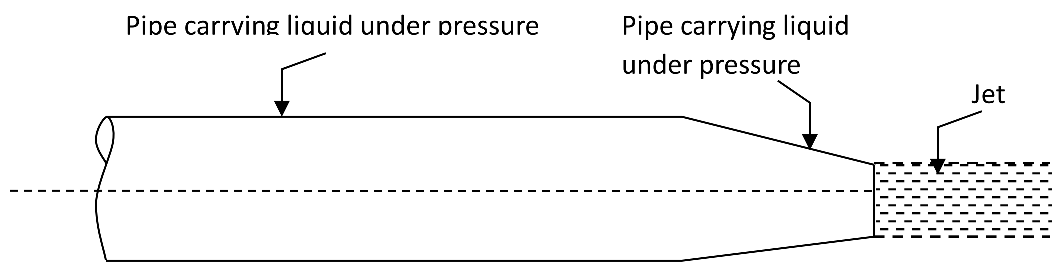
Module Five

Introduction:

The study of forces resulting from the impact of fluid jets and when fluids are diverted round pipe bends involves the application of Newton's second law in the form of F = m.a. The forces are determined by calculating the change of momentum of the flowing fluids. In nature these forces manifest themselves in the form of wind forces, and the impact forces of the sea on the harbour walls. The operation of hydro-kinetic machines such as turbines depends on forces developed through changing the momentum of flowing fluids.

Jet of Liquid



A nozzle is a tube of reducing cross section. As the water under pressure in the pipe passes through the nozzle, the area of cross section of flow decreases leading to increase in velocity and decreases in pressure. The jet of liquid comes out to the atmosphere.

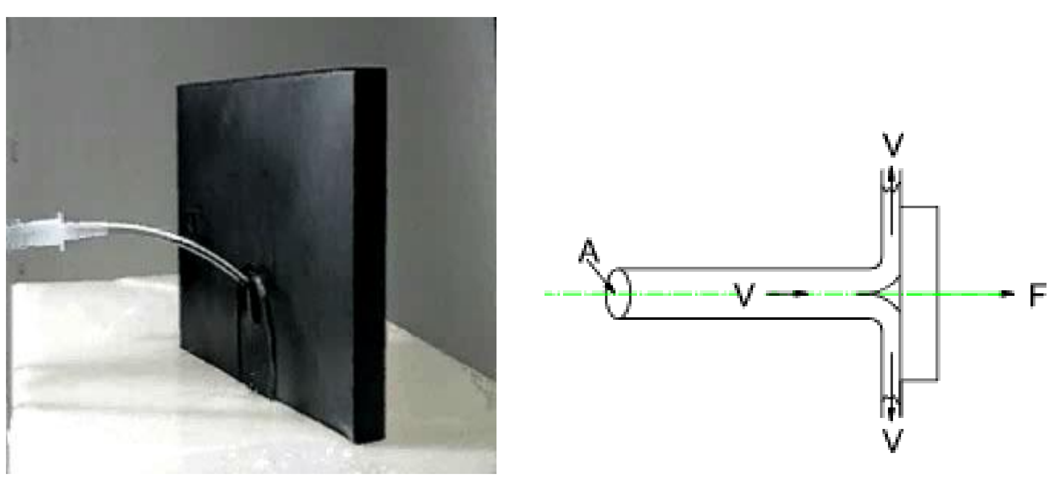


Fig. 4.1 Impact of Jet on Stationary Flat Plate

Net force experienced by fluid along x-direction.

$$\sum F_x = \frac{m}{t}(V_x - U_x)$$

1. $F_x = m\,(V_x - U_x)$

2. $F_y = m\,(V_y - U_y)$

Where m is the mass flow rate $m = \rho Q$

V = Final velocity of fluid along the direction.

U = Initial velocity of fluid along the direction.

Table 4.1 Capability of Momentum and Energy Equations

	Momentum Equation	Energy Equation
Applicable	To any fluid flow	To steady flow where energy changes are zero or known
Information required	• Velocity distribution at one end of control volume • Control distribution at the other end.	• Velocity and pressure at one point on the stream line with independent knowledge of energy changes • Pressure variation or velocity variation along stream line
Solution gives	Average final velocity of stream or total force	Velocity variation or pressure variation along stream lime
Solution will not give	Actual velocity distribution or pressure distribution	Tangential forces due to friction
Best application	When energy changes are unknown and only overall knowledge of flow is required. Eg: Total force, Mean velocity	When energy changes are known and detailed information of flow is required. Eg: Velocity and Pressure distribution.

The force exerted **by** the fluid **on** the solid body touching the control volume is opposite to F_R . S
force, R, is given by $R = -F_R$

4.3 Direct impact of a jet on a stationary flat plate (U=0):

To compute the impact of field jet on stationary flat plate held normal to the jet.

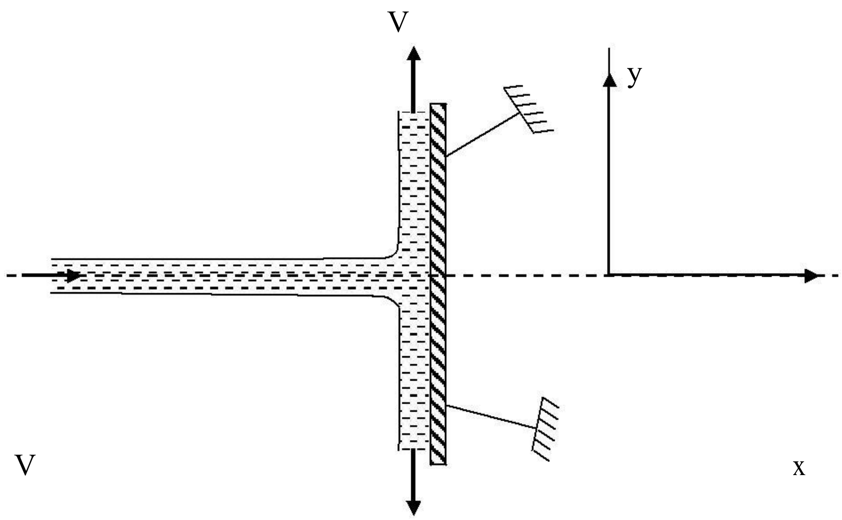


Fig. 4.3 Direct impact of a jet on a Stationary flat plate (U=0):

V – Velocity of jet striking the plate a – Area of cross section of jet.

$$\boxed{\therefore\ m = \boldsymbol{\rho a V}}$$

Force exerted by plate on fluid jet along x – direction = $F_X = m\,[V_X - U_X]$

Force exerted by the jet on the plate along x – direction will be equal and opposite to that of force exerted by plate on the jet.

$\therefore$ Force exerted by jet on plate along x – direction = $F_X = m\,(U_X - V_X)$

$$F_X = \rho a v\,[V - 0]$$

$$\boxed{\mathbf{F_X = \rho a v^2}}$$

Work done by the jet W.D = Force x Velocity of plate

= Force x 0

= 0

4.4 Oblique impact of a jet on a stationary flat plate (U=0):

To compute the impact of jet on a stationary flat plate held inclined to the direction of jet.

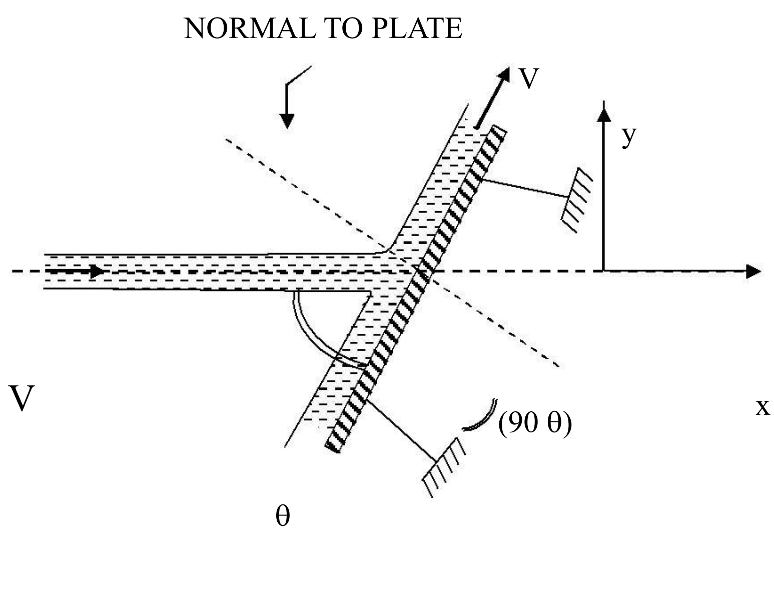


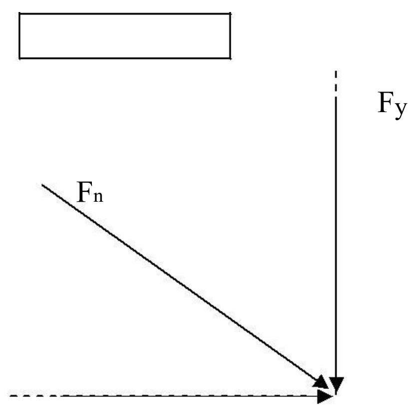
Fig. 4.4 Oblique impact of a jet on a stationary flat plate (U=0)

Force exerted by jet on vane along normal direction.

1. $F_n = m\,[U_n - V_n]$

$F_n = \rho a V\,[(V\sin\theta) - O]$

$$\mathbf{F_n = \rho a V^2 \sin\theta]}$$



(90 - θ) F_x

$F_x = F_n \cos(90 - \theta)$

$F_x = [\rho a V^2 \sin\theta]\,\sin\theta$

$$\boxed{\mathbf{F_x = \rho a V^2 \sin^2\theta}}$$

$F_y = F_n \sin(90 - \theta)$

$F_y = [\rho a V^2 \sin\theta]\,\cos\theta$

$$\boxed{\mathbf{F_y = \rho a V^2 \sin\theta\cos\theta}}$$

Work done by the jet on vane = Force x Velocity of vane

= Force x 0

W.D. $= 0$

4.5 Direct impact on a moving plate (U = U)

To compute the impact of jet on a moving flat plate held normal to the jet.

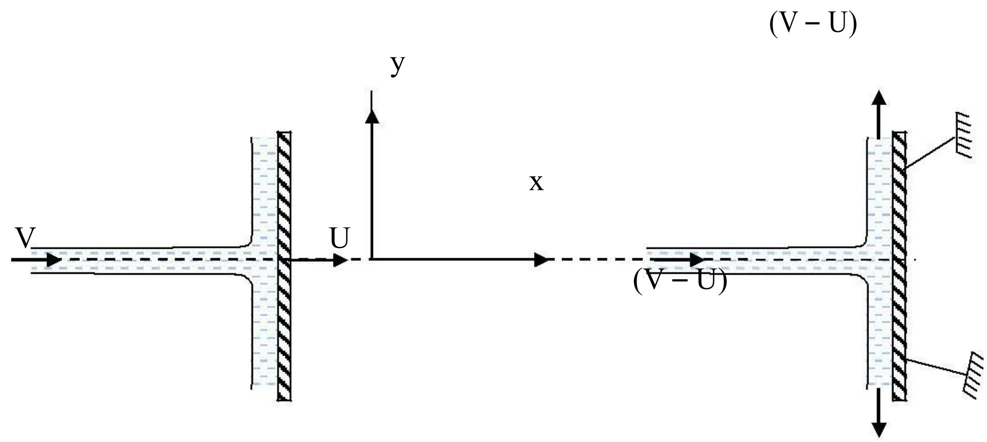


Fig. 4.5 Direct impact on a moving plate

V = Velocity of jet striking the plate

U = Velocity of vane along the direction of vane.

Adopting the concept of relative velocity, the system can be considered to be a stationary plate, the jet striking the vane with a relative velocity (V – U).

- m =ρQ

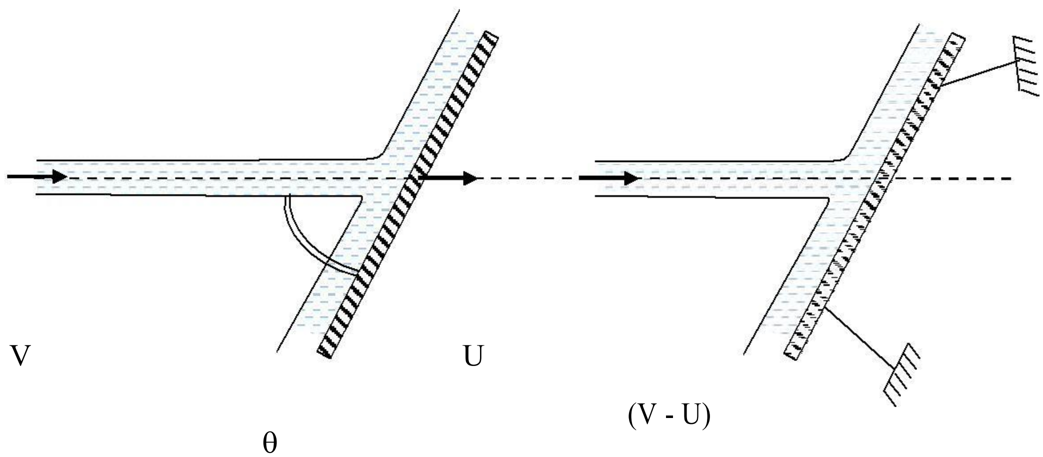
$\rho a(V- U)\ F_x = \rho a(V - U)^2$

Work done by the jet on plate = Force x Velocity of plate

W.D. $= \rho a\ (V - U)^2 \times U$

Direct impact of a jet on a series of a jet on a series of flat vanes on a wheel

To compute the impact of jet on a moving flat plate held inclined to the direction of jet.



V = Velocity of jet

U = Velocity of plate along the direction of jet.

Adopting the concept of relative velocity, the above case can be considered to be fixed vane with a jet velocity of (V – U).

$$\therefore F_n = \rho a\,(V - U)^2 \sin\theta$$

$$F_x = \rho a\,(V - U)^2 \sin^2\theta$$

$$F_y = \rho a\,(V - U)^2 \sin\theta\cos\theta$$

Work done by the jet on vane plate along x – direction

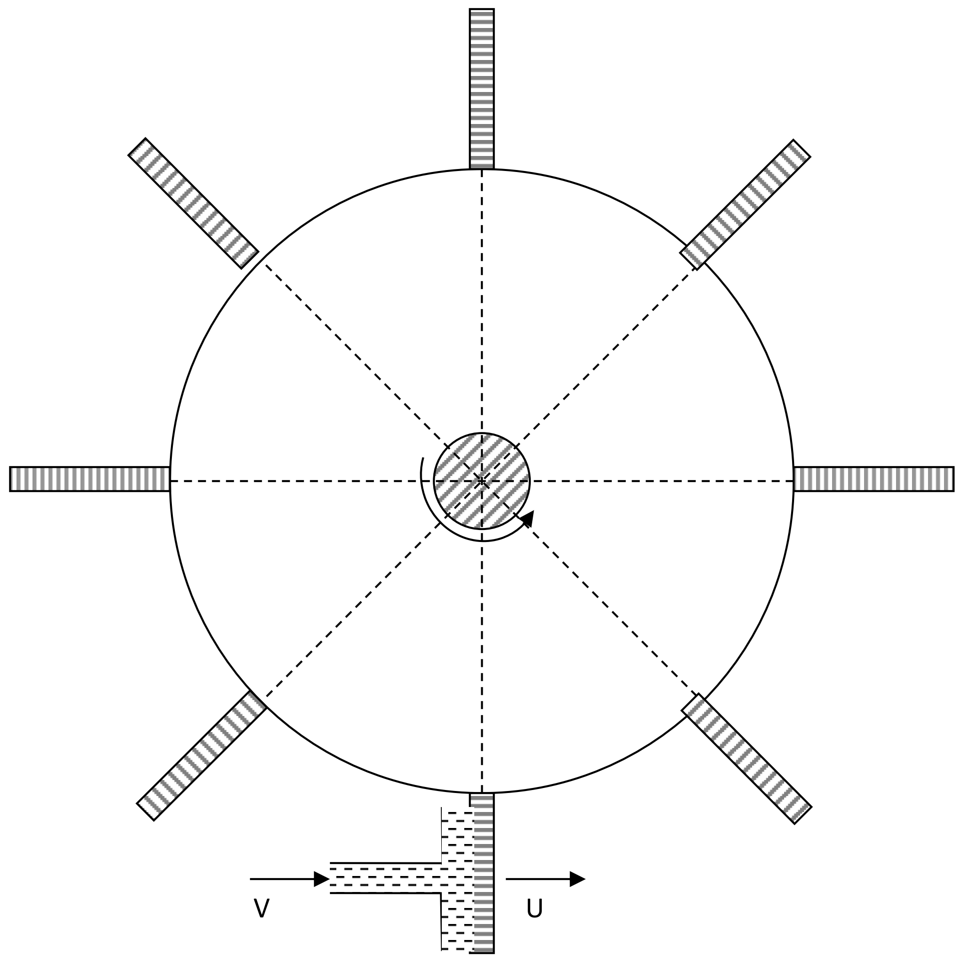
$= F_x$ x Velocity of plate along x – direction

$= \rho a\,(V - U)^2 \sin^2\theta\, U$

4.6 Conditions for maximum hydraulic efficiency for a jet impinging on a series of flat vanes on a wheel:

To derive expressions for the force exerted, work done and efficiency of impact of jet on a series of flat vanes mounted radially on the periphery of a circular wheel.

Let us consider flat vanes mounted radially on the periphery of a circular wheel. V is the velocity of jet and 'U' is the velocity of vane. The impact of jet on vanes will be continuous since vanes occupy one after another continuously.



