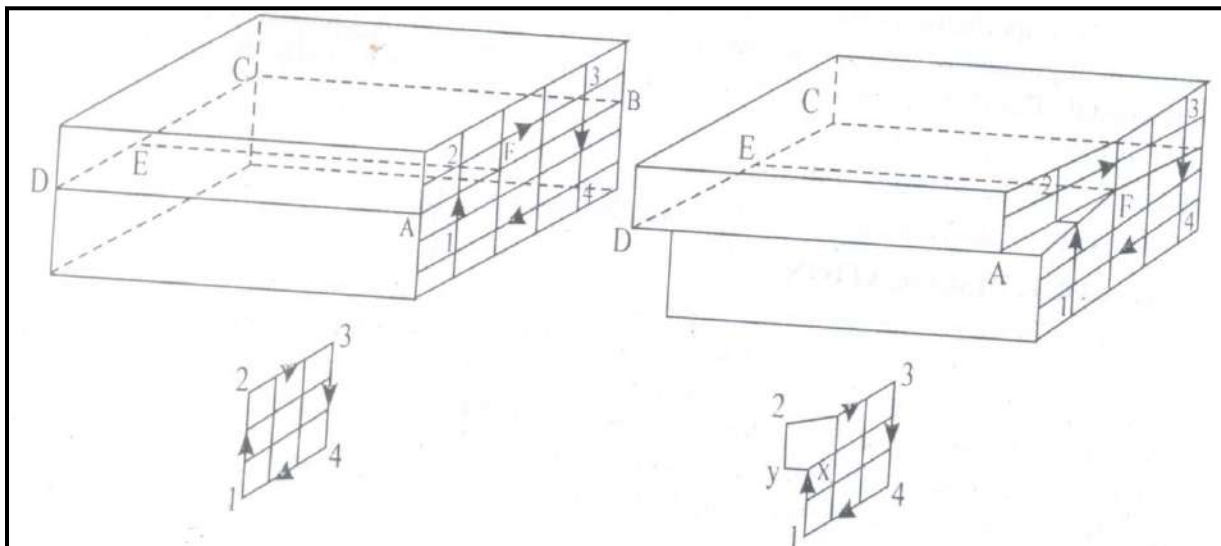
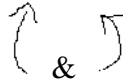


Fig: (a) Ideal Crystal

Fig: (b) Crystal with screw dislocation

Edge dislocation	Screw dislocation
1] An edge dislocation is a line defect where there is a discontinuity in a line of atoms. The discontinuous line of atoms can also be considered as an extra plane of atoms.	1] A Screw dislocation is also a line defect formed when a part of the crystal displaces angularly over the remaining part. The plane of atoms converts into a helical surface, or a screw.
2] Edge dislocation are symbolically represented as \perp & \top , which are positive & negative dislocations respectively	2] Screw dislocation are shown as  & are reference to as 'clockwise' & 'anticlockwise' or positive & negative Screw dislocations respectively.
3] Burger's Vector 'b' is always perpendicular to edge dislocation line.	3] Burger's Vector 'b' is always parallel to screw dislocation respectively
4] Atomic bonds around a dislocation line undergo tensile & compressive stresses.	4] Burger's Vector 'b' is always parallel to screw dislocation line
5] Force required to form edge dislocation is less.	5] Atomic bonds around a dislocation line undergo shear distortion.
6] Edge dislocation travel faster (≈ 50 times) under load.	6] Force required to form screw dislocation is more. Screw dislocations travel slowly under load.

(iii) Surface imperfections:

There are 4 types:

1. Grain boundaries, 2. Tilt boundaries, 3. Twin boundaries, 4. Stacking faults

1. Grain boundaries:

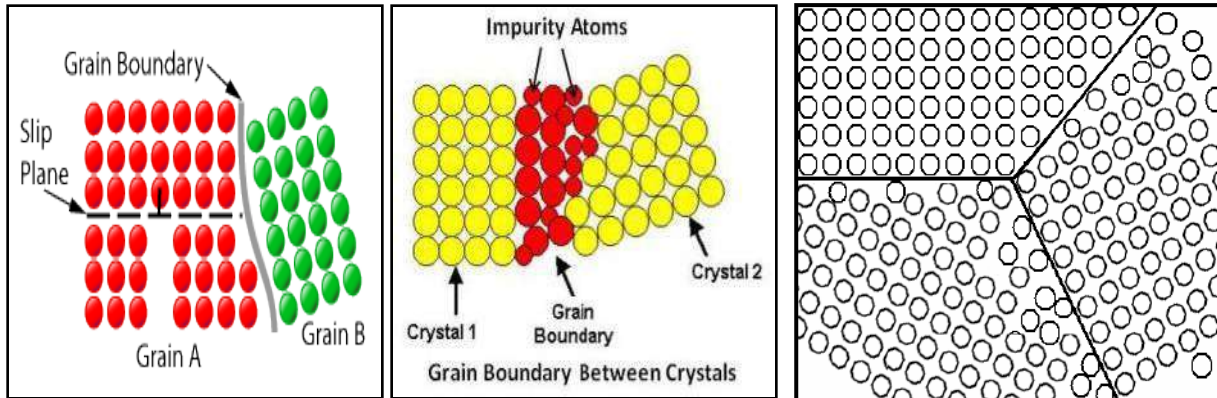
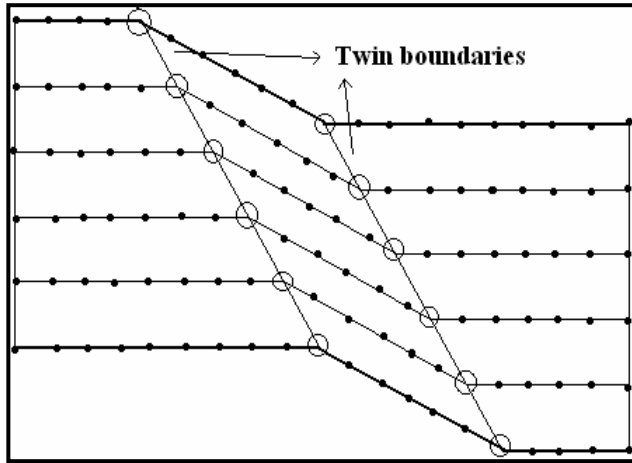
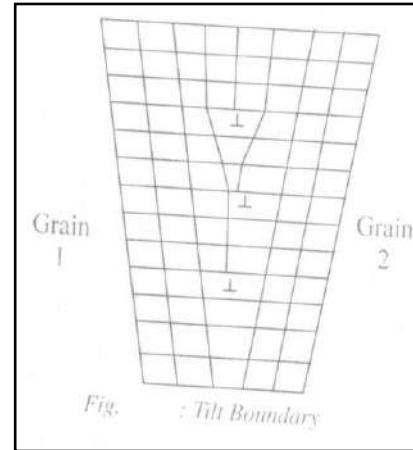


Figure-3.6: Schematic presentation of grain boundaries

- ✓ Grain boundaries are those regions which separate crystals of different conditions. A Grain boundary is formed when two adjoining growing crystals (grains) meet at their surfaces.
 - ✓ The thickness of these regions is only a few atomic diameters.
 - ✓ The atoms in these regions are highly distorted & are caught between the two crystals & pulled apart by each to its own configuration.
- ### 2. Tilt / Twin boundaries:
- ✓ It is a type of low angle grain boundary where the orientation difference between two neighbouring crystals is less than 10° .
 - ✓ The distortion in the boundary is less, & is limited to a few edge dislocations, located one below the other.



Twin boundary



Tilt boundary

3) Twin boundaries:

- ✓ In this type the atomic arrangement on one side of the twin boundary is a mirror reflection of the arrangement on the other side.
- ✓ Twin boundaries occur in pairs so that change in orientation of two grains introduced by one boundary is restored by the other grain boundary.
- ✓ Twins are generally formed during annealing or mechanical working of metals.

4) Stacking faults: They are surface imperfections created by an error in the stacking sequence of atomic planes in the crystals. Consider the stacking arrangement in an FCC crystal.

.....ABC ABC (A) BC ABC.....

If suppose the plane within the bracket is missing, then the stacking becomes

.....ABC ABC (BC) ABC.....

The two planes BC found in the middle of FCC stacking in then termed as a stacking fault. In other words stacking faults can be called as discrepancies in the packing sequence of the layers of the crystal structure.

5) Volume defects: Volume imperfections are those defects like blow holes, cracks, foreign inclusions etc. Which are 3 dimensional & are much larger than other types of imperfections. They are normally introduced into solids during processing & fabrication techniques & have a considerable effect on the properties of materials.

DIFFUSION

The migration of atoms from their original lattice sites in a crystal structure to other sites is known as Diffusion.

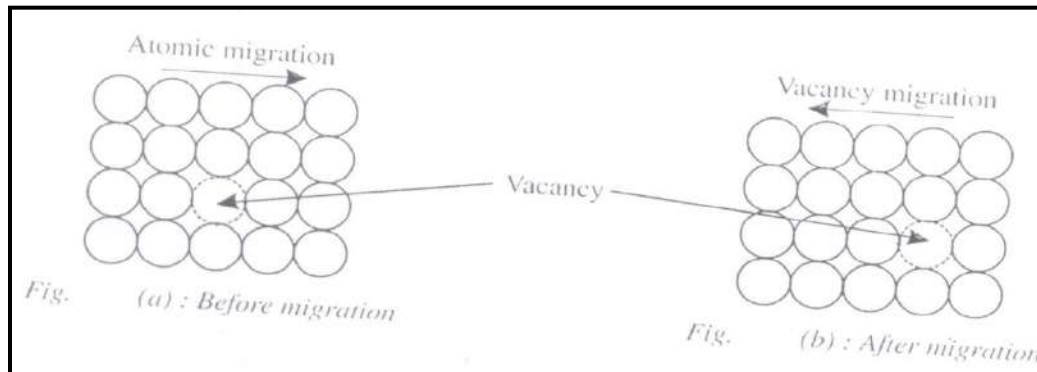
Diffusion involves the movement of atoms, ions or molecules from one position to another position & occurs mainly due to another position & occurs mainly due to thermal agitation or the presence of concentration gradients. Diffusion in gaseous state is more than the liquid & solids.

DIFFUSION MECHANISMS

There 3 types of diffusion mechanisms can be explained in solids:

- I] Atomic diffusion by vacancy migration
- II] Atomic diffusion by Interstitialcy migration
- III] Diffusion by interchange of atoms

I] Atomic diffusion by vacancy migration:



When an atom moves into a vacancy, a new vacancy or a hole is created. In turn, this vacancy may receive an atom from any of the neighbouring sites. As a result the vacancy takes a *random walk* through the crystal.

II] Atomic diffusion by Interstitial migration:

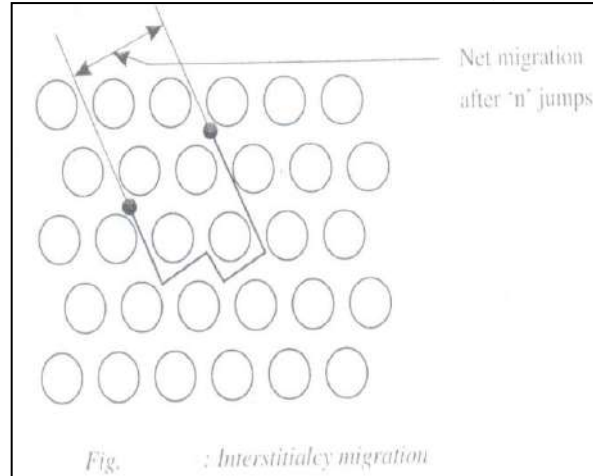


Fig: Atomic diffusion by Interstitial migration

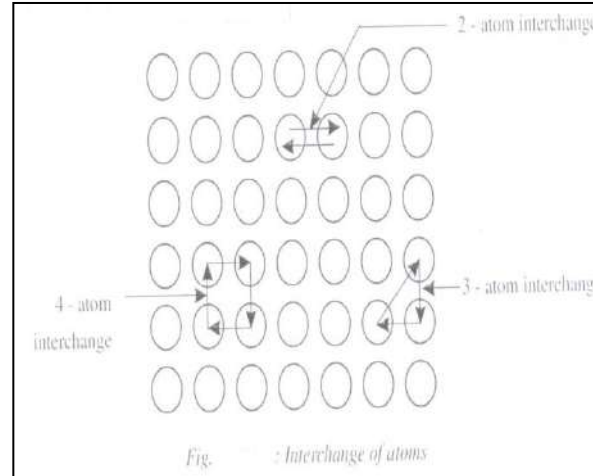


Fig: Diffusion by interchange of atoms

III] Diffusion by interchange of atoms:

In this type of diffusion, atoms exchange their positions & hence movement of atoms take place.

FICKS I LAW OF DIFFUSIION:

It states that, the flux of atoms J moving across a unit surface area in unit time is proportional to the concentration gradient $\frac{dc}{dx}$.

Under steady state flow, $\mathbf{J} = -D \frac{dc}{dx}$

Where, J = No of atoms/unit area of diffusion per unit time (atoms/m².sec)

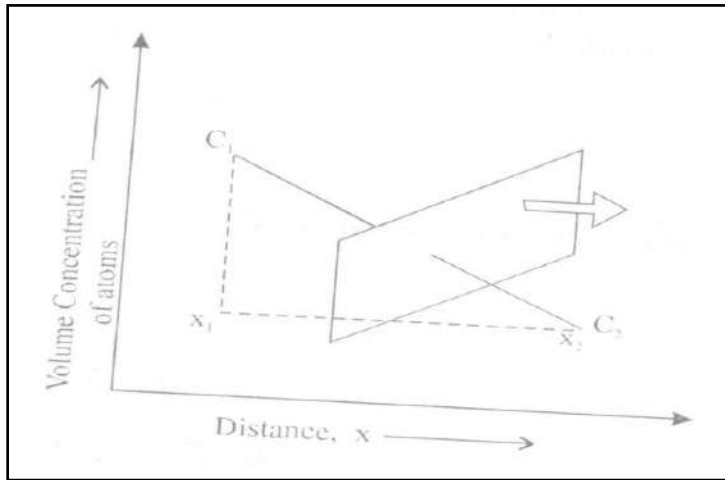
C = Volume concentration of atoms (atom/m³)

X = Distance between the planes in the direction of flow of atoms (m)

D = diffusion co-efficient or diffusivity (m²/sec)

-^{ve} sign indicates that flux moves from higher to lower concentration.

The geometry of Fick's Ist law is illustrated below.



J = flux of atoms across plane
with unit area

$$J = -D \frac{dc}{dx} = -D [C_2 - C_1 / X_2 - X_1]$$

$\frac{dc}{dx}$ = Concentration gradient

FICKS II LAW OF DIFFUSION

This law relates to the rate of change in concentration with time. This non-steady state condition is represented by a second order differential equation.

It states that, $dc/dt = D [d^2c/dx^2]$

Where $\frac{dc}{dx}$ = Concentration gradient

D = diffusion coefficient

$\frac{dc}{dt}$ = the rate of accumulation of diffusing atoms at a point where the concentration gradient is

dc/dx .

i.e., rate of compositional change

Fick's IInd law is also stated as, “the rate of compositional change is equal to the diffusivity times the rate of change of the concentration gradient”.

In general, steady state diffusion in which conditions do not change with time is not commonly encountered in engineering materials. In most cases non-steady state diffusion in which the concentration of solute atoms at any point in the material changes with time takes place. Ex: If carbon is being diffused into the surface of a steel camshaft to harden its surface, the concentration of the carbon under the surface at any point will change with time as the diffusion process progresses. It is for these cases of non-steady state diffusion in where diffusivity is independent of time. Fick's IInd law of diffusion is applied.

Diffusivity

Diffusivity or the coefficient of diffusion (D) is defined as *the amount of substance diffusing in unit time across unit area through a unit concentration gradient* & its unit is m²/sec.

The coefficient of diffusion of various materials vary with crystal structure & temperature.