$F_x = \dot{m} \, (U_x - V_x)$

$F_x = \rho aV \, [(V - U) - 0]$

$\mathbf{F_x = \rho aV \, (V - U)}$

Work done/s or Power = $F_x \cdot U$

$$\textbf{Power} = \boldsymbol{\rho aV (V - U) U}$$

Efficiency = $\eta = \dfrac{O/P}{I/P}$

$$\eta = \frac{\rho aV(V-U)U}{\frac{1}{2}(\rho aV)V^2}$$

$$\eta = \frac{2(V-U)U}{V^2}$$

Condition for maximum efficiency:

$$\eta = \frac{2}{V^2}\left(VU - U^2\right)$$

For maximum efficiency

$$\frac{d\eta}{dU} = 0$$

$$\frac{d\eta}{dU} = 0 = \frac{2}{V^2}(V - 2U)$$

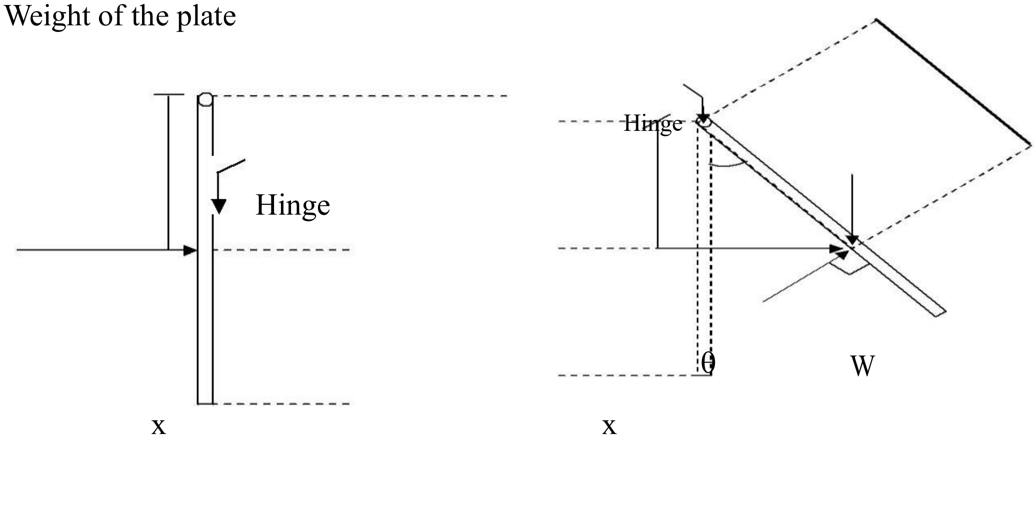
$$\therefore U = \frac{V}{2}$$

$\therefore$ Efficiency is maximum when the vane velocity is 50% of velocity of jet.

$$\eta_{max} = \frac{2\left(V - \frac{V}{2}\right)\frac{V}{2}}{V^2} = \frac{1}{2} \quad = \mathbf{50\%}$$

4.8 Impact of a jet on a hinged flat plate: A vertical flat plate is hinged at its top. A jet of water strikes at the centre of the plate. Due to the impact of jet, the plate attains equilibrium at an angle 'θ' with the plate. To determine the inclination in terms of velocity of flow, density of liquid, weight and area of the plate

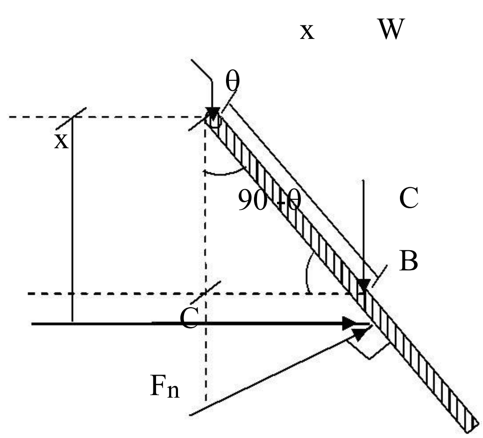
Let. ρ – Mass density of fluid, a – area of cross section of jet V – Velocity of jet , W – Weight of the plate



F_n INITIAL

FINAL

A



$$\Sigma M_{Hinge} = 0 = -F_n x\ AB + W\ x\ CA \sin\theta = 0$$

$$-\rho a V^2 \sin(90-\theta)\cdot \frac{x}{\cos\theta} + W \cdot x \sin\theta = 0$$

$$\rho a V^2 = W \sin\theta$$

$$\sin\theta = \rho\ a\ V^2/W$$

4.9 Solved Problems:

Q.1A jet of water of 22.5 cm diameter impinges normally on a flat plate moving at 0.6 m/s in the same direction as the jet. If the discharge is 0.14 m^3/s, find the force and the work done per second on the plate.

Solution

Density of water, ρ = 1000 kg/m^3

Area of jet, A = $\pi*(0.225)^2/4\ m^2$

$= 0.0398\ m^2$

velocity of jet, v $= Q/A$

$= 0.014/0.0398\ m/s$

$= 3.52\ m/s$

plate moving, u $= 0.6\ m/s$

$F_2 = 0$

$F3 = 0$ (free jet)

$F1 = \rho A(v - u)^2\cos\theta$

$= 1000*0.0398*(3.52-0.6)^2\ N$

$= 339\ N$ (force on jet, ←)

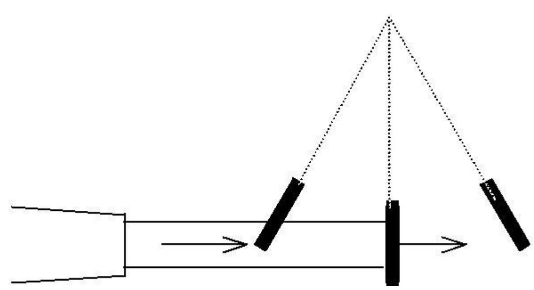
Force on the plate, $R = -F_1 = -339\ N$ (→)

Work done on plate / sec = F * u= 339 * 0.6 Nm/s or J/s or W

= 204 W

Q.2 In an undershot waterwheel the cross-sectional area of the stream striking the series of radial flat vanes of the wheel is $0.1\ m^2$ and the velocity v of the stream is 6 m/s. The velocity 'u' of the vanes is 3 m/s. Calculate the force 'F' exerted on the series of vanes by the stream.

Solution:



$A= 0.1m^2$, $v = 6m/s$ $u = 3m/s$

Since there are a series of vanes on the wheel, the average length of the jet from the nozzle to the point of impact remains constant and all the water from the nozzle strikes one or other of the vanes. Assuming that the diameter of wheel is large so that impact is approximately normal,

Mass of water striking vanes/sec = $\rho \times A \times v$

Initial velocity of water = v

= 6 m/s

Final velocity of water = velocity of vanes

= u = 3 m/s

change of velocity on impact = (v – u) = (6-3) = 3m/s

$F_2 = F_3$

Force of water on vanes, F = $\rho \times A \times v \times (v-u)$

Density of water, ρ = 1000 kg/m^3

Area of jet, A = 0.1 m^2

F = 1000*0.1*6*(6-3) N

= 1800 N

= 1.8 kN

Q.3 A jet of water 50 mm diameter strikes a flat plate held normal to the direction of jet. Estimate the force exerted and work done by the jet if.

a. The plate is stationary
b. The plate is moving with a velocity of 1 m/s away from the jet along the line of jet.
c. When the plate is moving with a velocity of 1 m/s towards the jet along the same line.

The discharge through the nozzle is 76 lps.

Solution:

$d = 50 \text{ mm} = 50 \times 10^{-3}$ m

$a = {}^{\Pi}\cancel{x}\,(50 \times 10^{-3})^{2}\ 4$

$\mathbf{a = 1.9635 \times 10^{-3} m^2}$

$Q = aV$

$76 \times 10^{-3} = 1.9635 \times 10^{-3} \times V$

$\mathbf{V = 38.70\ m/s}$

Case a) When the plate is stationary

$F_x = \rho a V^2$

$F_x = 1000 \times (1.9635 \times 10^{-3}) \times (38.70)^2$

$\mathbf{F_x = 2940.71\ N}$

Work done/s = $F_x \times U$

Work done/s = $F_x \times 0$

Work done/s = 0

Case b) V = 38.70 m/s (→)

U = 1 m/s (→)

$F_x = \rho a\,(V - U)^2$

$F_x = 1000 \times 1.9635 \times 10^{-3} \times (38.7 - 1)^2$

$\mathbf{F_x = 2790\ TN}$

Work done/s = $F_x \times U$

Work done/s = 2790.7 x 1

Work done/s = 2790.7 Nm/s or J/s or W

Case c) V = 38.70 m/s (→)

$U = 1$ m/s (←)

$F_x = \rho a\,(V - U)^2$

$F_x = 1000 \times 1.9635 \times 10^{-3} \times (38.7 + 1)^2$

$\mathbf{F_x = 3094.65\ N}$

Work done/s = $F_x \times U$

Work done/s = 3094.65×1

Work done/s = 3094.65 Nm/s

Q.4 A jet of water 50 mm diameter exerts a force of 3 kN on a flat vane held perpendicular to the direction of jet. Find the mass flow rate.

Solution:

$d = 50$ mm $= 50 \times 10^{-3}$ m

$a = \frac{\pi}{4} \times (50 \times 10^{-3})^2$

$\mathbf{a = 1.9635 \times 10^{-3} m^2}$

$F_x = \rho \times a \times V^2$

$3000 = 1000 \times 1.9635 \times 10^{-3} \times V^2$

V = 39.09 m/s

$m = \rho \times Q$

$m = \rho a V$

$m = 1000 \times 1.9635 \times 10^{-3} \times 39.09$

m = 76.75 kg/s

Q.5 A jet of data 75 mm diameter has a velocity of 30 m/s. It strikes a flat plate inclined at 45° to the axis of jet.

Find the force on the plate when.

- The plate is stationary
- The plate is moving with a velocity of 15 m/s along and away from the jet.

Also find power and efficiency in case (b)

Solution:

$d = 75 \times 10^{-3}$ m

$a = \frac{\Pi}{4} \times (75 \times 10^{-3})^2$

$V = 30$ m/s

$\mathbf{a = 4.418 \times 10^{-3}\ m^2}$

$\theta = 45^o$

Case a) When the plate is stationary (U = 0)

$F_X = \rho \times a \times V^2 \times \sin^2\theta$

$= 1000 \times 4.418 \times 10^{-3} \times 30^2 \times \text{Sin}^2 45° \quad \mathbf{= 1988\ N}$

Case b) When the plate is moving (U = 15m/s)

$V = 30$ m/s ($\rightarrow$)

$U = 15$ m/s ($\rightarrow$)

$F_X = \rho a\ (V - U)^2 \sin^2\theta$

$F_X = 1000 \times 4.418 \times 10^{-3} \times (30 - 15)^2 \sin^2 45°$

$\mathbf{F_X = 497.03\ N}$

Output power = Work done/s

Output power = F_X x U

Output power = 497.03 x 15

Output power = 7455.38 W

Input Power = Kinetic Energy of jet per second = $\frac{1}{2} \times \boldsymbol{m} \times \boldsymbol{V}^2 = \frac{1}{2} \times (\boldsymbol{\rho} \times \boldsymbol{a} \times \boldsymbol{V}) \times \boldsymbol{V}^2$

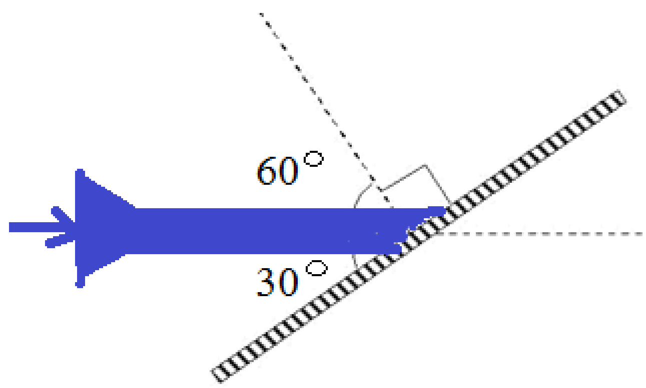
Input Power = Kinetic Energy of jet per second = $\frac{1}{2} \times (1000 \times 4.418 \times 10^{-3} \times 30) \times 30^2$

Input Power = 59643 W

$$\boldsymbol{Effciency\ of\ the\ System}\ '\boldsymbol{\eta}' = \frac{\boldsymbol{Output}}{\boldsymbol{Input}} \times 100 = \frac{7455.38}{59643} = 12.5\%$$

Q.6 A 75 mm diameter jet having a velocity of 12 m/s impinges a smooth flat plate, the normal of which is inclined at 60^o to the axis of jet. Find the impact of jet on the plate at right angles to the plate when the plate is stationery.

What will be the impact if the plate moves with a velocity of 6 m/s (a) in the direction of jet (b) Away from it. What will be the force if the plate moves towards the plate?



Solution

$d = 75 \text{ x } 10^{-3}$ m

$a = \frac{\pi}{4} \text{x} (75 \text{ x} 10^{-3})^2$

$\mathbf{a = 4.418 \text{ x } 10^{-3} m^2}$

When the plate is stationery (U = 0)

$F_n = \rho a V^2 \sin\theta$

$F_n = 1000 \text{ x } (4.418 \text{ x } 10^{-3})\ 12^2 \sin 30°$

F_n= **318.10 N**

When the plate is moving away from the jet (**U = 6m/s**)

$F_n = \rho a\ (V - U)^2 \sin\theta$

$F_n = 1000 \times 4.418 \times 10^{-3} (12 - 6)^2 \sin 30°$

$F_n =$ **79.52 N**

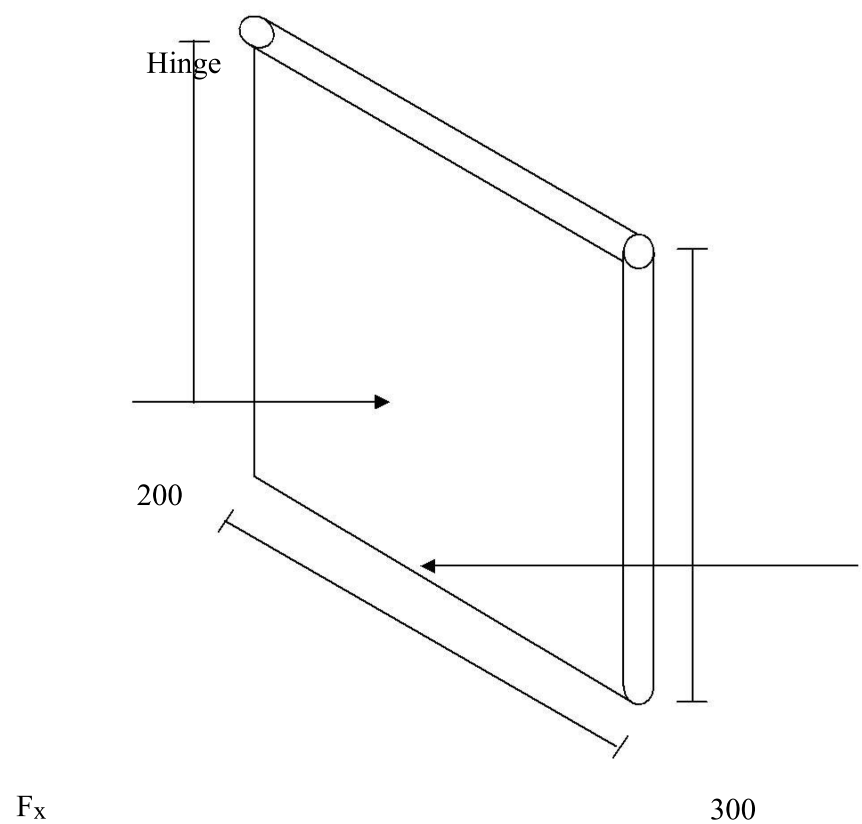
When the plate is moving towards the jet (**U = -6m/s)**

$F_n = \rho a (V + U)^2 \sin\theta$

$F_n = 1000 \times 4.418 \times 10^{-3} (12 + 6)^2 \sin 30°$

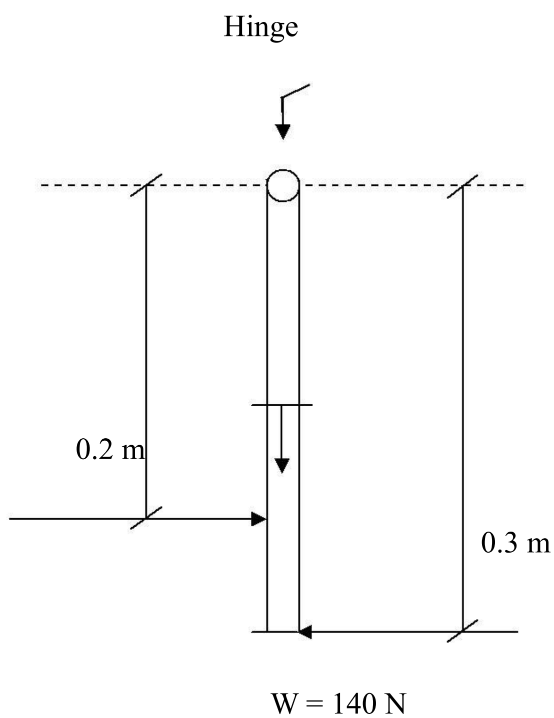
$F_n =$ **715.72 N**

Q. 7 A square plate weighing 140 N has an edge of 300 mm. The thickness of the plate is uniform. It is hung so that it can swing freely about the upper horizontal edge. A horizontal jet of 20 mm diameter having 15 m/s velocity impinges on the plate. The centre line of jet is 200 mm below. The centre line of jet is 200 mm below the upper edge of plate. Find what force must be applied at the lower edge of plate in order to keep it vertical.



P

300



F_n

P

Area of jet $\boldsymbol{a} = \frac{\boldsymbol{\pi}}{4} \times \left(\frac{20}{1000}\right)^2 = 3.1416 \times 10^{-4}$

V = 15m/s

$\Sigma M_{Hinge} = 0$

F_x x 0.2 + P x 0.3 = 0

$\rho \times a \times V^2 \times 0.2 = P \times 0.3$

$1000 \times 314.16 \times 10^{-6} \times 0.2 \times 15^2 = P \times 0.3$

P = 47.124 N

PART – B

UNIT-5: IMPACT OF JET ON CURVED VANES

5.1 Introduction

5.2 Force exerted by a jet on a fixed curved vane

5.3 Force exerted by a jet on a moving curved vane

5.4 Introduction to concept of velocity triangles

5.5 Impact of jet on series of curved vanes

5.6 Problems

5.1 Introduction:

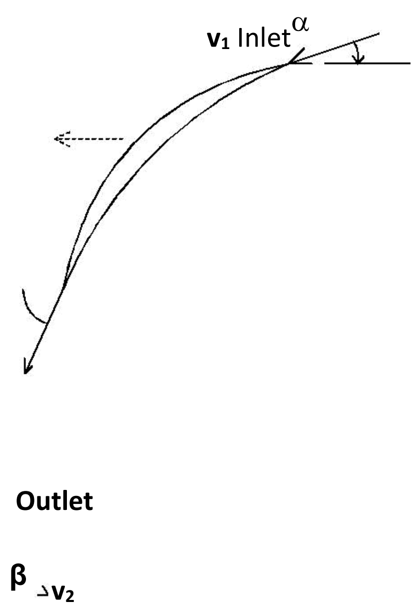
Hydraulic machinery makes use of the ‘force’ of a liquid. The liquid impinges on a series of blades or vanes connected to the periphery of a wheel, thus driving the wheel. The vanes are usually curved for greater effect as illustrated later.

When a **flat plate is used, the momentum normal to the plate is destroyed**. **It is more effective to change the direction of the momentum. It can be arranged for the jet to impinge tangentially on a curved vane,** so that no momentum is destroyed and the jet is merely deflected. The efficiency of energy conversion increases considerably.

In the following case, the forces parallel to and perpendicular to the dotted line will be calculated. The blade velocity is stationary.