ACTIVATION ENERGY

- In any diffusion mechanism, work must be done if the atoms are to move from one location to another.
- To do this work, an energy barrier must be overcome for the atoms to begin a jump from one site to another site.
 - ✓ *The minimum energy required by the atoms to overcome this energy barrier is called the Activation energy of diffusion.*
- In a vacancy mechanism, energy is required to pull the atoms from their positions to vacant atomic sites

- In the Interstitialcy mechanism energy is required to force the atoms between other atoms to different interstitial positions.
Activation energy depends on a number of factors. They are:
- A small atom has lower activation energy than a large atom or molecule.
- Interstitial movements require more energy than vacancy movements.
- High activation energies are required for atomic diffusion for those materials which are strongly bonded & have higher melting points.
Ex: Tungsten, boron-carbide, silicon carbide.

Arrhenius equation

An Arrhenius type equation for atomic diffusion is given by,

$$D = D_0 e^{-Q/RT}$$

Where D = Diffusion coefficient (m²/sec)

D₀ = Constant (m²/sec)

Q = Activation energy for diffusion (J/mol)

R = Universal gas constant (8.314 J/mol-K)

T = Absolute temperature (K)

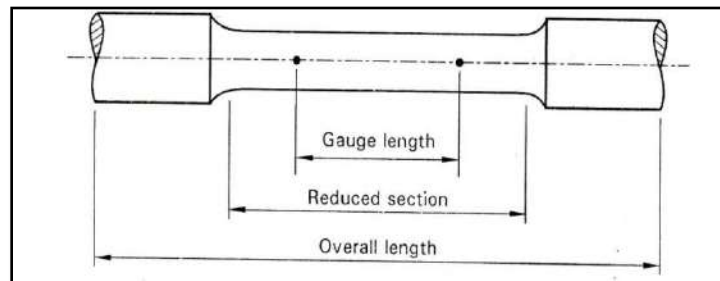
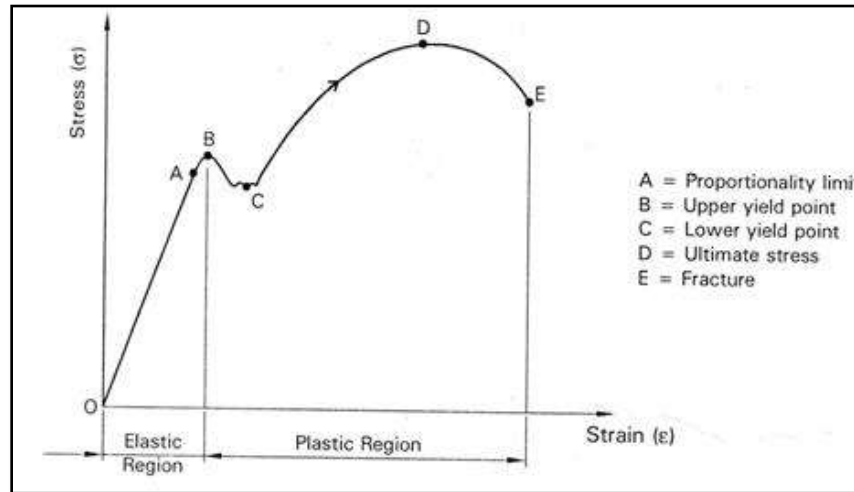
The values of D₀ & Q are determined from a plot of ln D along y-axis & 1/T along x-axis. This plot yields a straight line, the slope of which is -Q/R. The intercept on the y-axis is ln D₀

FACTORS AFFECTING ATOMIC DIFFUSION

- 1] TEMPERATURE: High temperature provides the necessary activation energy to the atoms to begin diffusion. So a higher temperature initiates diffusion faster.
- 2] CRYSTAL STRUCTURE: If a crystal structure is distorted, i.e., if there are more imperfections the rate of diffusion is increased.
- 3] ATOMIC PACKING FACTOR: If APF is high; the rate of diffusion will be decreased. Diffusion is much slower in FCC-iron than in BCC-iron.
- 4] GRAIN BONDARIES: The diffusion process proceeds more rapidly along the grain boundaries since it is a zone of crystal imperfections.
- 5] GRAIN SIZE: Since diffusion through grain boundaries is faster than through the grains themselves, a material with finer grains will have a faster rate of diffusion.
- 6] ATOMIC SIZE: Diffusion occurs more readily when the size of the diffusing atom is less. Ex : Carbon in iron.
- 7] CONCENTARTION GRADIENT: Higher the concentration gradient higher will be the rate of diffusion.

MECHANICAL BEHAVIOUR

Stress-strain curve for a ductile material:



Standard tensile specimen

a) Point A — Limit of Proportionality

The graph OA on the stress-strain curve is a long straight line, which indicates that stress is proportional to strain during the initial period of the test. In other words, the material obeys Hook's law till point A is reached. Beyond point A, the graph deviates from the straight line indicating that stress is no longer proportional to strain.

Elastic Limit

The graph OB on the stress-strain curve represent that the material behaves in an elastic manner, i.e., on removal of stress, the deformation vanishes, and the material springs back completely to its original shape. This happens at a point slightly beyond the limit of proportionality, and is called the *elastic limit*. Beyond the elastic limit, any additional loading will result in **small permanent deformation in the material**, and hence the material will not return to its original shape when the load is removed.

In other words, beyond the elastic limit, the material is said to deform plastically to any further increase in load. Hence, elastic limit is defined as the limiting value of stress up to which, if the material is stressed and then released (unloaded), strain disappears completely in the material, and the original shape is regained.

Point B & C— Upper yield point and lower yield point

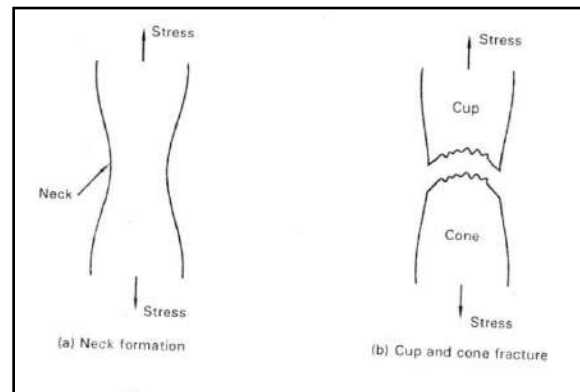
At point B, there is a small dip in the stress-strain curve. This point is called the upper yield point, and is defined as the stress at which the load starts reducing and the extension increases. This phenomenon is called yielding of material, and continues till point C is reached. The point C is called the lower yield point, where the stress remains same, but strain increases for some time. In the region of upper and lower yield points, the material becomes perfectly plastic, which indicates that it can deform without an increase in applied load.

Point D — Ultimate stress

After yielding, any further increase in stress will cause considerable increase in strain, and the curve rises to the maximum point D as shown in the stress-strain graph. The stress corresponding to this point is called ultimate stress, and is defined as the maximum stress that any material can resist without fracture.

Point E — Fracture (Failure)

When the load is continued beyond the ultimate stress (point D), the cross-sectional area decreases rapidly in the localized region of the test specimen. This process is called *necking* or *neck formation*. The load carrying capacity of this region also decreases rapidly. The specimen finally reaches a point (point E) where fracture takes place. The fracture is in the form of a *cup and cone* type as shown in figure . The cup and cone appearance of the fracture surface is caused by the ductility of the material. The point E on the stress-strain graph is called fracture point, and the corresponding stress, fracture stress or failure stress or breaking stress.