5.2 Force exerted by a Jet on a Fixed Curved Vane (U= 0):

Let the inlet velocity of the jet is V_1 and it enters the curved blade with an angle α and the jet leaves with an outlet velocity V_2 and outlet angle ‘β’



Force on jet = $\rho \times Q \times$ change of velocity of jet $= \rho \times Q \times \Delta U$

$= \rho \times Q \times$(final velocity of jet - initial velocity of jet)

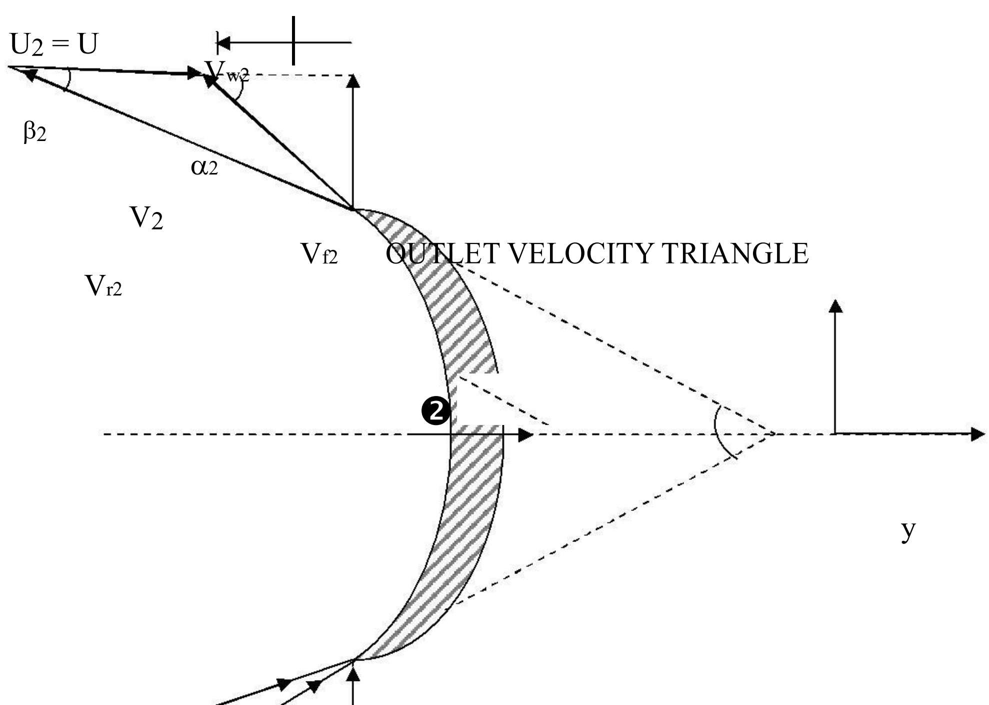
Force on vane = - Force on jet

$= \rho \times Q \times$(Initial velocity of jet - final velocity of jet)

$R_x = \rho \times Q \times (v_1\cos\alpha - v_2\cos\beta)$ Eq….5.1

$R_y = \rho \times Q \times (v_1\sin\alpha - v_2\sin\beta)$ Eq….5.2

Force exerted by a jet of water on an asymmetrical curved vane whenthe jet strikes tangentially at one of the tips:



β_2

U β_1 x

V_1

V_{f1}

V_{r1}

α_1 β_1 INLET VELOCITY TRIANGLE

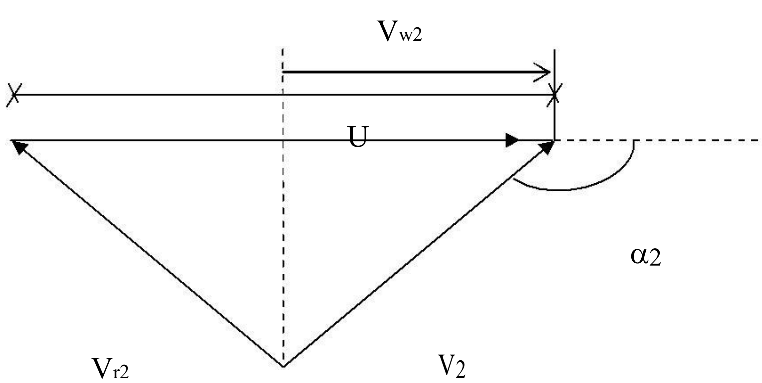
U

V_{w1}

$F_x = m\,[U_x - V_x]$

$F_x = \rho a V_{r1}\,[(V_{w1} - U) - (-(V_{w2} + U))]$

$$\mathbf{F_x = \rho a V_{r1}\,[V_{w1} + V_{w2}]}$$



$F_x = \rho a V_{r1}\,[(V_{w1} - U) - (-$

$(U - V_{w2}))]$

$$\mathbf{F_x = \rho a V_{r1}\,[V_{w1} - V_{w2}]}$$

$$\therefore \mathbf{F_x = \rho a V_{r1}\,[V_{w1} \pm V_{w2}]}$$

Work done/s $= F_x \times U$

$$\textbf{Work done/s} = \mathbf{\rho a V_{r1}\,[V_{w1} \pm V_{w2}]\,U}$$

Work done for unit mass flow rate $= [V_{w1} \pm V_{w2}]\,U$

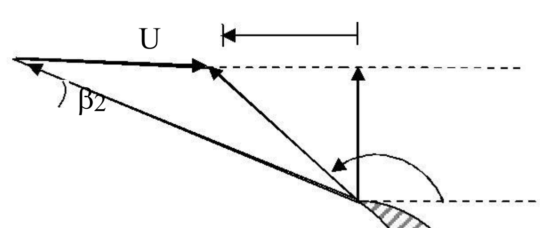
Work done for unit weight flow rate $= [V_{w1} \pm V_{w2}]/g$ $\overline{U}$

Example - Force of jet impinging normally on a fixed plate

Q.1A jet of water impinges a curved plate with a velocity of 20 m/s making an angle of 20° with the direction of motion of vane at inlet and leaves at 130° to the direction of motion at outlet. The vane is moving with a velocity of 10 m/s. Compute.

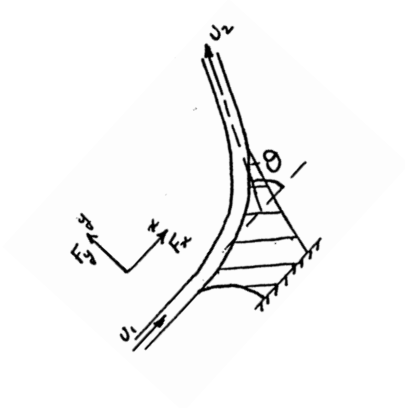
1. Vane angles, so that water enters and leaves without shock.
2. Work done/s

- **Solution:**



V_r

2



V

1

1 = 20o 1

U

V
w
1

V_{f1}

V_r

$V_1 = 20$ m/s

$U_1 = U_2 = 10$ m/s

Assuming number loss $V_{r1} = V_{r2}$

$V_{w1} = 20 \cos 20 = 18.79$ m/s

$V_{f1} = 20 \sin 20 = 6.84$ m/s

$$\tan \beta_1 = \frac{V_{f1}}{(V_{w1} - U)}$$

$$\tan \beta_1 = \frac{6.84}{(18.79 - 10)}$$

$\tan \beta_1 = 37.88^o$

$$\sin \beta_1 = \frac{V_{f1}}{V_{r1}}$$

$$\sin 37.88 = \frac{6.84}{V_{r1}}$$

$V_{r1} = 11.14$ m/s

$V_{r2} = V_{r1} = 11.14$ m/s

$$\frac{V_{r2}}{\sin 130} = \frac{U}{\sin (180 - 130 - \beta_2)}$$

$$\sin(150 - \beta_2) = \frac{10 \sin 130}{11.14}$$

$$\sin(50 - \beta_2) = 0.6877$$

$$\boldsymbol{\beta_2 = 6.55^o}$$

Work done per unit mass flow rate

$= (V_{w1} + V_{w2})\, U$

$= [18.79 + (V_{r2}\cos\beta_2 - U)]$

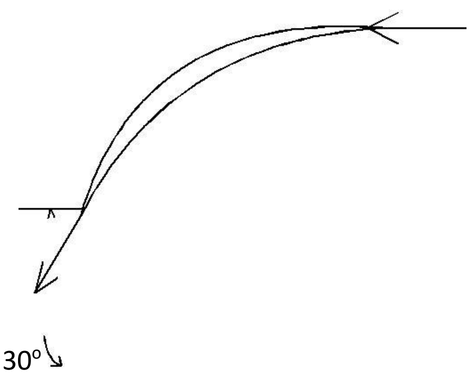
$= [18.79 + 11.14 \cos 6.55 - 10]\, 10$

$=$ **198.57 W/kg**

Example - Force of jet impinging normally on a fixed plate

Q.2 Find the forces on the blade parallel to and perpendicular to the water jet at the inlet.

The jet is 50 mm diameter.



Solution:

Density of water, ρ = $1000\ kg/m^3$

Area of jet, A = $\pi*(0.05)^2/4\ m^2$

= $1.96*10^{-3}\ m^2$

velocity of jet, v = 30 m/s

m = $\rho \times A \times v$

= $1000*1.96*10^{-3}*30$ kg/s

= 58.9 kg/s

Force on jet parallel to jet at inlet

= m × (30 cos30° - 30) (←)

= 58.9*30*(1 - cos30°) N (→)

= 236.8 N (→)

Force on jet perpendicular to jet at inlet

= m × (30 sin30° - 0) (↓)

= 58.9*30*0.5 N (↓)

= 883.5 N (↓)

Forces on the blade are 236.8 N (←) and 883.5 N (↑)

Q.3. A jet of water of 100 mm diameter impinges normally on a fixed plate with a velocity of 30 m/s. Find the force exerted on the plate.

Solution:

The cross sectional area of the jet,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.1^2 = 7.854 \times 10^{-3}\ m^2$$

Force exerted by the jet on the plane.

$$F = \frac{waV^2}{g} = \frac{9.81 \times (7.854 \times 10^{-3}) \times 30^2}{9.81}\ KN$$

$$\therefore F = 7.07\ KN$$

5.3 Force exerted by a Jet on a Moving Curved Vane:

Consider a jet of water entering and leaving a moving curved vane as shown in **Fig. 5.1**V = Velocity of the jet (AC), while entering the vane,

- V_1 = Velocity of the jet (EG), while leaving the vane,
- v_1, v_2 = Velocity of the vane (AB, FG)
- α= Angle with the direction of motion of the vane, at which the jet enters the vane,
- β= Angle with the direction of motion of the vane, at which the jet leaves the vane,
- V_r = Relative velocity of the jet and the vane (BC) at entrance (it is the vertical difference between V and v)

- V_{r1} = Relative velocity of the jet and the vane (EF) at exit (it is the vertical difference between v_1 and v_2)

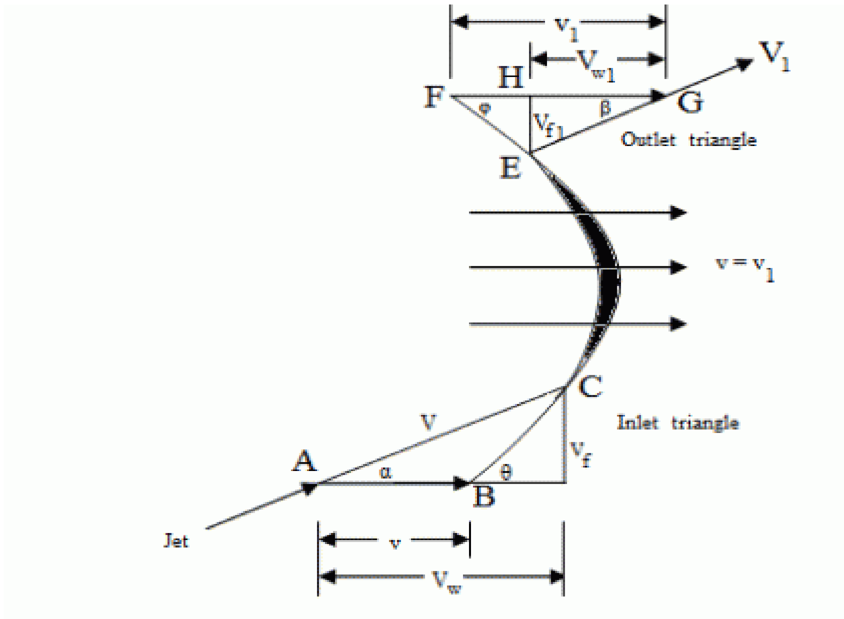


Fig. 5.1 Jet impinging on a moving curved vane

- θ = Angle, which V_r makes with the direction of motion of the vane at inlet (known as vane angle at inlet),
- ϕ = Angle, which V_{r1} makes with the direction of motion of the vane at outlet (known as vane angle at outlet),
- V_w = Horizontal component of V (AD, equal to $V\cos\alpha$). It is a component parallel to the direction of motion of the vane (known as velocity of whirl at inlet),
- V_{w1} = Horizontal component of V_1 (HG, equal to $V_1\cos\beta$). It is a component parallel to the direction of motion of the vane (known as velocity of whirl at outlet),
- V_f = Vertical component of V (DC, equal to $V\sin\alpha$). It is a component at right angles to the direction of motion of the vane (known as velocity of flow at inlet),
- V_{f1} = Vertical component of V_1 (EH, equal to $V_1\sin\beta$). It is a component at right angles to the direction of motion of the vane (known as velocity of flow at outlet),
- a = Cross sectional area of the jet.

As the jet of water enters and leaves the vanes tangentially, therefore shape of the vanes will be such that V_r and V_{r1} will be along with tangents to the vanes at inlet and outlet.

The relations between the inlet and outlet triangles (until and unless given) are:

(i) v=v1, and

(ii) Vr=Vr1

We know that the force of jet, in the direction of motion of the vane,

F= mass of water flowing per sec. X change of velocity of whirl

F= waV/g(Vw+Vw1) if β<90

F= waV/g(Vw-Vw1) if β>90

F= waV/g(Vw+Vw1) if β=90

Example - Force of jet impinging on a moving plate

Q.4 A jet of water of 50 mm diameter, moving with a velocity of 26 m/s is impinging normally on a plate. Determine the pressure on the plat, when (a) it is fixed and (b) it is moving with a velocity of 10 m/s in the direction of the jet.

Solution

Given,

- d = 50 mm = 0.05 m
- V = 26 m/s
- v = 10 m/s

a) Pressure on the plate when it is fixed (U = 0)

The cross sectional area of the jet,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.05^2 = 1.964 \times 10^{-3}\ m^2$$

Pressure on the plate,

$$P_1 = \frac{waV^2}{g} = \frac{9.81 \times (1.964 \times 10^{-3}) \times (26)^2}{9.81}$$

$$\therefore P_1 = 1.33\ KN$$

b) Pressure on the plate when it is moving (U = 10m/s)

The pressure on the plate when it is moving,

$$F = \frac{wa(V - v)^2}{g} KN$$

$$\Rightarrow F = \frac{9.81 \times (1.964 \times 10^{-3}) \times (26 - 10)^2}{9.81}$$

$$\therefore P_2 = 0.503 \ KN$$

5.4 Introduction to concept of Velocity Triangles:

Velocity triangle helps us in determining the flow geometry at the entry and exit of a blade. A minimum number of data are required to draw a velocity triangle at a point on blade. Some component of velocity varies at different point on the blade due to changes in the direction of flow. Hence an infinite number of velocity triangles are possible for a given blade. In order to describe the flow using only two velocity triangles we define mean values of velocity and their direction. Velocity triangle of any turbo machine has three components as shown:

VELOCITY – TRIANGLES IN TURBINES

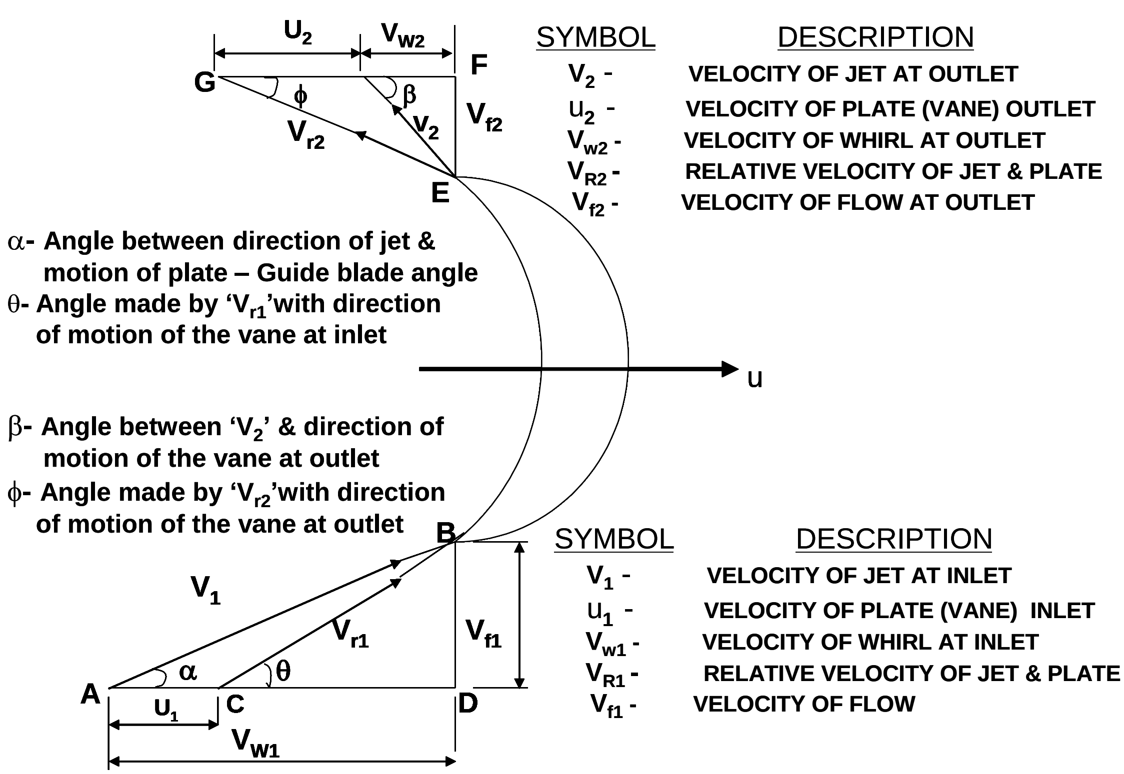


Fig 5.2 A Typical Velocity Trainable

These velocities are related by the triangle law of vector addition: -V=U+V_r

This relatively simple equation is used frequently while drawing the velocity diagram. Thevelocity diagram for the forward, backward face blades shown are drawn using this law. The angle α is the angle made by the absolute velocity with the axial direction and angle β is the angle made by blade with respect to axial direction.

Concept of Relative Velocity & Velocity Triangle:

The key idea in turbo machinery is concept of relative velocity. Suppose you are standing on this rotating turbo machine. The velocity of fluid you experience while moving with it is called as relative velocity. If fluid is having an absolute velocity V, and the blade is moving with a velocity U, then relative velocity experienced by you will be as follows.

$$\vec{W} = \vec{V} - \vec{U}$$

For a stationary device in order to have smooth operation, flow should be tangential to the blade. Similarly in a moving device relative velocity should be tangential to blade profile. With knowledge of direction of relative velocity and the vectorial representation of relative velocity, these 3 velocities could be drawn as shown below. This is known as a velocity triangle.

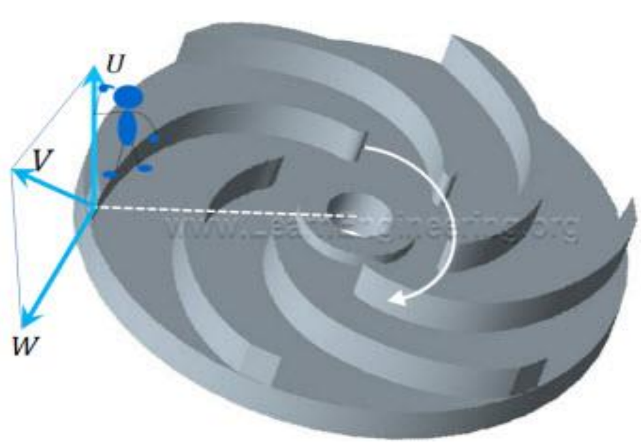


Fig. 5.3 Velocity triangle in a Turbo machine

Similar velocity triangle can be made on inlet of turbo machine. The beauty of turbo machinery is that using relatively simple analysis of inlet and outlet velocities you can predict performance of any turbo machine.

To develop turbo machinery fundamentals consider fluid flow through channel shown below. The inlet velocity, V1 gets changed to outlet velocity V2. Velocity of fluid can be split into tangential and radial components. This is shown in following figure

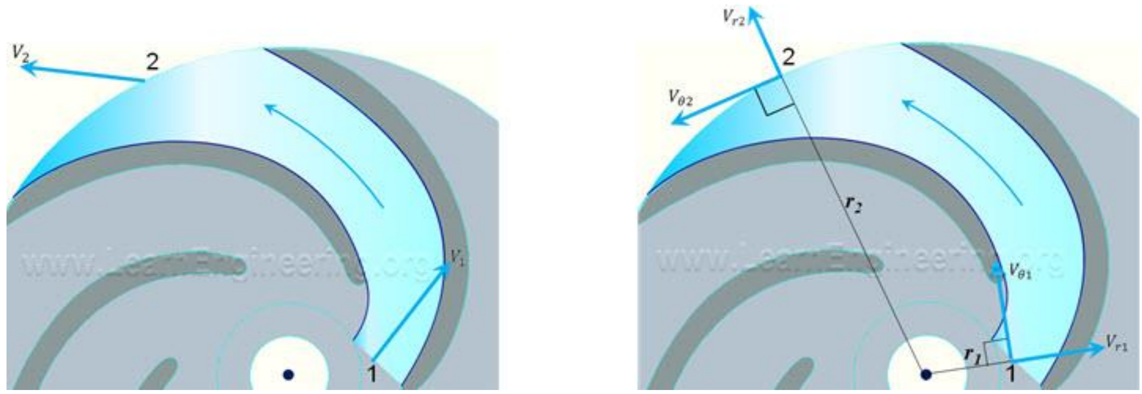


Fig. 5.4 Velocities at inlet and outlet can be split into tangential and Radial Components

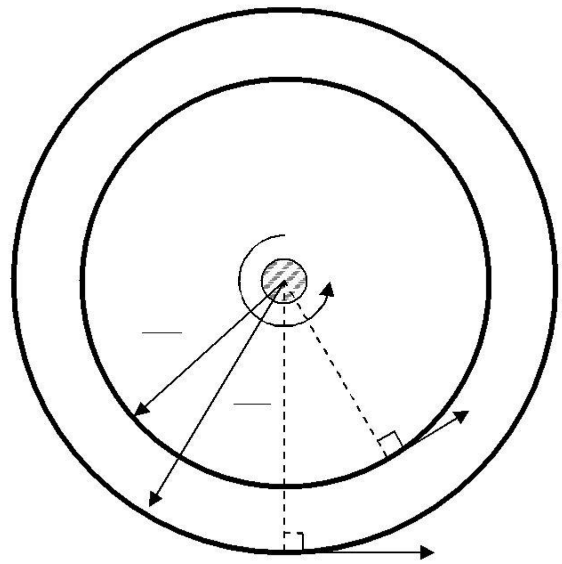
To make the fluid flow there should be external torque acting on it. This torque can be derived from Newton's 2nd law of motion, which acts as fundamental equation of turbo machinery. The torque is given by following equation, which is also called as Euler turbo machine equation.

$$Torque = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1})$$

….Eq. 5.3

5.5 Impact of jet on series of Curved Vanes:

Work done by water striking the vanes of a reaction turbine



$\frac{D_2}{2}$ R_2

$\frac{D_1}{2}$ U_2

2

R_1

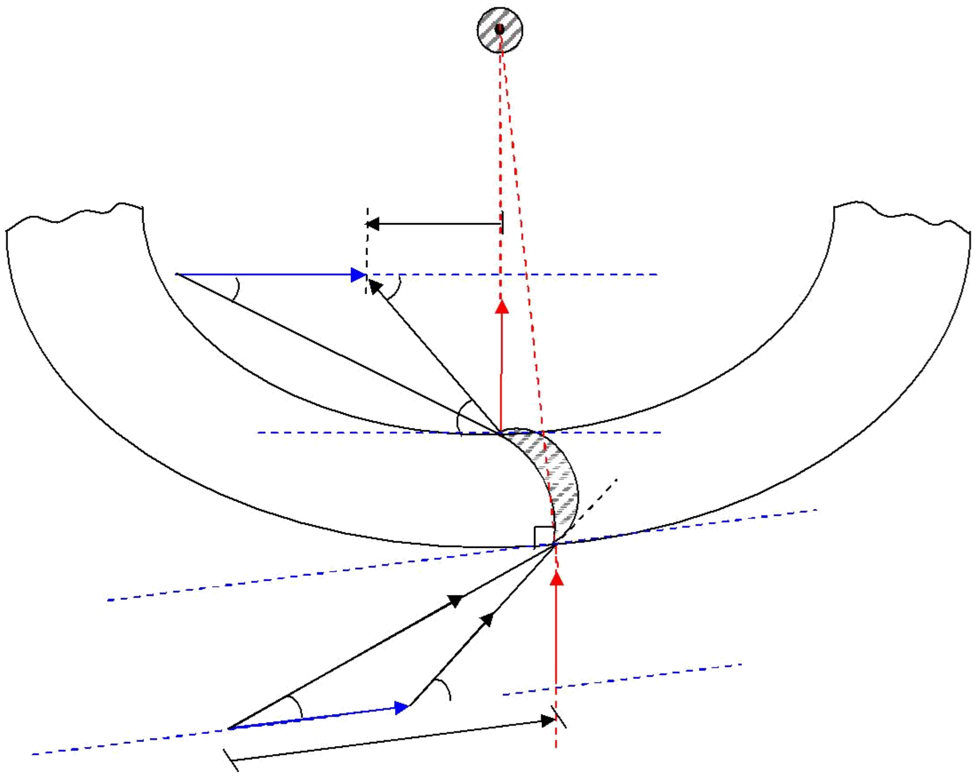
U_1

$U_1 = \frac{\pi D_1 N}{60}$ $\qquad U_1 = R\omega$

$U_2 = \frac{\pi D_2 N}{}$ $\qquad U_2 = R\omega$

Angular Momentum Principle:

Torque = Rate of change of angular momentum



$T = (\rho Q) \cdot [V_{w1} R_1 - V_{w2} R_2$

U_1- Tangential velocity of wheel at inlet $= \dfrac{\pi D_1 N}{60}$

U_2- Tangential velocity of wheel at inlet $= \dfrac{\pi D_2 N}{60}$

V_1 - Absolute velocity of fluid at inlet

V_2 - Absolute velocity of fluid at outlet

V_{w1} - Tangential component of absolute velocity at inlet – velocity of wheel at inlet = $V_1 \cos\alpha_1$.

V_{w2} - Tangential component of absolute velocity at outlet – velocity of wheel at outlet = $V_2 \cos\alpha_2$.

V_{f1} - Absolute velocity of flow at inlet

V_{f2} - Absolute velocity of flow at outlet

V_{r1} - Relative velocity at inlet

V_{r2} - Relative velocity at outlet

α_1 - Guide angle or guide vane angle at inlet

β_1

β_2

• •

r momentum equation

- Vane angle at inlet

$$T = m\,[V_{w1} R_1 - (-V_{w2} R_2)]$$

$$T = m\,[V_{w1} R_1 + V_{w2} R_2]$$

$$T = m\,[V_{w1} R_1 \pm V_{w2} R_2] \qquad \text{Eq.... 5.4}$$

Work done/s or power = T x Angle velocity

- Vane angle at outlet

Work done/s or power = $T \cdot \omega$

Work done/s or power = $m\,[V_{w1} R_1 \pm V_{w2} R_2]\,\omega$

Work done/s or power = $m\,[V_{w1}(\omega R_1) \pm V_{w2}(\omega R_2)]$

Work done/s or power = $m\,[V_{w1} U_1 \pm V_{w2} U_2]$

Work done per unit mass flow rate = $[V_{w1} U_1 \pm V_{w2} U_2]$

Work done per unit weight flow rate = $\frac{1}{g}(V_{w1} U_1 \pm V_{w2} U_2)$

By angula

Efficiency of the system $= \eta = \dfrac{m[V_{w1} U_1 \pm V_{w2} U_2]}{0.5\, m\, V_1^2}$

$$\eta = \frac{2[V_{w1} U_1 \pm V_{w2} U_2]}{V_1^2}$$

Q.5 A jet of water having a velocity of 35 m/s strikes a series of radial curved vanes mounted on a wheel. The wheel has 200 rpm. The jet makes 20° with the tangent to wheel at inlet and leaves the wheel with a velocity of 5 m/s at 130° to tangent to the wheel at outlet. The diameters of wheel are 1 m and 0.5 m. Find

- Vane angles at inlet and outlet for radially outward flow turbine.
- Work done
- Efficiency of the system

Solution

$V1$ - 35 m/s

N - 200 rpm

$\alpha 1$ - 20^o

$\beta 2$ - $180 – 130 = 50^o$

$V2$ - 5 m/s

$D1$ - 1 m

$D2$ - 0.5 m

$U1=\pi D1\ N/60$

$U1\quad \pi x1x\ 200/60$

$U1 = 10.47$ m/s

$U2\quad \pi D\ 2\ N\ /60$

$U2\quad \pi x\ 0.5\ x\ 200/60$

$U2 = 5.236$ m/s

$Vw1 = V1\ \cos\alpha 1$

$Vw1 = 35\ \cos\ 20$

$Vw1 = 32.89$ m/s

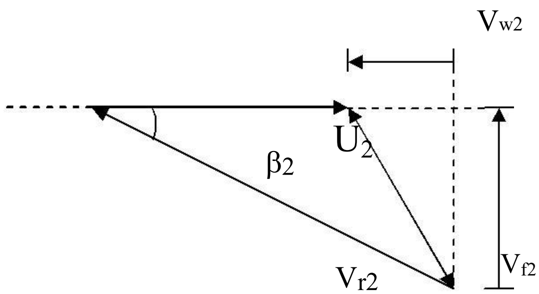
$Vf1 = V1\ \sin\ \alpha 1$

$Vf1 = 11.97$ m/s

$$\tan\beta_1 = \frac{V_{f1}}{(V_{w1} - U_1)}$$

$$\tan\beta_1 = \frac{11.97}{(32.89 - 10.47)}$$

$$\boldsymbol{\beta_1 = 28.10^0}$$



$V_{f2} = 5 \sin 50$

$\mathbf{V_{f2} = 3.83\ m/s}$

$V_{w2} = 5 \cos 50$

$\mathbf{V_{w2} = 3.214\ m/s}$

$$\tan\beta_2 = \frac{V_{f2}}{U_2 + V_{w2}}$$

$$\tan\beta_2 = \frac{3.83}{5.236 + 3.214}$$

$\tan \beta_2 = 24.38^0$

Work done per unit mass flow rate = $[V_{w1} U_1 + V_{w2} U_2]$

Work done per unit mass flow rate = [32.89 x 10.47 + 3.214 x 5.236] Work done per unit mass flow rate = 362.13 W/kg

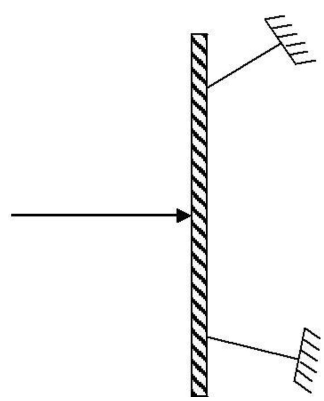
$$\text{Efficiency} = \eta = \frac{2\left[V_{\omega 1} U_1 + V_{\omega 2} U_2\right]}{V_1^2}$$

$$\text{Efficiency} = \eta = \frac{2\left[32.89 \times 10.47 + 3.214 \times 5.236\right]}{35^2}$$

Efficiency = η = 0.5896 or 58.96 %

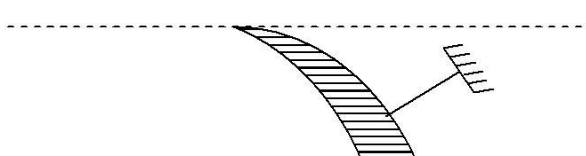
5.6 Solved Problems–

Q.6. Show that the force exerted by a jet on a hemispherical stationery vane is twice the force exerted by the same jet on flat stationery normal vane.



(DERIVE)

$F_x = \rho a V^2$



(DERIVE)

$$F_x = \rho a V^2 (1 + \cos\theta)$$

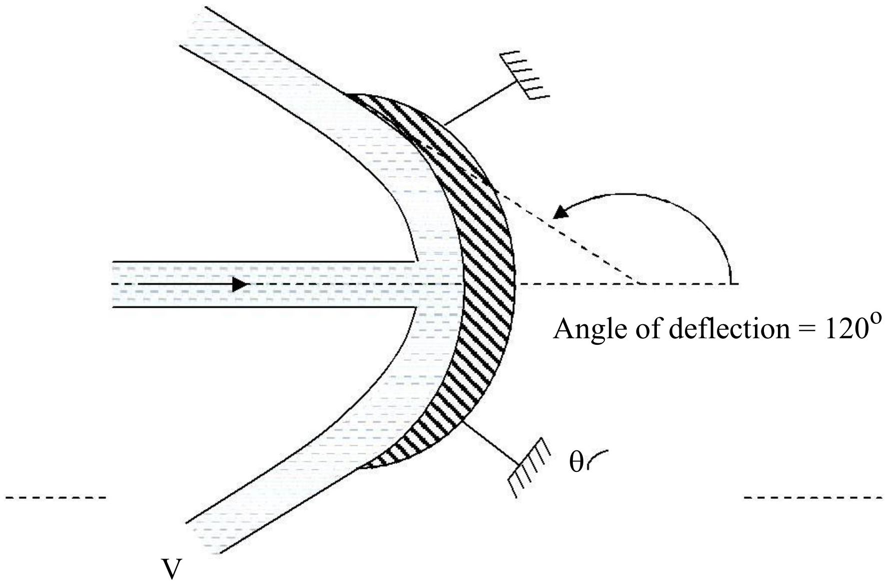
$$= \rho a V^2 (1 + \cos\theta)$$
$$= 2\rho a V^2$$

$$\frac{F_{x2}}{F_{x1}} = \frac{2\rho a V^2}{\rho a V^2}$$

$$\therefore F_{x2} = 2F_{x1}$$

Q. 7. A jet of water of diameter 50 mm strikes a stationary, symmetrical curved plate with a velocity of 40 m/s. Find the force extended by the jet at the centre of plate along its axis if the jet is deflected through 120° at the outlet of the curved plate.

Solution:



$d = 50 \times 10^{-3} m$

$a = \frac{\pi}{4} \times (50 \times 10^{-3})^2$

$\mathbf{a = 1.963 \times 10^{-3}\ m^2}$

$V = 40$ m/s

$\theta = 60^{o}$

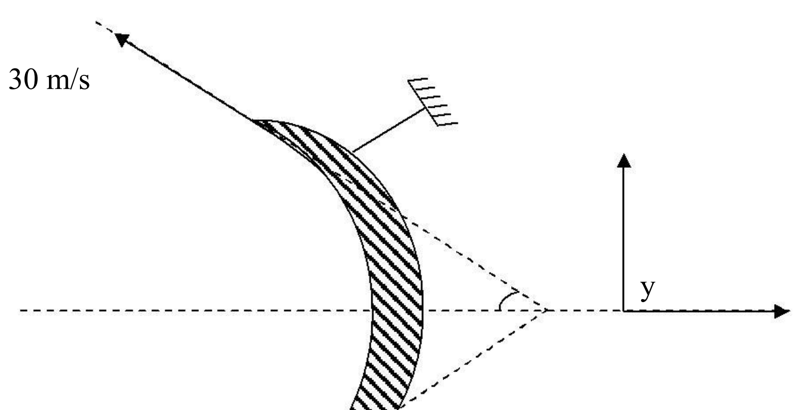
$F_x = \rho a V^2 (1 + \cos\theta)$

$F_X = 1000 \times 1.963 \times 10^{-3} \times 40^2\ (1 + \cos 60)$

$\mathbf{F_X = 4711\text{-}2\ N}$

Q. 8. A jet of water strikes a stationery curved plate tangentially at one end at an angle of 30°. The jet of 75 mm diameter has a velocity of 30 m/s. The jet leaves atthe other end at angle of 20°to the horizontal. Determine the magnitude of force exerted along 'x' and 'y' directions.

Solution:



20° x

30° (

30 m/s

$d = 50 \times 10^{-3} m$

$1.= \frac{\pi}{4} \times (75 \times 10^{-3})^2$

$\mathbf{a = 4.418 \times 10^{-3}\ m^2}$

$F_X = m\ [U_X - V_X]$

$F_X = \rho aV\ [30 \cos 30 - (-30 \cos 20)]$

$F_X = 1000 \times 4.418 \times 10^{-3} \times 30\ (30 \cos 30 + 30 \cos 20)$

$\mathbf{F_X = 7179.90\ N}$

Q.9 A jet of water of diameter 75 mm strikes a curved plate at its centre with a velocity of 25 m/s. The curved plate is moving with a velocity of 10 m/s along the direction of jet. If the jet gets deflected through 165° in the smooth vane, compute.

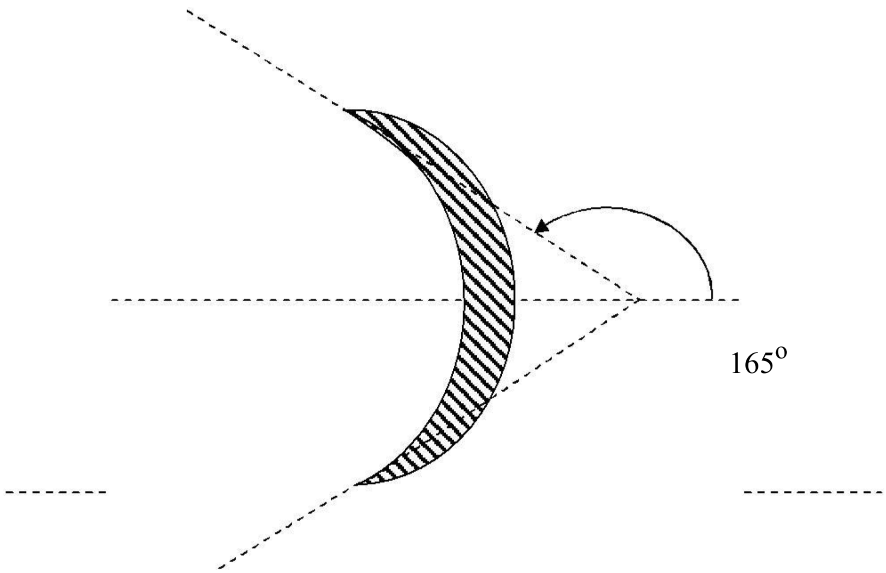
a Force exerted by the jet.
b Power of jet.
c Efficiency of jet.

$d = 75\ mm = 75 \times 10^{-3} m$

$a = \pi x \left(75\ x10^{-3}\right)^2 4$

$\mathbf{a = 4.418\ x\ 10^{-3}\ m^2}$

Solution:



$\theta = 15^{o}$

$V = 25$ m/s

$U = 10$ m/s

$F_x = \rho a\ (V - U)^2\ (1 + \cos\theta)$

$F_x = 1000\ x\ 4.418\ x\ 10^{-3}\ [25 - 10]^2\ x\ (1 + \cos 15)$

$\mathbf{F_x = 1954.23\ N}$

Power of jet = Work done/s

Power of jet = $F_x \times U$

Power of jet = 1954.23 x 10

Power of jet = 19542.3 W

Kinetic energy of jet/s = $\frac{1}{2} mV^2$

Kinetic energy of jet/s = $\frac{1}{2}\left[1000 \text{ x } 4.418 \text{ x} 10^3 \text{ x } 25\right] 25^2$

Kinetic energy of jet/s = 34515.63 W

η =Output/ In put

$\eta = \frac{19542.3}{34515.63}$

η **= 56.4 %**

PART – B

UNIT-6: PELTON WHEEL

6.1 Introduction to Turbines:

The word *turbine* was coined in 1828 by Claude Burdin (1788-1873) to describe the subject of an 1826 engineering competition for a water power source. It comes from Latin *turbo, turbinis*, meaning a "whirling" or a "vortex," and by extension a child's top or a spindle

Definition: The device which converts hydraulic energy into mechanical energy or vice versa is known as Hydraulic Machines. The hydraulic machines which convert hydraulic energy into mechanical energy are known as Turbines and that convert mechanical energy into hydraulic energy is known as Pumps.