To determine tensile properties:

The various properties that can be interpreted from the tensile test experiment can be classified as:

a) Properties in the Elastic region

- Linear elastic properties
- Non-linear elastic properties

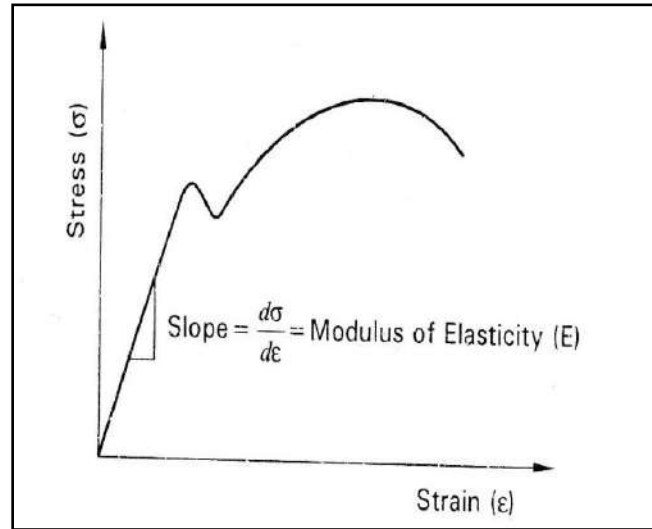
b) Properties in the Plastic region

→ **Properties in the Elastic Region:**
Linear Elastic Properties:

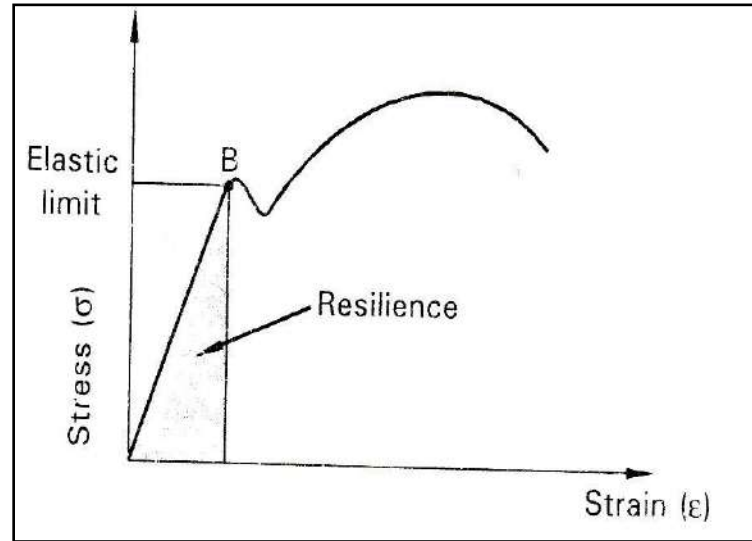
The properties that are classified under linear elastic category obey Hook's law, and include elastic strength, stiffness, and resilience

a) Elastic Strength: Elastic strength corresponds to the highest stress at which the behavior of the material remains elastic, i.e., the material springs back completely to its original shape upon removal of stress (load). Elastic strength cannot be measured accurately. It is generally measured by the stress at the end of proportionality limit. For most structural materials, the elastic limit and the proportionality limit almost coincide, and hence both have nearly the same numerical value.

b) Stiffness: Stiffness is defined as the ability of a material to resist elastic deformation. In other words, materials having high stiffness show less elastic deformation under load. Young's modulus or modulus of elasticity, obtained from the slope of the stress-strain curve is a measure of stiffness of a material. The greater the modulus of elasticity (E), the more stiffer is the material, and greater the stress required to produce a given strain.



c) Resilience : Resilience (strain energy) is the ability of a material to absorb energy when it is loaded within the elastic limit, and give back the same energy when the load is removed. Resilience is measured by the triangular area under the elastic region of the stress-strain curve shown in fig.



A material with greater resilience is capable of absorbing more energy without any plastic deformation. Resilience should be considered when the material is subjected to shock or impact loading. This property is important in the manufacture of springs, shock absorbers, and similar such parts.

Modulus of Resilience

The strain energy stored by the material per unit volume within the elastic limit is known as modulus of resilience. It is given by the equation :

$$\text{Modulus of resilience } (U_r) = \frac{\sigma_s^2}{2E}$$

where σ_s = stress at elastic limit

E = Young's modulus

Non — linear Elastic Properties:

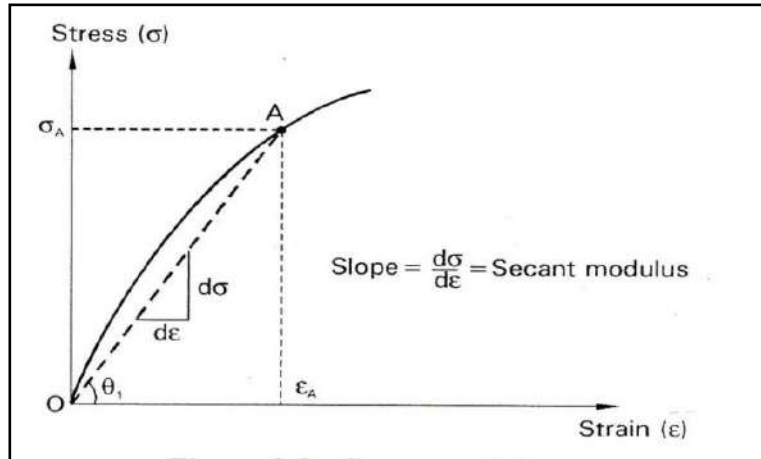
Materials such as cement, concrete, plastic, wood, rubber, soft copper, gray cast iron etc., do not obey Hook's law in the elastic range. Therefore, the stiffness (Young's modulus) of these materials is not constant; but varying with the magnitude of the applied stress. The stiffness in these materials is expressed by Secant modulus and Tangent modulus.

- a) **Secant modulus:** Secant modulus is the measure of average stiffness at any given stress value. Consider a non-linear elastic stress-strain curve as shown in figure.

Let A be the point on the curve, and σ_A & ϵ_A be the corresponding stress and strain. Draw a line from the origin O to intersect the point A on the curve. The line OA represents the average slope at the given stress σ_A .

The secant modulus is then obtained as : $E_{\text{sec}} = \left[\frac{\sigma}{\epsilon} \right]_{\sigma=\sigma_A} = \tan \theta$

Secant modulus is considered when the non-linearity of the curve is *not much*.

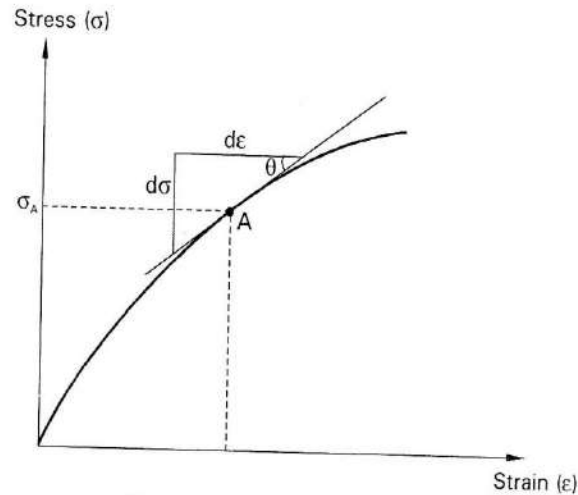


Tangent modulus:

When the non linearity of the curve is too much secant modulus cannot be used. Instead tangent modulus is suitably used.

Consider a non-linear elastic stress-strain curve as shown in figure 2.9. Let A be the point on the curve, and σ_A & ϵ_A be the corresponding stress and strain. Draw a line tangent to the non-linear curve and passing through the point A . The instantaneous stiffness may be found from the slope of the tangent drawn on the stress-strain curve at the given stress value (σ_A).

$$\text{Tangent Modulus} = E_{\text{tan}} = \left[\frac{\sigma}{\epsilon} \right]_{\sigma=\sigma_A} = \tan \theta$$



Properties in the Plastic Region

The properties which are classified under this category do not obey Hooks law, and include Yield strength, Offset yield strength, Ductility, Toughness, Ultimate tensile strength, and Fracture strength

- a) **Yield strength:** Yield strength is defined as the stress required to produce a small, specified amount of plastic deformation. It is also defined as the average stress at the lower yield point. It is determined using the relation:

$$\text{Yield stress } (\sigma_y) = \frac{F_y}{A_o}$$

where F_y = load at yield point

A_o = original cross-sectional area of the specimen.

Prior to the yield strength or yield point, the material will deform elastically and will return to its original shape when the applied load is removed. Once the yield point is passed, some fraction of the deformation will be permanent and non-recoverable. Knowledge of yield point or yield strength is vital when designing a component, since it generally represents an upper limit to the load that can be applied to avoid permanent deformation.