Hydro electricity is a reliable form of renewable energy. Water turbines are highly efficient and easily controlled to provide power as and when it is needed. A hydraulic turbine is a prime mover (a machine which uses the raw energy of a substance and converts into mechanical energy) that uses the energy of flowing water and converts it into the mechanical energy (in the form of rotating of the runner). This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the hydraulic turbine; from this electric generator, we get electric power which can be transmitted over long distances by means of transmission lines and transmission towers. The hydraulic turbines are also known as "Water turbines" since the fluid medium used in them is water.

First hydroelectric station was probably started in America in 1882 and thereafter development took place very rapidly. In India, the first major hydroelectric development of 4.5 MW-capacity named as Sivanasamudram scheme in Mysore was commissioned in 1902.

Hydro (Water) Power is a conventional renewable source of energy which is clean, free from pollution and generally has a good environmental effect.

The disadvantage of energy from water is that it is strictly limited, and widely distributed in small amounts that are difficult to exploit. Only where a lot of water is gathered in a large river, or where descent is rapid, is it possible to take economic advantage.

The following factors are major obstacles in the utilization of hydropower resources.

(i) Large investments.
(ii) Long gestation period, and
(iii) Increased cost of power transmission.

Turbines can be divided into two basic types. These are **Impulse Turbines** and**Reaction Turbines**.

6.2 Classification of Turbines:

The hydraulic turbines are classified as follows:

(a) According to the head and quantity of water available.
(b) According to the name of the originator
(c) According to the action of water on moving blades
(d) According to the direction of flow of water in the runner.
(e) According to the disposition of thee turbine shaft
(f) According to the specific speed N.

6.2.1. According to the head and quantity of water available:

(i) Impulse turbine ….. requires high head and small quantity of flow

(ii) Reaction, turbine … requires low head and high rate of flow

Actually there are two types of reaction turbines, one for medium head and medium flow and the other for low head and large flow.

<table>
<tr><th colspan="2">Turbine</th><th rowspan="2">Type of Energy</th><th rowspan="2">Head</th><th rowspan="2">Discharge</th><th rowspan="2">Direction of flow</th><th rowspan="2">Specific Speed</th></tr>
<tr><th>Name</th><th>Type</th></tr>
<tr><td>Pelton Wheel</td><td>Impulse</td><td>Kinetic</td><td>High Head > 250m to 1000m</td><td>Low</td><td>Tangential to runner</td><td>Low
5 Single jet
35Multiple jet</td></tr>
<tr><td>Francis Turbine</td><td rowspan="2">Reaction Turbine</td><td rowspan="2">Kinetic + Pressure</td><td>Medium
60 m to
150 m</td><td>Medium</td><td>Radial flow
Mixed Flow</td><td>Medium
60 to 300</td></tr>
<tr><td>Kaplan Turbine</td><td>Low
< 30 m</td><td>High</td><td>Axial Flow</td><td>High
300 to 1000</td></tr>
</table>

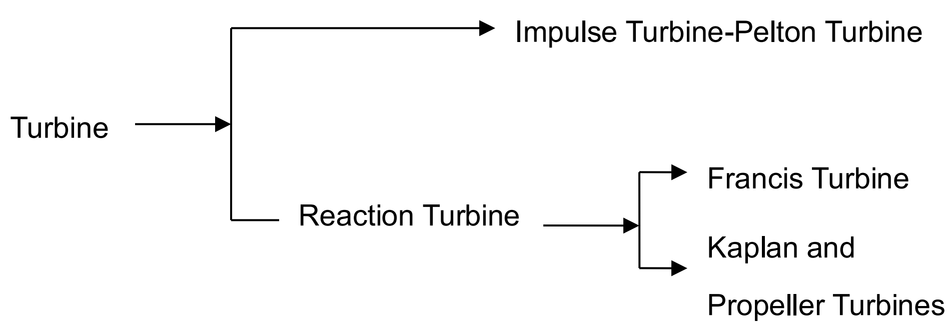
6.2.2. According to the name of the originator:

(i) Pelton turbine - Named after Allen Pelton of California (USA). It is an impulse type of turbine and is used for high head and low discharge.

(ii) Francis turbine - named after James Bichens Francis. It is a reaction type of turbine from medium high to medium low heads and medium small to medium large quantities of water.

(iii) Kaplan turbine - named after Dr.Victor Kaplan. It is a reaction type of turbine for low heads and large quantities of flow.

6.2.3. According to action of water on the moving blades:



6.2.4. According to direction of flow of water in the runner

(i) Tangential flow turbines (Pelton turbine)

(ii) Radial flow turbine (no more used)

(iii) Axial flow turbine (Kaplan turbine)

(iv) Mixed (radial and axial) flow turbine (Francis turbine).

In tangential flow turbine of Pelton type the water strikes the runner tangential to the path of rotation.

In axial flow turbine water flows parallel to the axis of the turbine shaft. Kaplan turbine is an axial flow turbine. In Kaplan turbine the runner blades are adjustable and can be rotated about pivots fixed to the boss of runner. If the runner blades of the axial flow turbines are fixed, these are called "Propeller turbines"

In mixed flow turbines the water enters the blades radially and comes out axially, parallel to the turbine shaft. Modern Francis turbines have mixed flow runners.

6.2.5 According to the disposition of the turbine shaft :

Turbine shaft may be either vertical or horizontal. In modern practice, Pelton turbines usually have horizontal shafts whereas the rest, especially the large units, have vertical shafts.

6.2.6 According to specific speed:

The specific speed of a turbine is defined as the speed of a geometrically similar turbine that would develop 1 KW under 1 m head. All geometrically similar turbines (irrespective of the size) will have the same specific speeds when operating under the same head.

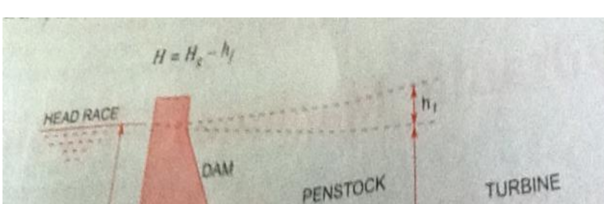
$$\text{Specific speed, } N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Where N = the normal working speed (rpm)

P = power output (Kw) of the turbine, and

H = the net or effective head in meters.

Turbines with low specific speeds work under high head and low discharge conditions, while high specific speed turbines work under low head and high discharge conditions.



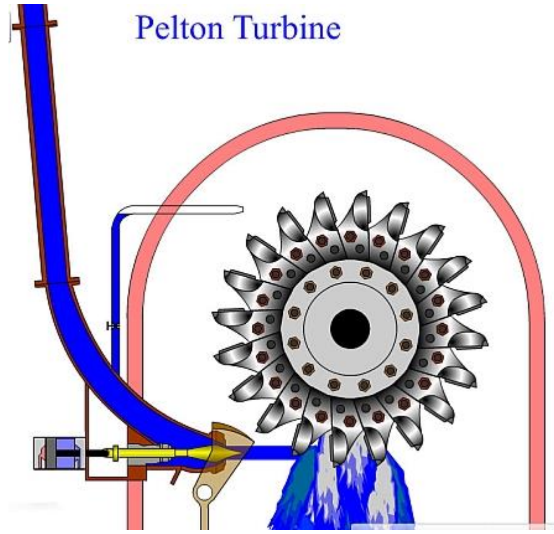


Fig. 6.1 General Lay-out of a Hydro-electric Plant

It consists of the following:

1. A ***Dam*** constructed across a river or a channel to store water. The reservoir is also known as ***Headrace.***

2. Pipes of large diameter called ***Penstocks*** which carry water under pressure from storage reservoir to the turbines. These pipes are usually made of steel or reinforced concrete.

3. ***Turbines*** having different types of vanes or buckets or blades mounted on a wheel called runner.

4. Tailrace which is channel carrying water away from the turbine after the water has worked on the turbines. The water surface in the tailrace is also referred to as tailrace.

Table 6.1 gives the comparison between the impulse and reaction turbines with regard to their operation and application.

Table - 6.1: Comparison between impulse and Reaction Turbines

SL No.	**Aspects**	**Impulse Turbine**	**Reaction turbine**

1	Conversion of fluid energy	The available fluid energy is converted into K.E by a nozzle.	The energy of the fluid is partly transformed in to K.E before it (fluid) enters the runner of the turbine.
2	Changes in pressure and velocity	The pressure remains same (atmospheric) throughout the action of water on the runner	After entering the runner with an excess pressure, water undergoes changes both in velocity and pressure while passing through the runner.
3	Admittance of water over the wheel	Water may be allowed to enter a part or whole of the wheel circumference.	Water is admitted over the circumference of the wheel
4	Water-tight causing	Required	Not necessary
5	Extent to which the water fills the wheel/turbine	The wheel/turbine does not run full and air has a free access to the buckets.	Water completely fills all the passages between the blades and while flowing between inlet and outlet sections does work on the blades.
6	Installation of Unit	Always installed above the tail race. No draft tube is used.	Unit may be installed above or below the tail race-use of a draft tube is made
7	Relative velocity of water	Either remaining constant or reduces slightly due to friction.	Due to continuous drop in pressure during flow through the blade, the relative velocity increases.
8	Flow regulation	• By means of a needle valve fitted into the nozzle. • Impossible without less	• By means of a guide – vane assembly. • Always accompanied by loss

6.3 Pelton wheel – components:

The Pelton wheel is an impulse turbine which is among the most efficient types of water turbines. It was invented by Lester Allan Pelton in the 1870s. **Pelton wheel is a high head turbine**. **It is used with heads of more than 300 m**. A head is the distance by which the water falls before it strikes the turbine blades the flow of water is tangential to the runner. So it is a tangential flow impulse turbine. A Pelton's runner consists of a single wheel mounted on a horizontal shaft. Waterfalls towards the turbine through a pipe called penstock and flows through a nozzle. The high speed jet of water coming out from the nozzle hits the buckets (vanes) on the wheel and causes the wheel to rotate producing torque and power. The Pelton wheel extracts energy from the impulse momentum of moving water as opposed to its weight like traditional overshot water wheel

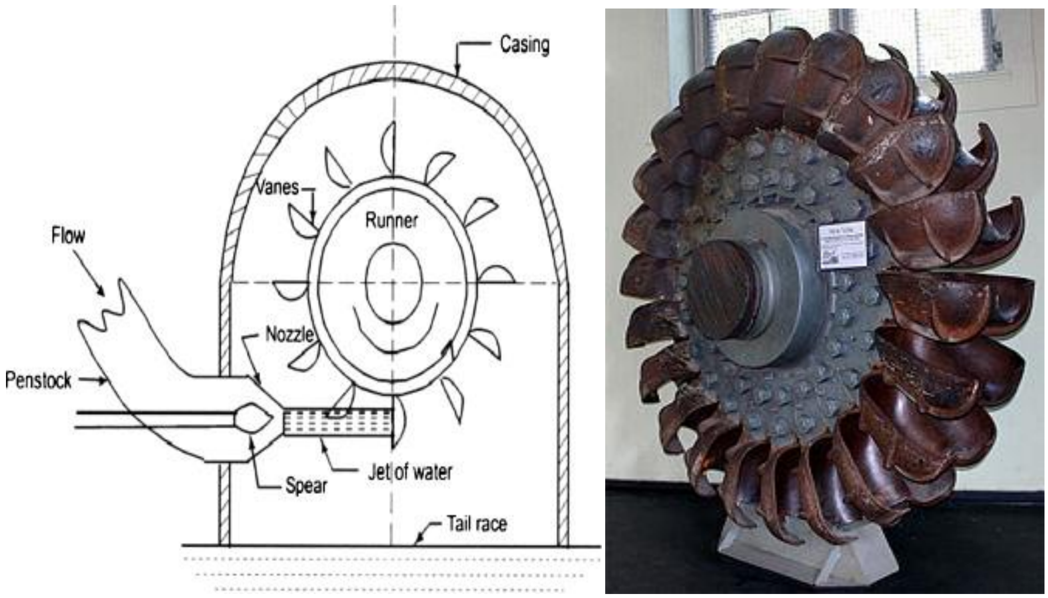


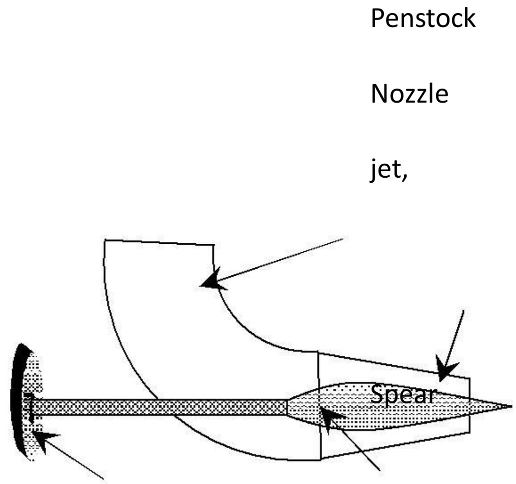
Fig. 6.2 Components of Pelton Wheel Turbine

The major parts of Pelton turbine identified in figure 6.2 are **penstock, nozzle, spear valve, runner, buckets and casing.** Penstock carries water from reservoir' to nozzle. The convergent nozzle is covert pressure energy of the incoming water into kinetic energy. Spear valve regulate the quantity of water striking the runner buckets. The spear valve is operated by a wheel in small machines or automatically by a governor in bigger machines. The high velocity water jet issuing out of the nozzle impinges on center of the double semi ellipsoidal shape bucket fixed evenly on the periphery of the runner. The jet of water impinges on the splitter, which divides the jet into two equal portions, each of which after flowing round the smooth inner surface of the bucket deflected through an angle of 160° to 170°. The advantage of double split bucket is that the axial thrust is equal and opposite, neutralize each other, and hence the bearings supporting the main shaft is not subjected any axial thrust. Further at the lower tip of the bucket a notch is cut which prevents the jet striking the preceding bucket and it also avoids the deflection of water towards the center of the runner. The impulsive force of the jet causes the wheel to rotate. The casing of Pelton turbine has no hydraulic function to perform. It prevents splashing of water and lead to tail race, support the main shaft and also safe guard against accidents.

The main components of a Pelton turbine are:

- ***Nozzle and flow regulating arrangement:***

Water is brought to the hydroelectric plant site through large penstocks at the end of which there will be a nozzle, which converts the pressure energy completely into Kinetic energy. This will convert the liquid flow into a high -speed which strikes the buckets or vanes mounted on the runner, Wheel which in turn rotates the runner of the turbine.



The amount of water striking the vanes is controlled by the forward and backward motion of the spear. As the water is flowing in the annular area between the annular area between the nozzle opening and the spear, the flow gets reduced as the spear moves forward and vice - versa.

- ***Runner with buckets:***

Runner is a circular disk mounted on a shaft on the periphery ofwhich a number of buckets are fixed equally spaced as shown in Fig . The buckets are made of cast -iron cast -steel, bronze or stainless steel depending upon the head at the inlet of the turbine. The water jet strikes the bucket on the splitter of the bucket and gets deflected through (α) 160 - 170 ^{0}.

- ***Casing:***

It is made of cast - iron or fabricated steel plates. The main function of the casing is to prevent splashing of water and to discharge the water into tailrace.

- ***Breaking jet:***

Even after the amount of water striking the buckets is completely y stopped, the runner goes on rotating for a very long time due to inertia. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of bucket with which the rotation of the runner is reversed. This jet i s called as breaking jet.

Applications of Pelton Wheel: The Pelton wheels are the preferred turbine for hydro-power, when the available water source has relatively high hydraulic head at low flow rates. Pelton wheels are made in all sizes. There exist multi-ton Pelton wheels mounted on vertical oil pad bearings in hydroelectric plants. The largest units can be up to 200 megawatts. The smallest Pelton wheels are only a few inches across, and can be used to tap power from mountain streams having flows of a few gallons per minute. Some of these systems utilize household plumbing fixtures for water delivery. These small units are recommended for use with thirty meters or more of head, in order to generate significant power levels. Depending on water flow and design, Pelton wheels operate best with heads from 15 meters to 1,800 meters, although there is no theoretical limit.

Thus, more power can be extracted from a water source with high-pressure and low-flow than from a source with low-pressure and high-flow, even though the two flows theoretically contain the same power. Also a comparable amount of pipe material is required for each of the two sources, one requiring a long thin pipe, and the other a short wide pipe.

6.4 Working Principle of Pelton Turbine:High speed water jets emerging from the nozzles (obtained by expanding high pressure water to the atmospheric pressure in the nozzle) strike a series of spoon-shaped buckets mounted around the edge of the Pelton wheel. High pressure water can be obtained from any water body situated at some height or streams of water flowing down the hills.

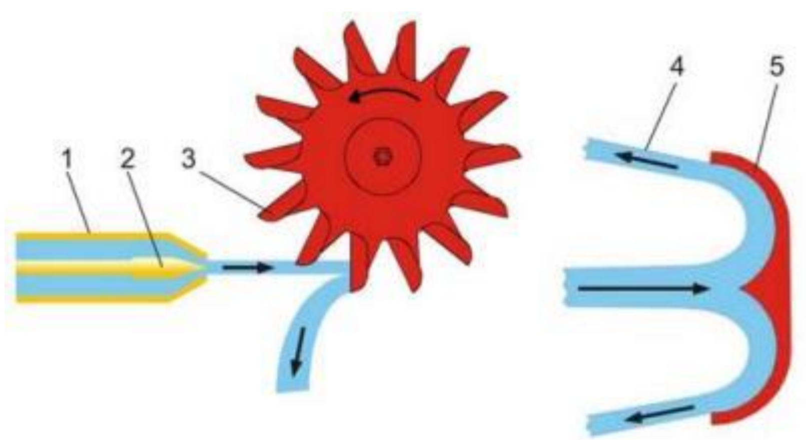


Fig. 6.3 Working of Pelton Wheel Turbine

As water flows into the bucket, the direction of the water velocity changes to follow the contour of the bucket. These jets flow along the inner curve of the bucket and leave it in the direction opposite to that of incoming jet. When the water-jet contacts the bucket, the water exerts pressure on the bucket and the water is decelerated as it does a "u-turn" and flows out the other side of the bucket at low velocity.

The change in momentum (direction as well as speed) of water jet produces an impulse on the blades of the wheel of Pelton Turbine. This "impulse" does work on the turbine and generates the torque and rotation in the shaft of Pelton Turbine.

To obtain the optimum output from the Pelton Turbine the impulse received by the blades should be maximum. For that, change in momentum of the water jet should be maximum possible. This is obtained when the water jet is deflected in the direction opposite to which it strikes the buckets and with the same speed relative to the buckets.

For maximum power and efficiency, the turbine system is designed such that the water-jet velocity is twice the velocity of the bucket. A very small percentage of the water's original kinetic energy will still remain in the water. However, this allows the bucket to be emptied at the same rate at which it is filled, thus allowing the water flow to continue uninterrupted.

Often two buckets are mounted side-by-side, thus splitting the water jet in half. The high speed water jets emerging from the nozzles strike the buckets at splitters, placed at the middle of the buckets, from where jets are divided into two equal streams.

This balances the side-load forces on the wheel, and helps to ensure smooth, efficient momentum transfer of the fluid jet to the turbine wheel.

Because water and most liquids are nearly incompressible, almost all of the available energy is extracted in the first stage of the hydraulic turbine. Therefore, Pelton wheels have only one turbine stage, unlike gas turbines that operate with compressible fluid.