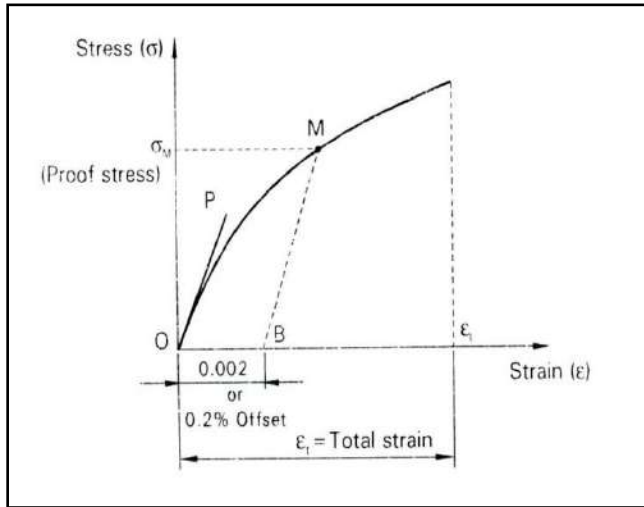
b) Offset yield strength:

Most non-ferrous materials, high-strength steels, and softer materials do not possess a well-defined yield point. For such materials, the yield strength is usually determined by the offset method, and hence the strength thus obtained is referred as offset yield strength or proof stress. The offset yield strength is defined as the stress corresponding to the intersection of the stress-strain curve, and a line drawn parallel to the linear (elastic) part of the curve offset by a specified strain.

The different percentage strain offset for materials are given below:

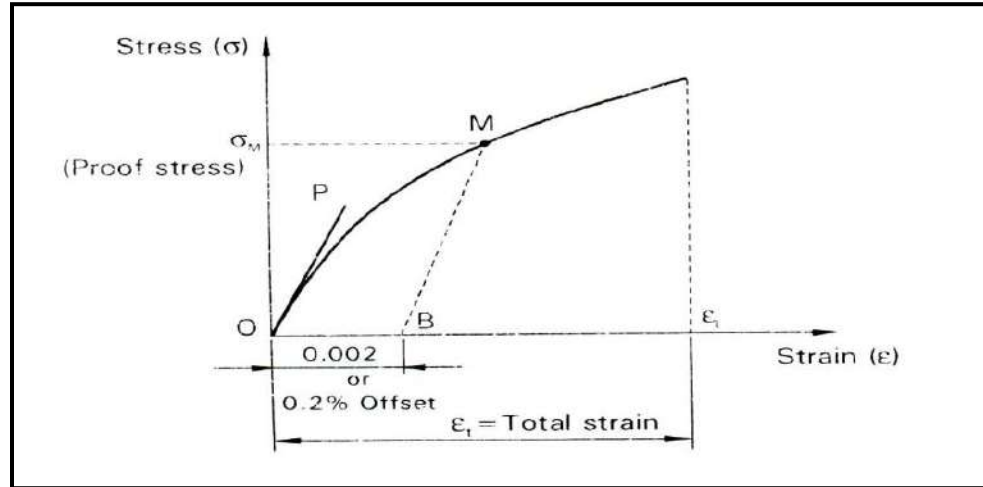
- 1) 0.2 % offset for harder materials
- 2) 0.1 % offset for less harder materials
- 3) 0.05 % offset for softer materials.

A plastic strain of 0.2 % offset is recommended by the Bureau of Indian Standards (BIS).



To determine offset yield strength

Consider the stress-strain curve for a certain material as shown in figure. A line OP tangent to the curve is drawn from the origin O. On the strain axis (x-axis), a point B is located at a distance of strain value equal to 0.2 % of total strain (ϵ_t). A line is drawn from the offset strain, i.e., from point B, parallel to line OP to intersect the curve at point M. The part OM of the curve represents the elastic range. The stress corresponding to the point M gives the yield strength, and is known as offset yield strength, or proof stress.



c) Ductility

Ductility refers to the ability of a material to undergo plastic deformation under tensile load. In other words, it indicates the extent to which a material can be deformed without fracture. Ductility is the property of a material by virtue of which it can be drawn into wires, or elongated before rupture takes place. The amount of ductility is an important factor when considering forming operations such as rolling and extrusion. Ductility is measured by the percentage elongation or percentage reduction in area before rupture of the specimen takes place.

$$\% \text{ Elongation} = \left[\frac{\text{Increase in length}}{\text{Original length}} \right] \times 100 = \left[\frac{L_f - L_o}{L_o} \right] \times 100$$

where L_f = final length of the specimen after the tensile test
 L_o = Original length of the specimen

$$\text{or } \% \text{ reduction in cross-sectional area} = \left[\frac{A_o - A_f}{A_o} \right] \times 100$$

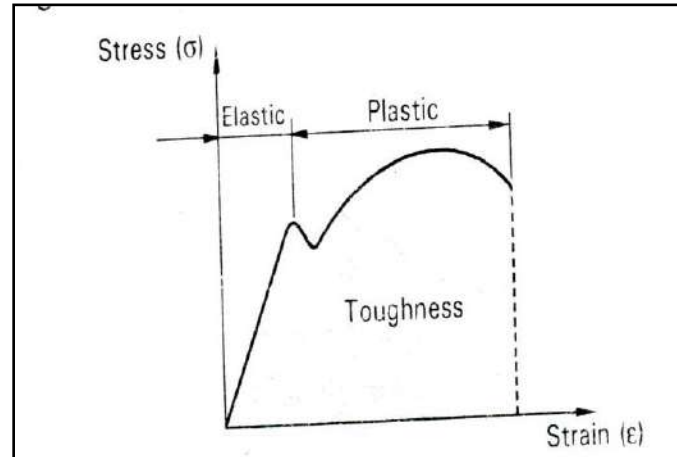
where A_o = Original cross-sectional area of the specimen = $\frac{\pi}{4} d_o^2$

A_f = Final cross-sectional area, i.e., after the tensile test = $\frac{\pi}{4} d_f^2$

d_o and d_f are the original and final diameter of the specimen respectively

d) Toughness

Toughness is defined as the ability of a material to withstand both elastic and plastic deformation. It is a measure of the energy, a material can absorb before it fractures. This is given by the area under the stress-strain curve generated to failure for a material. A tough material must therefore have good strength as well as ductility.



e) Ultimate tensile strength

Ultimate tensile strength, or simply referred as tensile strength, is defined as the maximum stress sustained by the material before fracture. It is determined by the relation:

$$\text{Ultimate stress } (\sigma_u) = \frac{\text{Ultimate load } (F_u)}{\text{Original cross-sectional area } (A_o)}$$

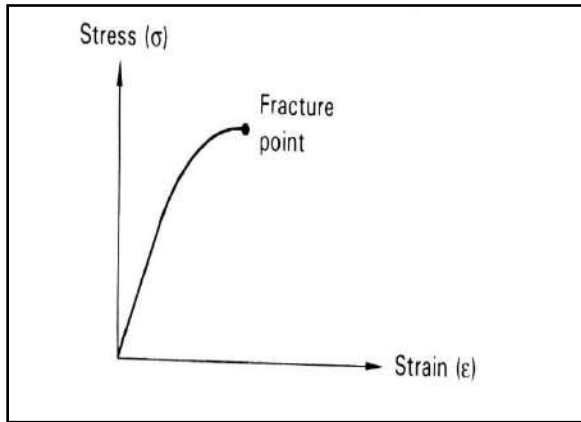
f) Fracture strength

Fracture strength is defined as the stress (load) at which the actual fracture takes place in a material. It is determined using the equation:

$$\text{Fracture stress } (\sigma_f) = \frac{\text{load at fracture point } (F_f)}{\text{Original cross-sectional area } (A_o)}$$

STRESS — STRAIN CURVE FOR BRITTLE MATERIAL

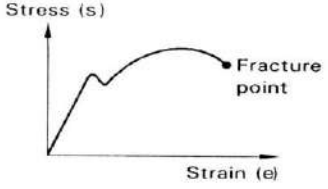
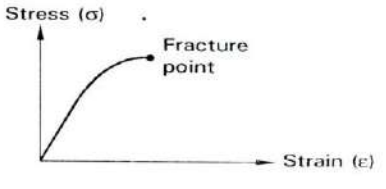
Figure shows the stress-strain relation for a brittle material like cast iron. In brittle materials, there is no appreciable change in rate of strain. There is no yield point, and no necking takes place. Ultimate point and breaking point are the same. Also, the strain at failure is very small.



BRITTLE & DUCTILE BEHAVIOUR

The behavior of engineering materials can be broadly classified into two categories: ductile and brittle. Steel, aluminum, gold, etc., are classified under ductile materials, while cast iron, glass, ceramics, etc., as brittle materials. The differences between the two types of materials with respect to the stress-strain curves are briefly given in table.

3.	Yield point exists in a ductile material.	There is no yield point.
4.	Ductile materials often have relatively low Young's modulus and ultimate stress.	Comparatively large.
5.	Ductile materials exhibit large strains and yielding before they fail.	Brittle materials fail suddenly, and without much warning.

Sl. No.	Ductile material (mild steel)	Brittle material (cast iron)
1.	<p>Stress-strain curve for a ductile material is shown in figure below.</p> 	<p>Stress-strain curve for a brittle material is shown in figure below.</p> 
2.	<p>Ductile materials can withstand large strains before the specimen ruptures. In other words, more strain energy is needed to cause fracture of the material</p>	<p>Brittle materials fracture at much lower strains.</p>

TRUE STRESS & STRAIN

- In the conventional or engineering stress-strain curve the various properties like stiffness, ductility, toughness etc., obtained were based on stress and strain, which make use of the original cross-sectional area and length in their calculations.
- This criteria is true within the elastic limit, since the elastic reduction in cross-sectional area and length is negligible (very small).
- But beyond the elastic limit, the properties obtained from the stress-strain curve must be interpreted with caution, since the dimensions of the specimen (area and length) experience substantial change from their original values.
- Thus, the engineering stress-strain curve does not give a true indication of the deformation characteristics of a material. To overcome this limitation, it is necessary to consider the **instantaneous dimensions of the specimen at any instant of load to calculate stress and strain**. The values thus obtained are called true stress and true strain.

True Stress

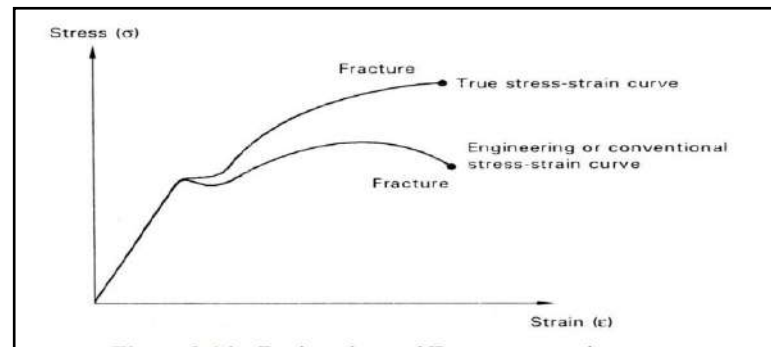
True stress is defined as the load acting on the specimen at any instant, divided by the cross-sectional area of the specimen *at that instant*.

$$\text{True stress } (\sigma') = \frac{\text{Force (F)}}{\text{Instantaneous cross-sectional area (A')}}$$

$$\text{where } A' = \frac{\pi}{4} (d')^2$$

d' = diameter of the specimen at any instant of loading.

Figure shows the comparison of the true stress-strain curve with the corresponding engineering stress-strain curve. It is clear from the two curves, that the true stress and strain are practically indistinguishable from the engineering stress and strain at small deformations. However, as the strain becomes large, and the cross-sectional area of the specimen decreases, the true stress is much larger than the engineering stress.



Conventional and true stress-strain diagrams have their own merits and demerits, and find applications in engineering practice. However, the decision as to which diagram to be used in any given analysis depends on several factors. For instance, conventional stress-strain diagrams are used in the elastic range, while true stress-strain diagrams are used in the plastic range.

Conventional stress-strain diagrams are largely used in the specification of mechanical properties for design purpose (except for certain specific properties), while engineers dealing with materials processing require data related to the true stress-strain curves.

Plastic deformation

When a material is stressed within (below) the elastic limit, the resulting strain or deformation is temporary, i.e., on removal of stress (load), the deformation vanishes and the material returns back completely to its original shape. But,

beyond the elastic limit, as the stress increases, strain also increases considerably resulting in a small permanent deformation in the material. In other words, the material will not return to its original shape after the stress is removed. This permanent deformation is called inelastic deformation or **plastic deformation**.

The mechanism by which plastic deformation occurs in polycrystalline materials is somewhat more complex due to the random crystallographic orientation of the numerous grains. However, analysis can be made easier by initially studying the behavior of a single crystal (grain) under stress, and later applying this knowledge to polycrystalline materials.

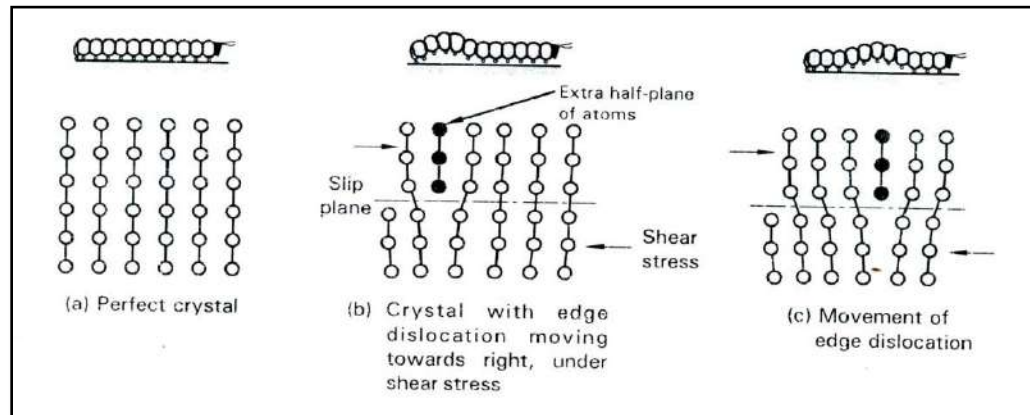
MODES OF PLASTIC DEFORMATION

Plastic deformation in a single crystal occurs by:

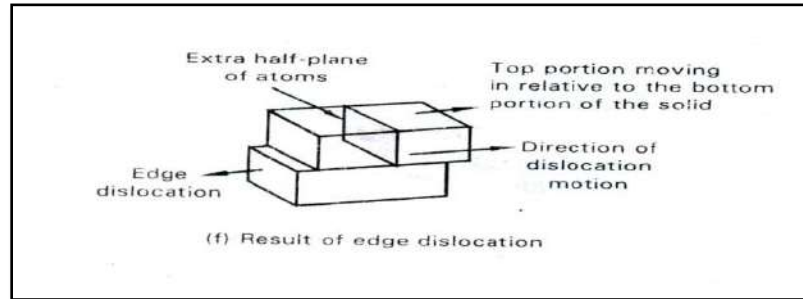
- Slip
- Twinning, or
- Combination of slip and twinning

1. Plastic Deformation by Slip:

The process by which plastic deformation is produced by the motion of a large number of dislocations is termed as slip.