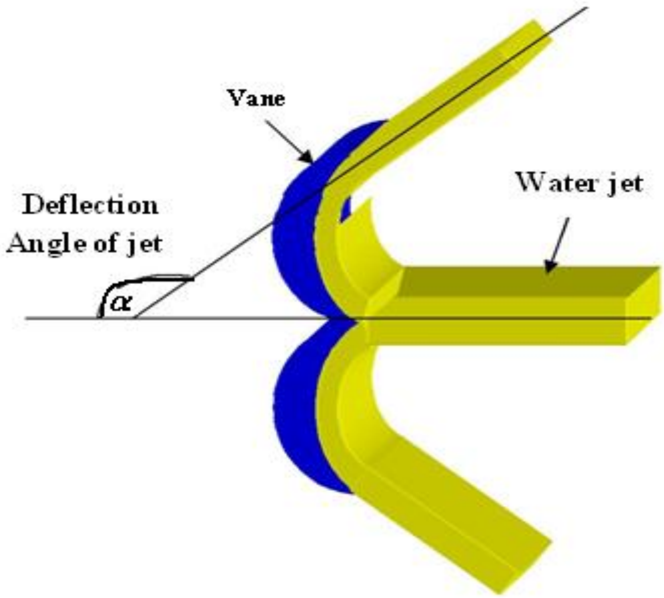


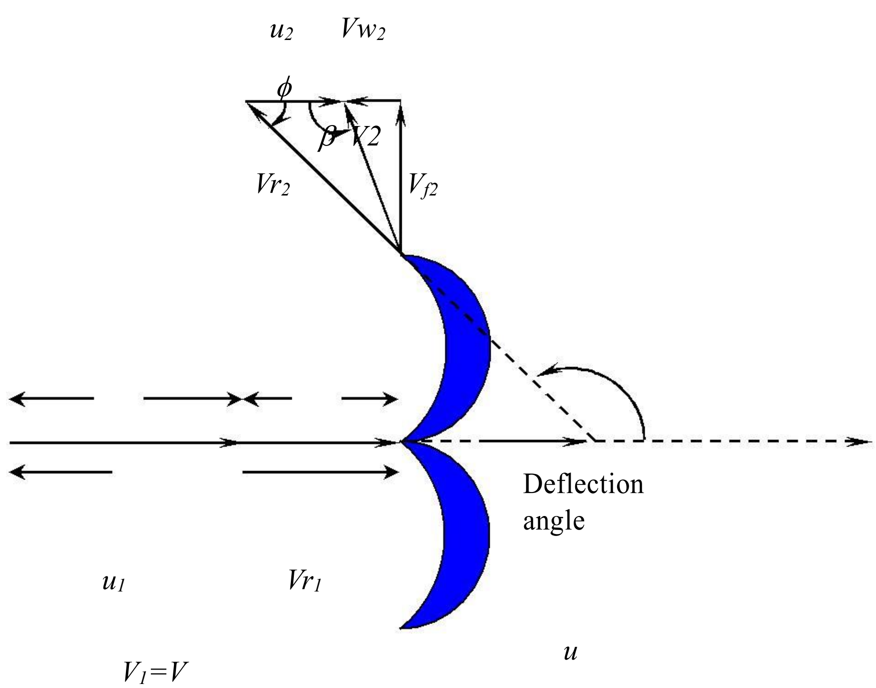
Fig. 6.4 3- D Picture of a jet striking the splitter and getting split in to two parts

6.5 Velocity triangles, Maximum power and Efficiency:

A typical velocity triangle for a Pelton wheel bucket is shown in Fig. 6.5. From the impulse - momentum theorem, the force with which the jet strikes the bucket along the direction of vane is given by,

F_x = rate of change of momentum of the jet along the direction of vanemotion

F_x = (Mass of water / second) × change in velocity along the *x*-direction



w_1

$V_{f1}=0$

Fig. 6.5 Velocity Triangles for the jet striking the Pelton Wheel Bucket

$$F_x = \rho a V_1 \left[V_{w1} - \left(-V_{w2} \right) \right]$$

$$F_x = \rho a V_1 \left[V_{w1} + V_{w2} \right]$$

Work done per second b y the jet on the vane is given b y the product of Force exerted on the vane and the distance moved b y the vane in one second

W . D . /Sec = $F_x \times u \; = \rho a V_1 \times \left[V_{w1} + V_{w2} \right] \times u$

Input to the jet per second = Kinetic energy of the jet per second

- $\frac{1}{2} \rho a V_1^3$

Efficiency of the jet $= \dfrac{Output / \text{second}}{Input / \text{second}} = \dfrac{Work\ done / \text{second}}{Input / second}$

$$\eta = \frac{\rho a V_1 \left[V_{w1} + V_{w2} \right] \times u}{\frac{1}{2} \rho a V_1^3}$$

$$\eta = \frac{2u \left[V_{w1} + V_{w2} \right]}{V_1^2}$$

From inlet velocity triangle, ***$V_{w1} = V_1$***

Assuming no shock and ignoring frictional losses through the vane, we have

$$V_{r1} = V_{r2} = (V_1 - u_1)$$

In case of Pelton wheel, the inlet and outlet are located at the same radial distance from the centre of runner and hence $u_1 = u_2 = u$

From outlet velocity triangle, we have $V_{w2} = V_{r2} \cos\phi - u_2 = (V_1 - u) \cos\phi - u$

$$F_x = \rho a V_1 \left[V_1 + (V_1 - u) \cos\phi - u \right]$$

$$F_x = \rho a V_1 (V_1 - u) \left[1 + \cos\phi \right]$$

Substituting these values in the above equation for efficiency, we have

$$\eta = \frac{2u \left[V_1 + (V_1 - u) \cos\phi - u \right]}{V_1^2}$$

$$\eta = \frac{2u \left[(V_1 - u) + (V_1 - u) \cos\phi \right]}{V_1^2}$$

$$= \frac{2u}{V_1^2} (V_1 - u) \left[1 + \cos\phi \right]$$

The above equation gives the efficiency of the jet striking the vane in case of Pelton wheel. To obtain the maximum efficiency for a given jet velocity and vane angle, from maxima -minima, we have

$$\frac{d\eta}{du} = 0$$

$$\Rightarrow \frac{d\eta}{du} = \frac{2}{V_1^2} \left[1 + \cos\phi \right]$$

$V_1 - 2u = 0$

$$U = \frac{V_1}{2}$$

When the bucket speed is maintained at half the velocity of the jet, the **efficiency of a Pelton wheel will be maximum Substituting we get,**

$\eta_{max} = [1 + \cos\phi] / 2$

From the above it can be seen that more the value of $\cos\phi$, more will be the efficiency. Form maximum efficiency, the value of $\cos\phi$ should be 1 and the value of ϕ should be 0^0. This condition makes the jet to completely deviate by 180^0 and this, forces the jet striking the bucket to strike the successive bucket on the back of it acting like a breaking jet. Hence to avoid this situation, at least a small angle of $\phi = 5^0$ should be provided.

6.6 Working proportions (Pelton wheel design):

1. Power from the turbine shaft (P)

$$P = (\gamma_w Q H)\eta_o$$

2. Theoretical velocity of jet

$$V_{th} = K_v\sqrt{2gH}$$

$$Coeffcient\ of\ Velocity(K_v) = 0.97\ to\ 0.99$$

3. The total discharge through the Pelton wheel must be equal to the discharge through the number of jets (n): $Q = n\frac{\pi}{4}d^2V$

4. Speed Ratio (Ku) or Peripheral Velocity: Represents the ratio of peripheral (linear) velocity of jet bucket at their mean diameter to the theoretical velocity

$$K_u = \frac{u}{\sqrt{2gH}}; \text{ Normally speed ratio has average value } 0.45 \text{ to } 0.47$$

5. Mean Diameter of the Pelton Wheel 'D' measured up to the centers of the buckets

$$u = \frac{\pi DN}{60} \text{ The mean diameter } (D) \text{ is also known as Pitch Diameter}$$

6. The jet ratio is defined as the ratio of diameters of Runner and jet diameter

$$m = \frac{D}{d}; \text{ The jet ratio } (m) \text{ lies between } 11 \text{ and } 15$$

7. Number of jets in a Pelton wheel: Normally one but to increase the power generation more number of nozzles are provided maximum up to 4

8. Working Proportions: The working proportions of the turbine buckets are generally in terms of jet diameter 'd'

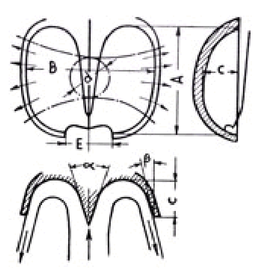
Axial Width **B** = 3d to 4d: Radial Length **A** = 2d to 3d : Depth **C** = 0.8d to 1.2d

9. Number of Buckets (Z): The number of buckets on the periphery is decided mainly on the two principles:

- The No. of buckets should be as few as possible so that there is little loss due to friction
- The jet of water must be fully utilised so that no water from the jet goes waste i.e. no water escapes without striking the buckets

10. The No. of desirable Number of Buckets (Z) is calculated by two relations

$$(i)\ Z = 0.5m + 15$$
$$(ii)\ Z = 5.4\sqrt{m}$$



6.7 Solved Example:

Q.1 The head at the base of the nozzle of a Pelton wheel is 640 m. The outlet vane angle of the bucket is 15 ^{o}. The relative velocity y at the outlet is reduced by 15% due to friction along the vanes. If the discharge at outlet is without whirl find the ratio of bucket speed to the jet speed. If the jet diameter is 100 mm while the wheel diameter is 1. 2 m, find the speed of the turbine in rpm, the force exerted by the jet on the wheel, the Power developed and the hydraulic efficiency. Take C_v=0.97.

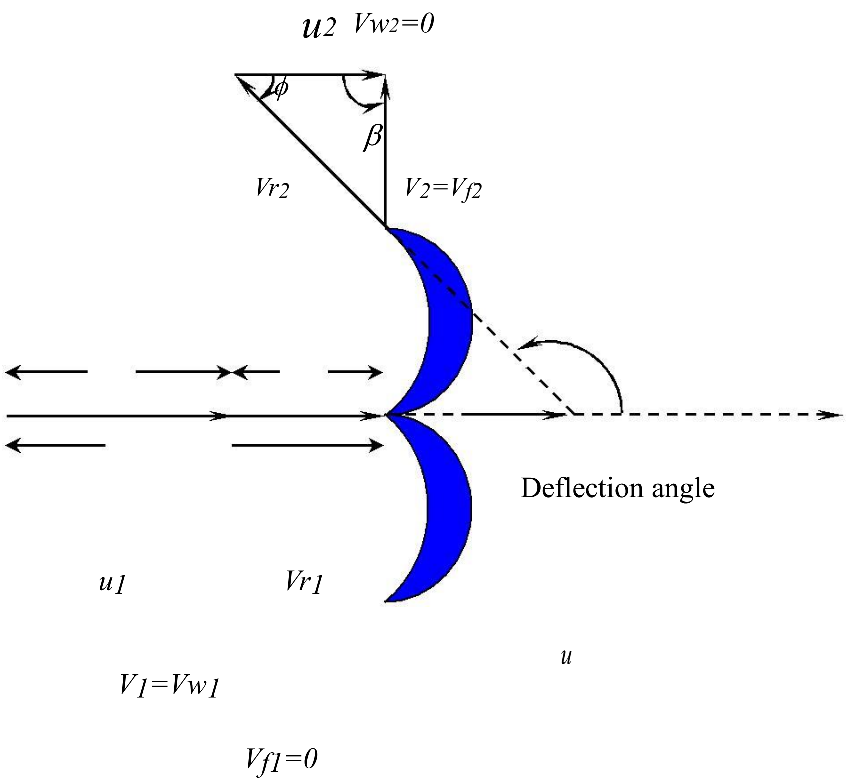
Solution:

$H = 640$ m; $\phi=15^{o}$; $V_{r1} = 0.85\,V_{r2}$; $V_{w2} = 0$; $d = 100$ mm; $D = 1.2$ m;

$C_v = 0.97$; $K_u = ?$; $N = ?$; $F_x = ?$; $P = ?$; $\eta_h = ?$

We know that the absolute velocity of jet is given b y

$V = C_v.(2\,g\,H)^{0.5} = 0.97(2\times10\times640)^{0.5} = 109.74$ m/s



Let the bucket speed be u

Relative velocity at inlet = $V_{r1} = V_1 - u = 109.74 - u$

Relative velocity at outlet = $V_{r2} = (1 - 0.15)V_{r1} = 0.85(109.74 - u)$ But $V_{r2}\cos\phi = u \Rightarrow 0.85(109.74 - u)\cos 15$

Hence $u = 49.48$ m/s

But $u = \frac{\pi D N}{60}$ and hence

$$N = \frac{60\,u}{\pi D} = \frac{60 \times 49.48}{\pi \times 1.2} = 787.5 \text{ rpm (Ans)}$$

Jet ratio = $m = \frac{u}{V} = \frac{49.48}{109.74} = 0.45$

Weight of water supplied = $\gamma Q = 10 \times 1000 \times \frac{\pi}{4} \times 0.1^2 \times 109.74^2 = 8.62$ kN/s

Force exerted = $F_x = \rho a V_1 \left(V_{w1} - V_{w2}\right)$

But $V_{w1} = V_1$ and $V_{w2} = 0$ and hence

$$F_x = 1000 \times \frac{\pi}{4} \times 0.1^2 (109.74)^2 = 94.58 \text{ kN}$$

Work done/second = $F_x x u$ = 94. 58 x 49. 48 = 4679. 82 kN/s

Kinetic Energy/second = $\frac{1}{2} \rho a V_1^3 = \frac{1}{2} \times 1000 \times \frac{\pi}{4} \times 0.1^2 \times 109.74^3 = 5189.85$ kN/s

hydraulic Efficiency = $\eta_h\% = \frac{\text{Work done/s}}{\text{Kinetic Energy/s}} = \frac{4679.82}{5189.85} \times 100 = 90.17\%$

Q. 2 A Pelton wheel turbine is having a mean runner diameter of 1. 0 m and is running at 1000 rpm. The net head is 100 m. If the side clearance is 20° and discharge is 0.1 m^3/s, find the power available at the nozzle and hydraulic efficiency of the Turbine

Solution:

D = 1. 0 m; N = 1000 rpm; H = 100 m; $\phi = 20^o$; Q = 0. 1 m^3/s; WD/s =? and η_h = ?

Assume C_v= 0. 98

We know that the velocity of the jet is given by

$$V = C_v \sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 100} = 44.3 m/s$$

The absolute velocity of the vane is given by,

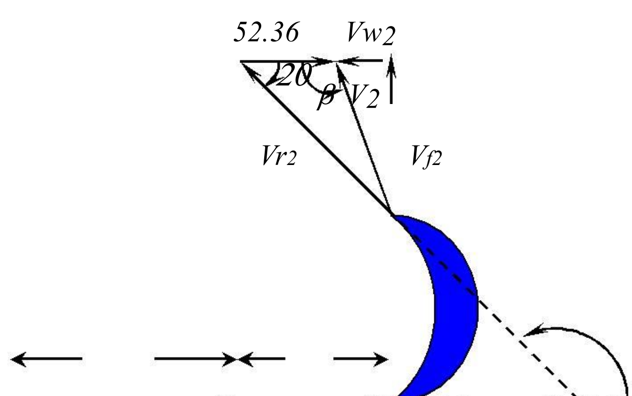
$$u = \frac{\pi D N}{60} = \frac{\pi \times 1 \times 1000}{60} = 52.36 \text{ m/s}$$

This situation is impracticable (**V<u) and** hence the data has to be modified. Clearly state the assumption as follows:

Assume $H = 700$ m

Absolute velocity of the jet is given by

$V = C_v \sqrt{2\,g\,H} = 0.98\ \sqrt{2 \times 10 \times 700} = 115.96$ m/s



Deflection angle

52.36 *Vr1*

u

V1=115.96

Vf1=0

Power available at the nozzle is the given b y work done per second WD/second $P = \gamma Q H = \rho g Q H$ = 1000 x 10x0. 1x700 **= 700 kW**

Hydraulic Efficiency is given b y

$$\eta_h = \frac{2u}{V_1^2}(V-u)\left[1+\cos\phi\right] = \frac{2\times 52.36}{115.96^2}\mathbf{(115.96-52.36)(1+\cos 20) = 96.07\%}$$

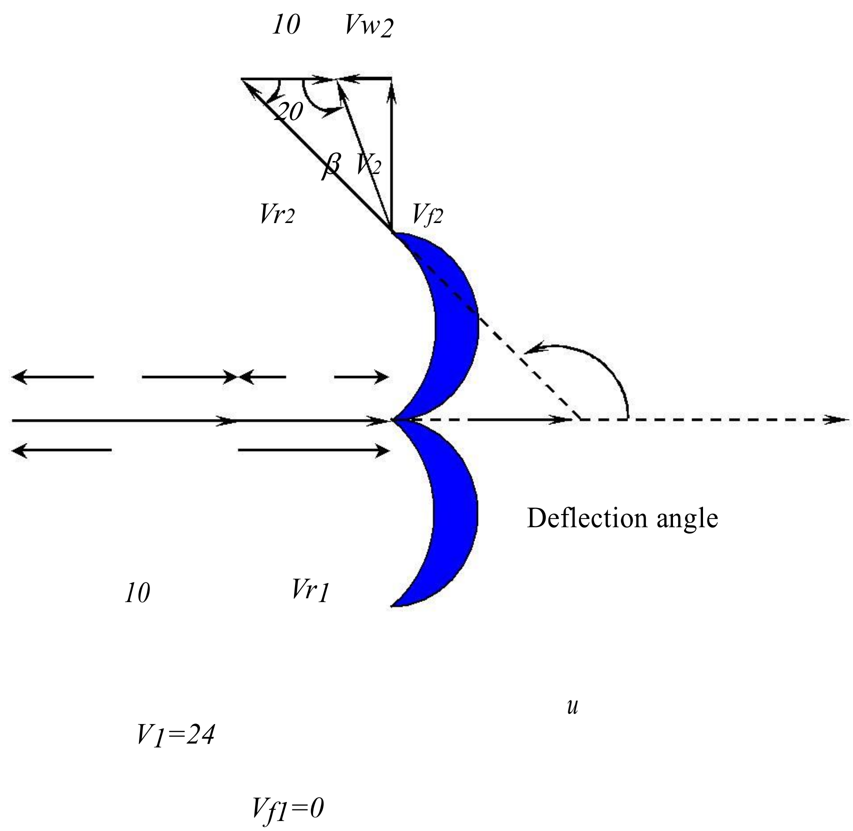
Q.3 A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 lps under a head of 30 m. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume the coefficient of nozzle as 0.98.

Solution:

u = 10 m/s; Q = 0.7 m^3/s; ϕ = 180 - 160 = 20^o; H = 30 m; C_v = 0 . 98;

WD/s =? and η_h = ? Assume g = 10 m/s^2

$$V = C_v\sqrt{2gH} = 0.98\sqrt{2\times 10\times 30} = 24 \text{ m/s}$$



$V_{r1} = V_1 - u = 24 - 10 = 14$ m/s

Assuming no shock and frictional losses we have $V_{r1} = V_{r2} = 14$ m/s

$V_{w2} = V_{r2}\ Cos\ \phi - u = 14$ x $Cos\ 20 - 10 = 3.16$ m/s

We know that the Work done b y the jet on the vane is given by

WD/s $= \rho a V_1 \left[V_{w1} + V_{w2} \right] u = \rho Q\, u \left[V_{w1} + V_{w2} \right]$ as $Q = aV_1$

$= 1000 \times 0.7 \times 10 \left[24 + 3.16 \right] = 190.12$ kN -m/s (Ans)

$$IP/s = KE/s = \frac{1}{2}\rho a V_1^3 = \frac{1}{2}\rho Q V_1^2 = \frac{1}{2}\times 1000 \times 0.7 \times 24^2 = 201.6 \text{ kN -m/s}$$

Hydraulic Efficiency = Output/ Input = 190.12/201. 6 = 94. 305%

It can also be directly calculated by the derived equation,

$$\eta_h = \frac{2u}{V_1^2}(V-u)[1+\cos\phi] = \frac{2\times 10}{24^2}(24-10)[1+\cos 20] = 94.29\% \text{ (Ans)}$$

Q.4 A Pelton wheel has to develop 13230 kW under a net head of 800 m while running at a speed of 600 rpm. If the coefficient of Jet $Cv = 0.97$, speed Ratio $\phi = 0.46$ and the ratio of jet Diameter (d) to wheel Diameter (D) = 1/16

i) Pitch circle diameter (D)

ii) The diameter of jet (d)

iii) The quantity of water supplied to the wheel (Q)

iv) The number of Jets required(n)

Assume overall efficiency as 85%.

Solution:

P = 13239 kW; H = 800 m; N = 600 rpm; $C_V = 0.97$; $\phi = 0.46$ (Speed ratio), $\eta_o = 85\%$

$d/D = 1/16$; $\eta_o = 0.85$; D =? d =? n =?

Assume $g = 10 \text{ m/s}^2$ and $\rho = 1000 \text{ kg/m}^3$

We know that the overall efficiency is given by

$$\eta_o = \frac{Output}{Input} = \frac{P}{\gamma Q H} = \frac{13239 \times 10^3}{10 \times 1000 \times Q \times 800} = 0.85$$

Hence $Q = 1.947 \text{ m}^3/\text{s}$ (Ans)

Absolute velocity of jet is given b y

$$V = C_v \sqrt{2gH} = 0.97\sqrt{2\times10\times800} = 122.696 \text{ m/s}$$

Absolute velocity of vane is given b y

$$u = \phi\sqrt{2gH} = 0.46\sqrt{2\times10\times800} = 58.186 \text{ m/s}$$

The absolute velocity of vane is also given b y

$u = \frac{\pi D N}{60}$ and hence

$$D = \frac{60u}{\pi N} = \frac{60\times58.186}{\pi\times600} = \mathbf{1.85\ m\ (Ans)}$$

$$d = \frac{1.85}{16} = \mathbf{115.\ 625\ mm\ (Ans)}$$

Discharge per jet = $q = \frac{\pi}{4}d^2\times V = \frac{\pi}{4}\times0.115625^2\times122.696$ $\mathbf{=1.288\ m^3/s}$

No . of jets = $n = \frac{Q}{q} = \frac{1.947}{1.288} \approx \mathbf{2}$ **(Ans)**

Q.5 Design a Pelton wheel for a head of 80m and speed of 300 RPM. The Pelton wheel develops 110 kW. Take co - efficient of velocity y= 0. 98, speed ratio= 0.48 and overall efficiency = 80%.

Solution:

H = 80 m; N = 300 rpm; P = 110 kW; C_v = 0. 98, K_u=0. 48; $\eta_o = 0.80$

Assume g = 10 m/s ^{2} and ρ = 1000 kg/m ^{3}

We know that the overall efficiency is given by

$$\eta_o = \frac{Output}{} = \frac{P}{} = \frac{110\times10^3}{} = 0.8$$

$Input \quad \gamma Q H \quad 10 \times 1000 \times Q \times 80$

Hence Q = 0. 171875 m^3/s

Absolute velocity of jet is given b y

$V = C_V \sqrt{2gH} = 0.98\sqrt{2 \times 10 \times 80} = 39.2$ m/s

Absolute velocity of vane is given by $u = \phi\sqrt{2\boldsymbol{gH}} = 0.48\sqrt{2 \times 10 \times 80}$

U = 19.2 m/s

The absolute velocity of vane is also given by,

$u = \frac{\pi D N}{60}$ and hence

$$D = \frac{60\,u}{\pi N} = \frac{60 \times 19.2}{\pi \times 300} = 1.22\ m \quad \text{(Ans)}$$

Single jet Pelton turbine is assumed

The diameter of jet is given by the discharge continuity equation for Q = 0.171875m^3/s

$$\boldsymbol{Q} = 0.171875 = \frac{\pi}{4} \times \boldsymbol{d}^2 \times \boldsymbol{V} = \frac{\pi}{4} \times \boldsymbol{d}^2 \times 39.2$$

Hence ***d* = 74. 72 mm**

The design parameters are

Single jet

Pitch Diameter = 1. 22 m

Jet diameter = 74. 72 mm

$D \qquad 1.22$

Jet Ratio = $m = \frac{}{d} = \frac{}{0.07472} = 16.33$

No. of Buckets = 0. 5x m + 15 = 23.165 = 24

Q.6 It is desired to generate 1000 kW of power and survey reveals that 450m of static head and a minimum flow of 0.3m^3/s are available. Comment whether the task can be accomplished by installing a Pelton wheel run at 1000 rpm and having an overall efficiency of 80%. Design the Pelton wheel assuming suitable data for coefficient of velocity and coefficient of flow.

Solution:

P = 1000 kW; H = 450 m; Q = 0. 3 m^3/s; N = 1000 rpm; η_o = 0. 8

Assume C_v= 0. 98; K_u = 0.45; ρ = 1000 kg/m^3 ; g = 10 m/s^2

$$\eta_o = \frac{Output}{Input} = \frac{P}{\gamma Q H} = \frac{1000 \times 10^3}{10 \times 1000 \times 0.3 \times 450} = 0.74 = 74\%$$

For the given conditions of P, Q and H, it is not possible to achieve the desired efficiency of 80%. To decide whether the task can be accomplished by a Pelton turbine computes the specific speed

$$\boldsymbol{N_s = \frac{N\sqrt{P}}{H^{5/4}}}$$

Where, 'N' is the speed of runner, 'P' is the power developed in 'kW' and 'H' is the head available in 'm' at the inlet.

$$N_s = \frac{1000\sqrt{1000}}{(450)^{1.25}} = 15.25 < 35$$

Hence the installation of single jet Pelton wheel is justified.

Absolute velocity of jet is given by,

$V = C_v \sqrt{2gH} = 0.98\sqrt{2 \times 10 \times 450} = 92.97$ m/s

Absolute velocity of vane is given by $u = \phi\sqrt{2gH} = 0.48\sqrt{2 \times 10 \times 450} = 45.54$ m/s

The absolute velocity of vane is also given b y

$u = \dfrac{\pi D N}{60}$ and hence

$$D = \frac{60\,u}{\pi N} = \frac{60 \times 19.2}{\pi \times 300}$$ **=1.22 *m* (Ans)**

Single jet Pelton turbine is assumed

The diameter of jet is given by the discharge continuity equation $Q = \dfrac{\pi}{4} d^2 \times V = \dfrac{\pi}{4} \times d^2 \times 92.97 \Rightarrow 0.3$

Hence *d* = 64.1 mm

The design parameters are

Single jet

Pitch Diameter = 1. 22 m

Jet diameter = 64.1 mm

Jet Ratio = $m = \dfrac{D}{d} = \dfrac{1.22}{0.0641} = 19.03$

No. of Buckets = 0.5× m + 15 = 0.5× 19.03+5 **= 24.5=25 buckets**

Q.7 A double jet Pelton wheel develops 895 MKW with an overall efficiency of 82% under a head of 60m. The speed ratio= 0.46, jet ratio 'm'= 12 and the nozzle coefficient C_v = 0. 97. Find the jet diameter, wheel diameter and wheel speed in RPM

Solution:

No.of jets = $n = 2$; $P = 895$ kW; $\eta_o = 0.82$; $H = 60$ m; $K_u = 0.46$; $m = 12$;

$C_v = 0.97$; D =? d =? N =?

We know that the absolute velocity of jet is given by

$$V = C_v \sqrt{2\,g\,H} = 0.97\sqrt{2 \times 10 \times 60} = 33.6 \text{ m/s}$$

The absolute velocity of vane is given b y

$$u = K_u \sqrt{2\,g\,H} = 0.46\sqrt{2 \times 10 \times 60} = 15.93 \text{ m/s}$$

Overall efficiency is given b y

$$\eta_o = \frac{P}{\gamma Q H} \text{ and hence } Q = \frac{P}{\gamma \eta H} = \frac{895 \times 10^3}{10 \times 10^3 \times 0.82 \times 60} = 1.819 \text{ m}^3/\text{s}$$

Discharge per jet = $q = \dfrac{Q}{n} = \dfrac{1.819}{2} = 0.9095 \text{ m}^3/\text{s}$

From discharge continuity y equation, discharge per jet is also given by

$$q = \frac{\pi d^2}{4} V = \frac{\pi d^2}{4} \times 33.6 \Rightarrow 0.9095$$

d* =0.186 *m

The jet ratio $m = 12 = \dfrac{D}{\ }$

d

Hence D = 2. 232 m

Also $u = \dfrac{\pi D N}{60}$ and hence $N = \dfrac{60\,u}{\pi D} = \dfrac{60 \times 15.93}{\pi \times 2.232}$ **=136 rpm**

Note: Design a Pelton wheel: Width of bucket = 5d and depth of bucket is 1.2 d

Q.8 The following data is related to a Pelton wheel:

Head at the base of the nozzle = 80m; Diameter of the jet = 100 mm; Discharge of the nozzle = 0. 3m ^{3}/s; Power at the shaft = 206 kW; Power absorbed in mechanical resistance = 4. 5kW. Determine (i) Power lost in the nozzle and (ii) Power lost due to hydraulic resistance in the runner.

Solution:

H = 80 m; d = 0. 1m; $a = \frac{1}{4}\pi d^2$ = 0.007854 m^2; Q= 0.3 m^3/s; SP = 206kW; Power absorbed in mechanical resistance = 4.5 kW.

From discharge continuity equation, we have, $Q = a \times V$ = 0.007854 x V ⇨0.3

V = 38.197 m/s

Power at the base of the nozzle = $\rho g\, Q\, H$

= 1000 x 10 x 0.3 x 80 = 240 kWPower corresponding to the kinetic energy of the jet = $\frac{1}{2}\rho a\, V^3$ = 218.85 kW

(i) Power at the base of the nozzle = Power of the jet + Power lost in the nozzle

Power lost in the nozzle = (240 — 218.85) = **21.15 kW (Ans)**

(ii) Power at the base of the nozzle = Power at the shaft + Power lost in the

(Nozzle + runner + due to mechanical resistance)

Power lost in the runner = 240 – (206 + 21.15 + 4.5) = **5.35 kW** (Ans)

Q.9 The water available for a Pelton wheel is $4 m^3/s$ and the total head from reservoir to the nozzle is 250 m. The turbine has two runners with two jets per runner. All the four jets have the same diameters. The pipeline is 3000 m long and the efficiency if power transmission through the pipeline and the nozzle is 91%. The efficiency of each runner is 90%. The velocity coefficient of each nozzle is 0.975 and coefficient of friction '*4f*' for the pipe is 0.0045. Determine: (i) The power developed by the turbine; (ii) The diameter of the jet and (iii) The diameter of the pipeline.

Solution: Given : $Q = 4\ m^3/s$; $H_g = 250$ m; No. of jets = $n = 2 \times 2 = 4$; Length of pipe = $l = 3000$ m;Efficiency of the pipeline and the nozzle = 0.91 and Efficiency of the runner =

$\eta_h = 0.9$; $C_v = 0.975$; $4f = 0.0045$

Efficiency of power transmission through pipelines and nozzle

$$\eta = \frac{H_g - h_f}{H_g} \Rightarrow 0.91 = \frac{250 - h_f}{250}$$

Hence $h_f = 22.5$ m

Net head on the turbine = $H = H_g - h_f = 227.5$ m

Velocity of jet = $V_1 = C_v \sqrt{2gH} = 0.975\sqrt{2 \times 10 \times 227.5} = 65.77$ m/s

(i) Power at inlet of the turbine = WP = Kinetic energy/second = $\frac{1}{2}\rho a V^3$

WP = ½x 4 x 65.77^2= 8651.39 kW

$$\eta_h = \frac{\text{Power developed by turbine}}{\text{WP}} = \frac{\text{Power developed by turbine}}{8651.39} \Rightarrow 0.9$$

Hence power developed by turbine = 0.9 x 8651.39 = **7786.25 kW (Ans)**

(ii) Discharge per jet = $q = \dfrac{\text{Total discharge}}{\text{No. of jets}} = \dfrac{4.0}{4} = 1.0\ m^3/s$

But $q = \frac{\pi}{4} d^2 \times V_1 \quad \Rightarrow 1.0 = \frac{\pi}{4} d^2 \times 65.77$

4 4

Diameter of jet = ***d* = 0.14 m (Ans)**

(iii) If D is the diameter of the pipeline, then the head loss through the pipe is given by = h_f

$$h_f = \frac{4fLV^2}{2gD} = \frac{fLQ^2}{3D^5}$$ (From $Q=aV$)

$$h_f = \frac{0.0045 \times 3000 \times 4^2}{3D^5} \Rightarrow 22.5$$

Hence *D* = 0.956 m (Ans)

Q.10 The three jet Pelton wheel is required to generate 10,000 kW under a net head of 400 m. The blade at outlet is 15^{o} and the reduction in the relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, C_v = 0.98 and the speed ratio = 0.46, then find: (i) the diameter of the jet, (ii) total flow (iii) the force exerted by a jet on the buckets (iv) The speed of the runner.

Solution:

No of jets = 3; Total Power P = 10,000 kW; Net head H = 400 m; Blade angle = ϕ = 15^{o}; V_{r2}= 0.95 V_{r1}; Overall efficiency = η_o = 0.8; C_v = 0.98; Speed ratio = K_u = 0.45; Frequency = f = 50 Hz/s.

We know that $\eta_o = \frac{P}{\rho g Q H} \Rightarrow 0.8 = \frac{10{,}000 \times 10^3}{1000 \times 10 \times Q \times 400}$

***Q* = 3.125 m^3/s (Ans)**

Discharge through one nozzle = ***q*=** $\frac{Q}{n} = \frac{3.125}{3} = 1.042\ m^3/s$

Velocity of the jet = V_1 $= C_v\sqrt{2gH} = 0.98\sqrt{2 \times 10 \times 400} = 87.65\ m^3/s$

But $q = 1.042 = (\pi/4)d^2 \times V_1 \Rightarrow (\pi/4)d^2 \times \mathbf{87.65}$

***d* = 123 mm (Ans)**

Velocity of the Vane = $u = K_u \sqrt{2gH} = 0.46\sqrt{2 \times 10 \times 400} = 41.14\ \text{m}^3/\text{s}$

$V_{r1} = (V_1 - u_1)$ =87.65–41.14 = 46.51 m/s $V_{r2} = 0.95\ V_{r1}$= 0.95 x 46.51 = 44.18 m/s

$V_{w1} = V_1$= 87.65 m/s

$V_{w2} = V_{r2}\cos\phi - u_2$= 44.18 cos 15–41.14 = 1.53 m/s

Force exerted by the jet onthe buckets $F_x = \rho q\,(V_{w1}+V_{w2})$

F_x = 1000 x 1.042 (87.65+1.53) **= 92.926 kN (Ans)**

Jet ratio = $m = \dfrac{D}{d} \Rightarrow 10$ (Assumed)

***D* = 1.23 m**

$$u = \frac{\pi D N}{60}$$

Hence $N = \dfrac{60\,u}{\pi D} = \dfrac{60 \times 41.14}{\pi \times 1.23}$ =**638.8 rpm (Ans)**

Force exerted by the jet onthe buckets $F_x = \rho q\,(V_{w1}+V_{w2})$

F_x = 1000 x 1.042 (87.65+1.53) **= 92.926 kN (Ans)**

Jet ratio = $m = \dfrac{D}{d} \Rightarrow 10$ (Assumed)

***D* = 1.23 m**

$$u = \frac{\pi D N}{60}$$

Hence $N = \dfrac{60\,u}{} = \dfrac{60 \times 41.14}{}$ =**638.8 rpm (Ans)**

πD $\pi \times 1.23$

UNIT-7: KAPLAN TURBINES

7.1 Introduction:

In Francis turbines as the specific speed (N_s) increases (more due to increased discharge 'Q') the shape of the runner changes so that the flow tends towards axial direction. **This trend when continued, the runner becomes purely axial flow type.** There are many locations where large flows are available at low head. In such a case the specific speed increases to a higher value. In such situations Kaplan Turbine (axial flow turbines) are gainfully employed.

A sectional view of a kaplan turbines in shown in figure 14.8.1. These turbines are suited for head in the range 5 – 80 m and specific speeds in the range 350 to 900. The water from supply pipes enters the spiral casing as in the case of Francis turbine. Guide blades direct the water into the chamber above the blades at the proper direction. The speed governor in this case acts on the guide blades and rotates them as per load requirements. The flow rate is changed without any change in head. The water directed by the guide blades enters the runner which has much fewer blades (3 to 10) than the Francis turbine. The blades are also rotated by the governor to change the inlet blade angle as per the flow direction from the guide blades, so that entry is without shock. As the head is low, many times the draft tube may have to be elbow type. The important dimensions are the diameter and the boss diameter which will vary with the chosen speed. At lower specific speeds the boss diameter may be higher.

The popular axial flow turbines are the Kaplan turbine and propeller turbine. In propeller turbine the blades are fixed. In the Kaplan turbines the blades are mounted in the boss in bearings and the blades are rotated according to the flow conditions by a servomechanism maintaining constant speed. In this way a constant efficiency is achieved in these turbines.

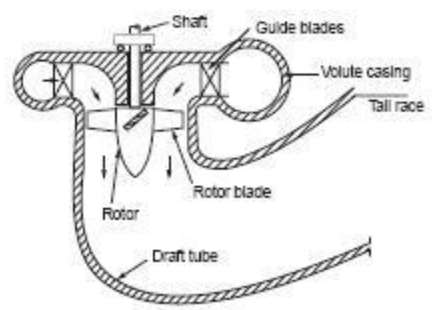


Fig. 7.1 Sectional view of Kaplan Turbine

7.1.1 Reaction Turbines

Reaction turbines are those turbines which operate under hydraulic pressure energy and part of kinetic energy. In this case, the water reacts with the vanes as it moves through the vanes and transfers its pressure energy to the vanes so that the vanes move in turn rotating the runner on which they are mounted. The main types of reaction turbines are

Radially outward flow reaction turbine: This reaction turbine consist a cylindrical disc mounted on a shaft and provided with vanes around the perimeter. At inlet the water flows into the wheel at the centre and then glides through radially provided fixed guide vanes and then flows over the moving vanes. The function of the guide vanes is to direct or guide the water into the moving vanes in the correct direction and also regulate the amount of water striking the vanes. The water as it flows along the moving vanes will exert a thrust and hence a torque on the wheel thereby rotating the wheel. The water leaves the moving vanes at the outer edge. The wheel is enclosed by a water-tight casing. The water is then taken to draft tube.

Radially inward flow reaction turbine: The constitutional details of this turbine are similar to the outward flow turbine but for the fact that the guide vanes surround the moving vanes. This is preferred to the outward flow turbine as this turbine does not develop racing. The centrifugal force on the inward moving body of water decreases the relative velocity and thus the speed of the turbine can be controlled easily.