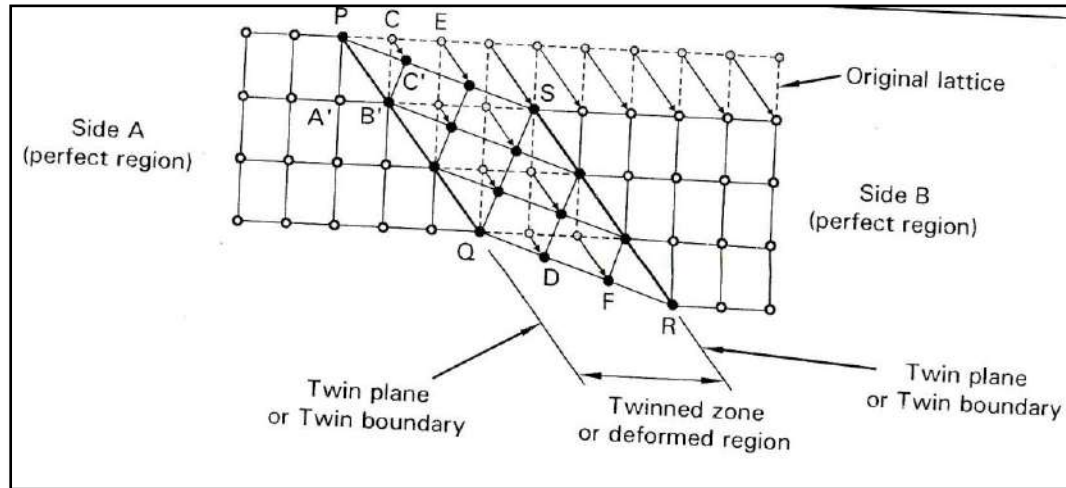


- ▶ Figure shows **an edge dislocation**, which helps in explaining slip under the action of shear stress. The figure illustrates how an edge dislocation gets glided along the slip plane under the action of shear stress, and how the slip is caused by the movement of edge dislocation.
- ▶ The dislocation motion is analogous to the movement of a caterpillar, which is also shown in figure above the crystal lattice for clear understanding. The caterpillar would have to exert a large force to move its entire body at once. Instead, it moves the rear portion of its body forward by a small amount and creates a hump. The hump then moves forward, and eventually moves all of the body forward by a small amount.
- ▶ Under the influence of shear stress, the dislocation in a crystal structure moves similarly by a small amount as that of a caterpillar. It can be seen in figure that, the dislocation in the top half of the crystal is slipping one plane at a time as it moves to the right from its initial position as is shown in figure (a), figure (b), and finally (c). In the process of slipping, the dislocation propagates across the crystal.
- ▶ The movement of the dislocation across the plane eventually causes the top half of the crystal to move with respect to the bottom half. Due to the actual movement of the atomic blocks, no return to the original lattice shape is possible, even after withdrawal of the straining force. This results in permanent deformation, which is clear from figure (e) when compared to figure (a)
- ▶ Figure (f) shows the 3-D view of the formation of a step on the surface of the crystal due to the motion of an edge dislocation



2. Plastic Deformation by Twinning:

Twinning is the movement of atoms in the lattice resulting in a division of lattice into two parts: twinned region and untwined region, which are symmetrical, but differently oriented. Twinning occurs when a material is permanently or plastically deformed, or during the heat treatment of certain metals. As shown in figure, PQ and RS are called twin boundaries or twin planes, which separate the un-deformed and the deformed parts of the metal lattice. The region PQRS is known as twinned zone or twinned region.



- ▶ The part of the atomic lattice deformed in the twinned region is a mirror image of the un-deformed lattice. The mechanism of twinning may thus be **described as the simple sliding of one plane of atoms over the next**; the extent of the movement of each plane in the twinned region being proportional to its distance from the twinning plane. For example, the first plane CD moves one-third of an inter-atomic distance, the second plane EF moves two-thirds of the inter-atomic distance, and the third plane SR moves an entire spacing. Notice that the atom C gets displaced to C' exactly the same distance from either side of the twin plane, i.e., $B'C' = B'A'$.
- ▶ In simple words, PC'B' in the twinned region is mirror image of PA'B' in the undeformed region. Similar is the case with respect to other atoms in the twinned region.
- ▶ Twin boundaries interfere with the slip process, and increase the strength of the metal. Movement of twin boundaries causes a metal to deform. Twinning is observed as wide bands under the microscope, and is shown in figure.

Difference between slip and twinning

Slip

1. All atoms in one block move over same distance
2. Slip appears in thin lines
3. There is little change in lattice orientation
4. Requires lower shear stress
5. More pronounced at high temperature and gradual loadings
6. Occurs in metals having more number of slip systems. Eg. Cu (FCC) (12 slip system)

Twinning

1. Different planes of atoms move fractional distance
2. They appear in broad lines
3. Lattice orientation change in the twinned region
4. Require higher shear stress
5. More pronounced at low temperature and impact loadings.
6. Occurs in metals having less number of slip systems. Eg. Zn (HCP) (3 slip system)

Mechanism of strengthening in metals

Among different properties that are resident in polycrystalline materials, we have two sets which are in opposite camps.

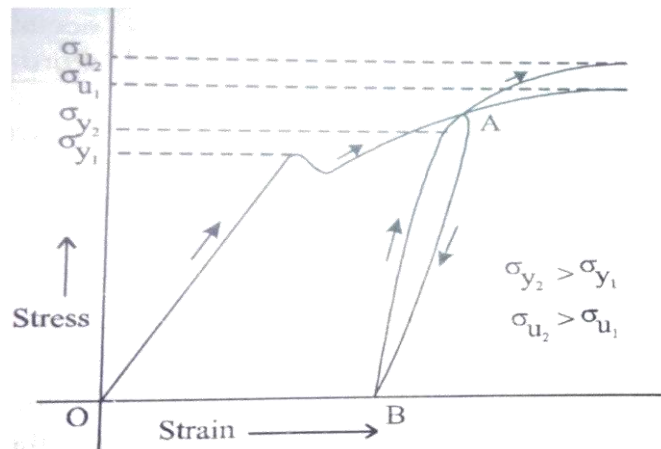
Strength with cost of ductility

Wear resistance – Toughness

Challenge for material engineer to balance between opposing properties³ broad mechanism through which metals can be strengthened

- Strengthening by grain size reduction
- Strain hardening or work hardening
- Age hardening or precipitation hardening or solution hardening (Module -3)

Strain Hardening (Work Hardening)



Strain Hardening:

It is the phenomenon where by a metal becomes harder and stronger when it is plastically deformed.

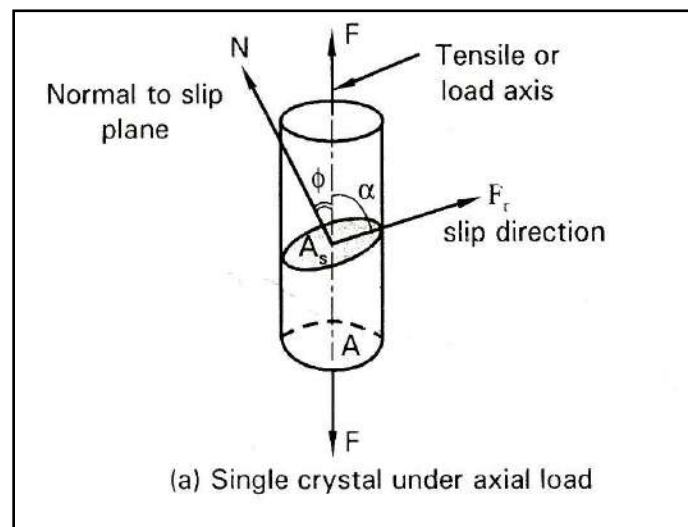
For example,, in a rolling plant sheet metal becomes harder and stronger when it is passed through rollars to reduce thickness.

This is a result of strain hardening or work hardening. And is more pronounced in ductile material which undergo large plastic deformation.

CRITICALLY RESOLVED SHEAR STRESS (CRSS)

The tensile test is widely used to study deformation mechanisms. Although the applied stress is pure tensile, shear components exist at all, but parallel or perpendicular alignments to the stress direction. These are termed as resolved shear stress, and their magnitude depend not only on the applied stress, but also on the **orientation of both the slip plane and direction within that plane.**

The concept of critically resolved shear stress (CRSS) was conceived by Schmidt, and hence is also known as **Schmidt's law**. According to this law, when the value of the resolved shear stress reaches a critical value on the slip plane in the slip direction, plastic deformation by slip takes place. Hence the name critical resolved shear stress. Consider a cylindrical shaped single crystal subjected to an axial tensile load as shown in figure (a).



$$\text{w.k.t. stress} = \sigma = \frac{F}{A} \quad \text{or } F = \sigma A \quad \text{----- (5)}$$

Substituting (5) in (4), gives, $\tau_r = \frac{\sigma \cdot A \cos \phi \cos \alpha}{A}$

$$\therefore \tau_r = \sigma \cos \phi \cos \alpha \quad \text{----- (6)}$$

Equation (6) is known as Schmidt's law, and the term " $\cos \phi \cos \alpha$ " is known as Schmidt's factor.

In equation (6), i.e., $\tau_r = \sigma \cos \phi \cos \alpha$, the resolved shear stress (τ_r) is *maximum*, when

$$\phi = \alpha = 45^\circ.$$

$$\therefore \tau_r = \frac{\sigma}{2} \quad \text{----- (7)}$$

Let F = magnitude of tensile load

A = cross-sectional area of the crystal

As a result of the axial load, let the *slip* take place along the shaded plane as shown in figure 2.19 (a).

Let A_s = area of the slip plane, or shear area on which the resolved shear force F_r is acting.

α = angle between the slip direction and the load axis.

ϕ = angle between normal to the slip plane and the load axis.

$$\text{Resolved shear stress} = \tau_r = \frac{\text{resolved shear force}}{\text{shear area (or slip plane area)}} = \frac{F_r}{A_s} \quad \text{----- (1)}$$

The resolved shear force F_r is nothing but the component of the applied force in the slip direction.

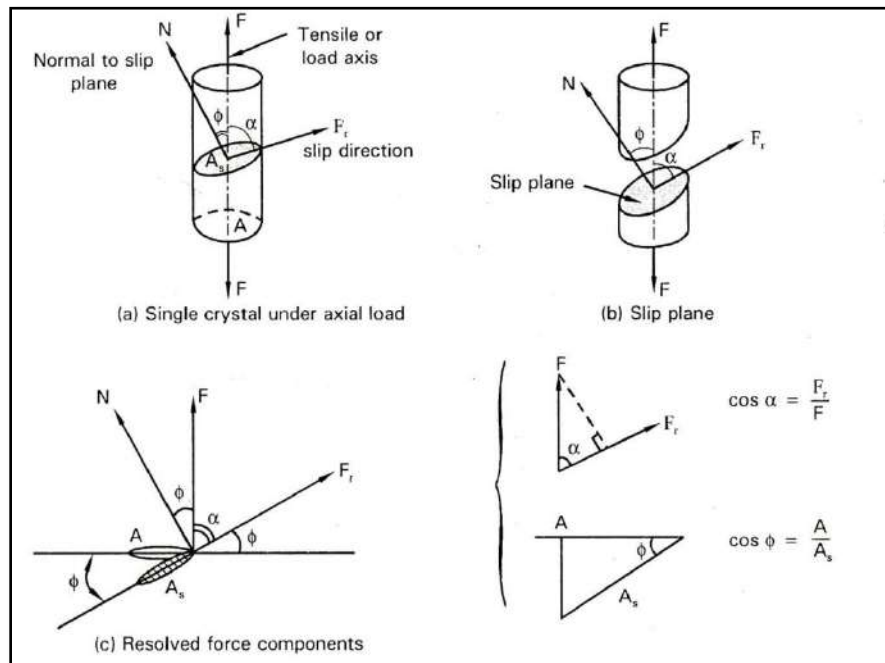
$$\text{i.e., } F_r = F \cos \alpha \quad \text{----- (2)}$$

$$\text{Also, } A_s = \frac{A}{\cos \phi} \quad \text{----- (3)}$$

Substituting equation (3) and (2) in (1), we have,

$$\tau_r = \frac{F_r}{A_s} = \frac{F \cos \alpha}{\frac{A}{\cos \phi}} = \frac{F \cos \phi \cos \alpha}{A} \quad \text{----- (4)}$$

Thus, the critical resolved shear stress will always be equal to half the tensile stress. When the tensile axis is normal to the slip plane, i.e., $\alpha = 90^\circ$, or, when the tensile axis is parallel to the slip plane, i.e., $\phi = 90^\circ$, the resolved shear stress (τ_r) defined in equation (6) will be equal to zero. Thus, slip will not occur for these extreme orientations, as there is no shear stress on the slip plane. Crystals close to these orientations tend to fracture rather than to slip.



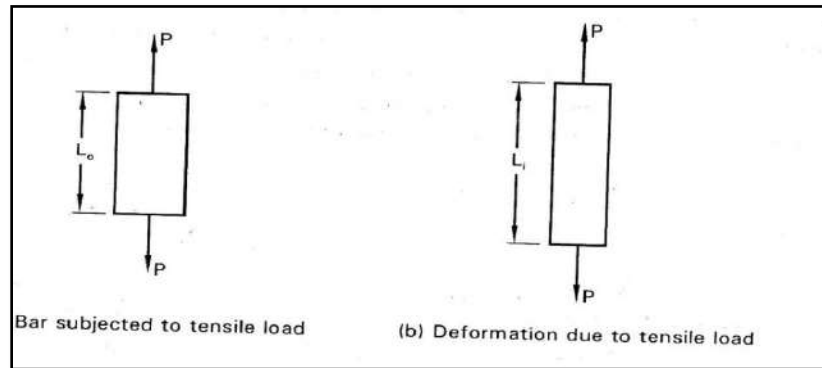
Relation between these two strains from fundamentals

Consider a bar subjected to a tensile load P as shown in figure (a). Due to the tensile load, the length of the bar increases, while its diameter decreases. Refer figure (b).

Let, L_o = original length of the bar

L_i length of the bar at any instant of load application

P = Tensile load



w.k.t., Conventional or engineering strain $\epsilon = \frac{\delta L}{L_0} = \frac{L_1 - L_0}{L_0}$

$$\epsilon = \frac{L_1}{L_0} - 1 \quad \text{or} \quad \epsilon + 1 = \frac{L_1}{L_0} \quad \text{----- (1)}$$

w.k.t., True strain $\epsilon' = \int_{L_0}^{L_1} \frac{dL}{L}$

$$\epsilon' = \ln \left(\frac{L_1}{L_0} \right) \quad \text{----- (2)}$$

Substituting equation (1) in (2), we have, $\epsilon' = \ln(\epsilon + 1)$

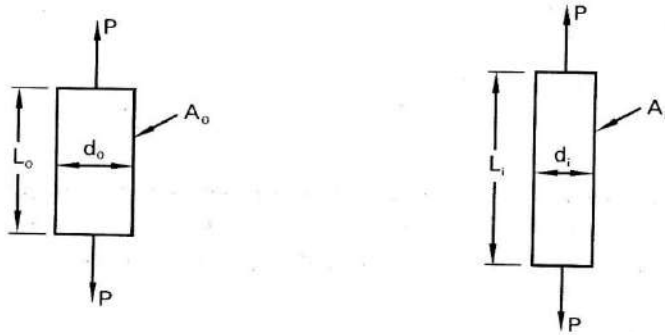
$$\text{or } \epsilon' = \ln(1 + \epsilon)$$

where ϵ' = True strain and ϵ = conventional strain

Derive an expression for true strain in the form $\epsilon^t = \ln \left(\frac{L_i}{L_o} \right)$. Show that the expression for true strain may also be represented by $\epsilon^t = \ln \left(\frac{A_o}{A_i} \right)$

Solution :

Consider a bar subjected to a tensile load P as shown in figure 2.21 (a).



(a) Bar subjected to tensile load

(b) Deformation due to tensile load

Figure 2.21

Let L_o = original length of the bar

L_i = length of the bar at any instant of load application

A_o = original cross-sectional of the bar

A_i = cross-section of the bar at any instant of load application

P = Tensile load

Due to the tensile load, the length of the bar increases, but its diameter decreases. Refer figure 2.21 (b).

Assuming that the volume in the gauge length of the test specimen remain unchanged before and after loading, we have,

$$\begin{aligned}
 V_0 &= V_i \\
 A_0 L_0 &= A_i L_i \\
 \text{or } \left(\frac{L_i}{L_0} \right) &= \frac{A_0}{A_i} \quad \text{----- (1)}
 \end{aligned}$$

$$\text{w.k.t. true strain} = \epsilon' = \sum \left[\left(\frac{L_1 - L_0}{L_0} \right) + \left(\frac{L_2 - L_1}{L_1} \right) + \left(\frac{L_3 - L_2}{L_2} \right) + \dots \right]$$

$$\text{or } \epsilon' = \int_{L_0}^{L_i} \frac{dL}{L}$$

$$\epsilon' = \ln \left(\frac{L_i}{L_0} \right) \quad \text{----- (2)}$$

Substituting equation (1) in (2), we have, $\epsilon' = \ln \left(\frac{A_0}{A_i} \right)$