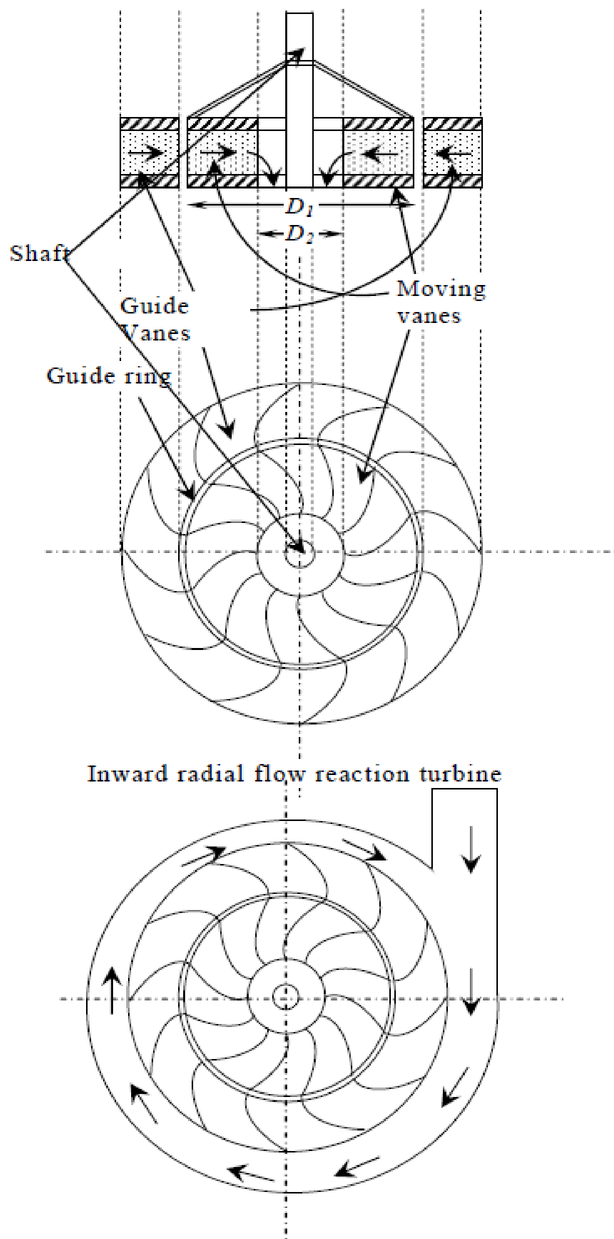


Fig. 7.2 Inward Radial Flow Reaction Turbine

Mixed flow reaction turbine: This is a turbine wherein it is similar to inward flow reaction turbine except that when it leaves the moving vane, the direction of water is turned from radial at entry to axial at outlet. The rest of the parts and functioning is same as that of the inward flow reaction turbines.

Axial flow reaction turbine: This is a reaction turbine in which the water flows parallel to the axis of rotation. The shaft of the turbine may be either vertical or horizontal. The lower end of the shaft is made larger to form the ***boss*** or the ***hub***. A number of vanes are fixed to the boss. When the vanes are composite with the boss the turbine is called ***propeller turbine***. When the vanes are adjustable the turbine is called a ***Kaplan turbine***.

7.2 Components:

The main component parts of a Kaplan Turbine (reaction turbine) are:

(1) Casing, (2) Guide vanes (3) Runner with vanes (4) Draft tube

Casing: This is a tube of decreasing cross -sectional area with the axis of the tube being of geometric shape of volute or a spiral. The water first fills the casing and then enters the guide vanes from all sides radially inwards. The decreasing cross -sectional area helps the velocity of the entering water from all sides being kept equal. The geometric shape helps the entering water avoiding or preventing the creation of eddies.

Guide vane or Wicket Gates: They guide flow smoothly from penstock pipe to the turbine runner without causing much disturbance or turbulence during the entry of water

Runner with vanes: The runner is mounted on a shaft and the blades are fixed on the runner at equal distances. The vanes are so shaped that the water reacting with the m will pass through them thereby passing their pressure energy to make it rotate the runner.

Draft tube: This is a divergent tube fixed at the end of the outlet of the turbine and the other end is submerged under the water level in the tail race. The water after working on the turbine, transfers the pressure energy there by losing all its pressure and falling below atmospheric pressure. The draft tube accepts this water at the upper end and increases its pressure as the water flows through the tube and increases more than atmospheric pressure before it reaches the tailrace.

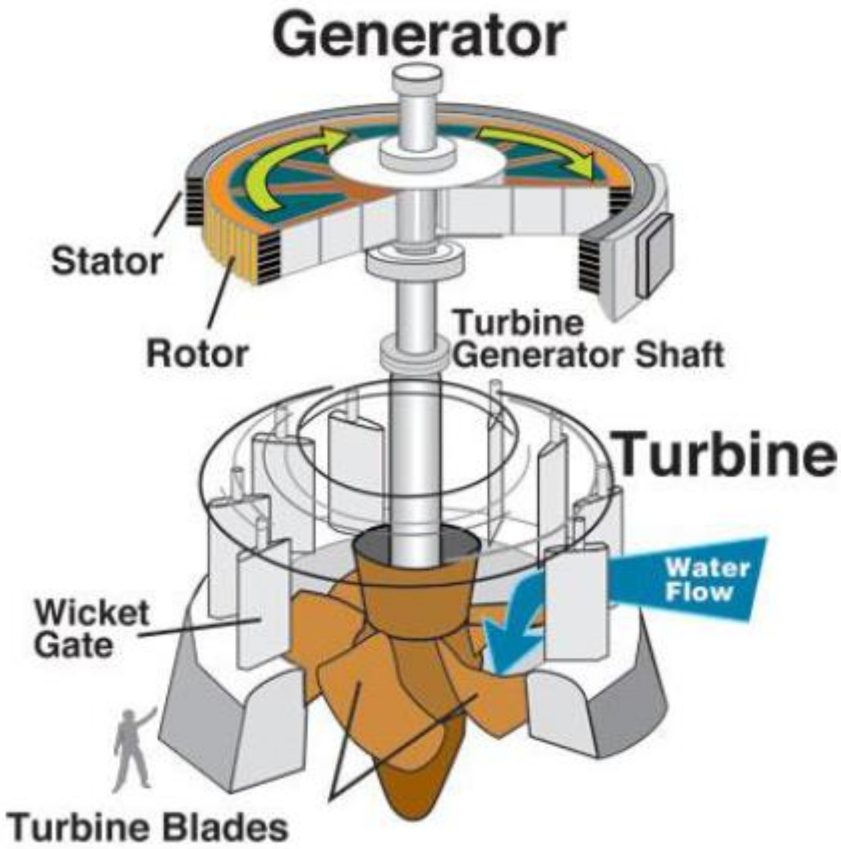


Fig.7.3 Components of Kaplan Turbine

7.3 Working and Velocity Triangles:

WORKING OF A KAPLAN TURBINE

The reaction turbine developed by Victor Kaplan (1815-1892) is an improved version of the older propeller turbine. It is particularly suitable for generating hydropower in locations where large quantities of water are available under a relatively low head. Consequently the specific speed of these turbines is high, viz., 300 to 1000. The Kaplan turbine is provided with a spiral casing, guide vane assembly and a draft tube. *The blades of a Kaplan turbine, three to eight in number are pivoted around the central hub or boss, thus permitting adjustment of their orientation for changes in load and head.* This arrangement is generally carried out by the governor which also moves the guide vane suitably. For this reason, while a fixed blade propeller turbine gives the best performance under the design loadconditions, **a Kaplan turbine gives a consistently high efficiency over a larger range of heads, discharges and loads.** The facility for adjustment of blade angles ensures shock-less flow even under non-design conditions of operation. Water entering radially from the spiral casing is imparted a substantial whirl component by the wicket gates.

Subsequently, the curvature of the housing makes the flow

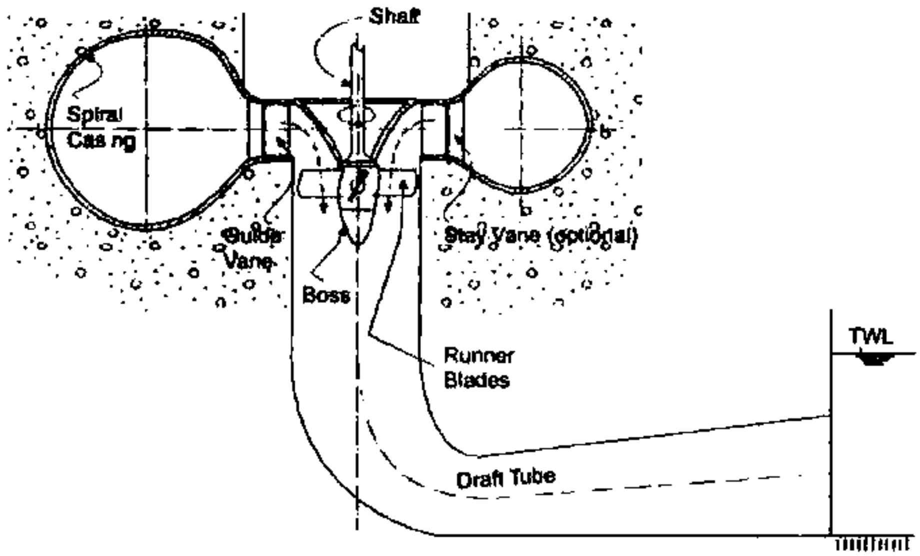


Fig. 7.4 Kaplan Turbine

become axial to some extent and finally then relative flow as it enters the runner, is tangential to the leading edge of the blade as shown in Fig 7.4.The energy transfer from fluid to runner depends essentially on the extent to which the blade is capable of extinguishing the whirl component of fluid. In most Kaplan runners as in Francis runners, water leaves the wheel axially with almost zero whirl or tangential component. The velocity triangles shown in Fig 7.5 at the inlet and outlet tips of the runner vane at mid radius, i.e., midway between boss periphery and runner periphery

Velocity Triangles: The number of blades in a Kaplan Turbine depends on the head available and varies from 3 to 10 for heads from 5 to 70 m. As the peripheral speed varies along the radius blade inlet angle should also vary with the radius. Hence twisted type or Airfoil blade section has to be used. The speed ratio is calculated on the basis of the tip speed as $\phi = u/ 2gH$ and varies from 1.5 to 2.4. The flow ratio lies in the range 0.35 to 0.75. Typical velocity diagrams at the tip and at the hub are shown in Figure 14.8.2. The diagram is in the axial and tangential plane instead of radial and tangential plane as in the other turbines.

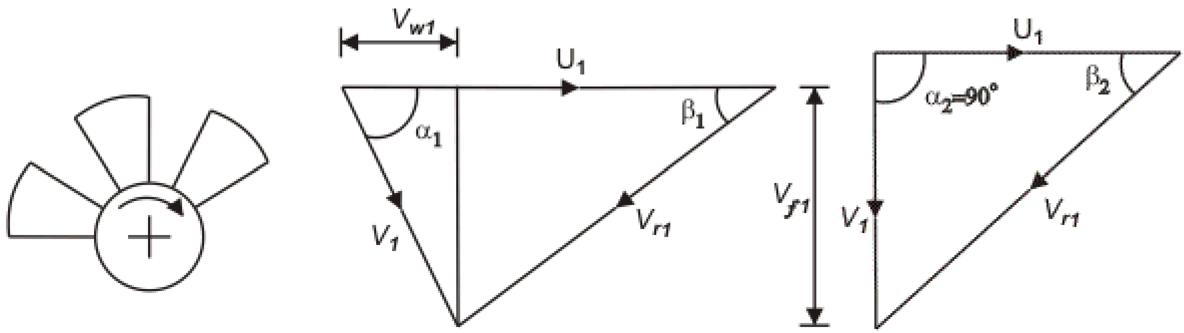


Fig. 7.2 Typical velocity diagrams for Kaplan Turbine

Work done = u_1V_{w2} (Taken at the mean diameter)

$$\eta_h = \frac{u_1 Vu_1}{gH} \times 100$$

All other relations defined for other turbines hold for this type also. The flow velocity remains constant with radius. As the hydraulic efficiency is constant all along the length of the blades, u_1V_{u1} = Constant along the length of the blades or V_{u1} decreases with radius.

Kaplan turbine has flat characteristics for variation of efficiency with load. Thus the part load efficiency is higher in this case. In the case of propeller turbine the part load efficiency suffers as the blade angle at these loads are such that entry is with shock.

KAPLAN TURBINE - SUMMARY

1. Peripheral velocities at inlet and outlet are same and given by,

$$u_1 = u_2 = \frac{\pi D_0 N}{60}$$

Where, 'D_o' is the outer diameter of the runner

2. Flow velocities at inlet and outlet are same. i.e. $V_{f1} = V_{f2}$

3. Area of flow at inlet is same as area of flow at outlet $Q = \frac{\pi}{4}\left(D_0^2 - D_b^2\right)$

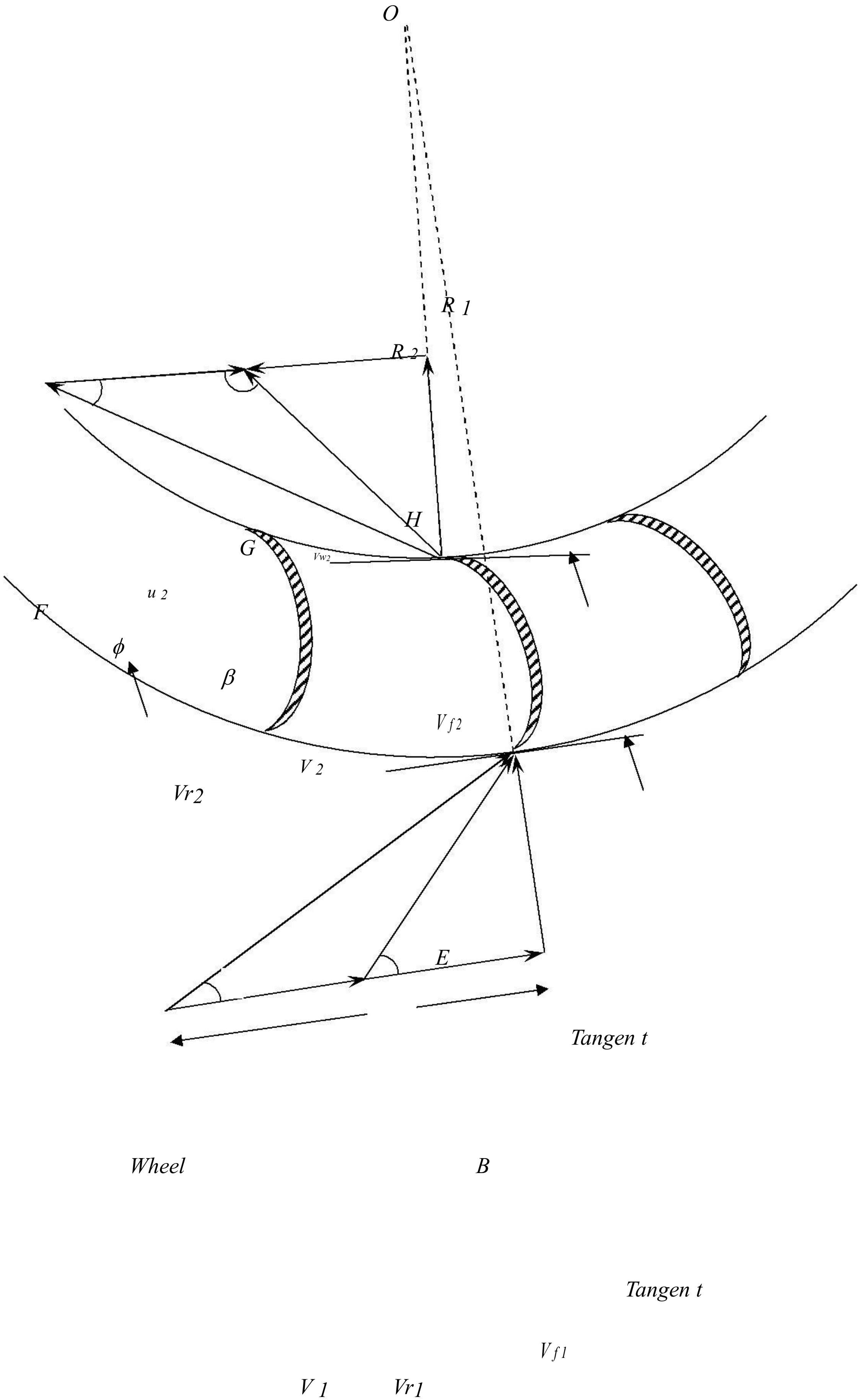
Where,'D_b'is the diameter of the boss

Comparison between Reaction and Impulse Turbines

SN	*Reaction turbine*	*Impulse turbine*
1	Only a fraction of the available hydraulic energy is converted into kinetic energy before the fluid enters the runner.	All the available hydraulic energy is converted into kinetic energy by a nozzle and it is the jet so produced which strikes the runner blades.
2.	Both pressure and velocity change as the fluid passes through the runner. Pressure at inlet is much higher than at the outlet.	It is the velocity of jet which changes, the pressure throughout remaining atmospheric.
3	The runner must be enclosed within a watertight casing (scroll casing).	Water-tight casing is not necessary. Casing has no hydraulic function to perform. It only serves to prevent splashing and guide water to the tail race
4.	Water is admitted over the entire circumference of the runner	Water is admitted only in the form of jets. . There may be one or more jets striking equal number of buckets simultaneously.
5.	Water completely fills at the passages between the blades and while flowing between inlet and outlet sections does work on the blades	The turbine does not run full and air has a free access to the buckets
6.	The turbine is connected to the tail race through a draft tube which is a gradually expanding passage. It may be installed above or below the tail race	The turbine is always installed above the tail race and there is no draft tube used
7.	The flow regulation is carried out by means of a guide-vane assembly. Other component parts are scroll casing, stay ring, runner and the draft tube	Flow regulation is done by means of a needle valve fitted into the nozzle.

7.4 Properties of the Turbine Discharge of the Turbines, Number of Blades:

<u>Derivation of the efficiency of a reaction turbine</u>


O
R 1
R 2
H
G
Vw2
u 2
F
ϕ
β
Vf2
V 2
Vr2
E
Tangen t
Wheel
B
Tangen t
Vf1
V 1
Vr1

θ D

α

A u_1 C_{Vw1}

Fig. 7.3

Let,

R_1= Radius of wheel at inlet of the vane

R_2= Radius of wheel at outlet of the vane

ω = Angular speed of the wheel

Tangential speed of the vane at inlet = u_1 $= \omega R_1$

Tangential speed of the vane at outlet = $u_2 = \omega R_2$

The velocity triangles at inlet and outlet are drawn as shown in Fig .

α and β are the angles between the absolute velocities of jet and vane at inlet and outlet respectively θ and ϕ are vane angles at inlet and outlet respectively

The mass of water striking a series of vanes per second = $\rho a V_1$

where a is the area of jet or flow and V_1 is the velocity of flow at inlet . The momentum of water striking a series of vanes per second at inlet is given by the product of mass of water striking per second and the component of velocity of flow at inlet

= $\rho a V_1$ x V_{w1} (V_{w1} is the velocity component of flow at inlet along tangential direction)

Similarly momentum of water striking a series of vanes per second at outlet is given b y

= $\rho a V_1$ x $(-V_{w2})$ (V_{w2} is the velocity component of flow at outlet along tangential direction and – because the velocity component is acting in the opposite direction)

Now angular momentum per second at inlet is given by the product of momentum of water at inlet and its radial distance = $\rho a V_1$ x V_{w1} x R_1

And angular momentum per second at inlet is given b y = $-\rho a V_1$ x V_{w2} x

R_2

Torque exerted by water on the wheel is given by impulse momentum theorem as the rate of change of angular momentum

$T = \rho a V_1 \text{x} V_{w1} \text{x} R_1 - -\rho a V_1 \text{x} V_{w2} \text{x} R_2$

$T = \rho a V_1 (V_{w1} R_1 + V_{w2} R_2)$

Work done per second on the wheel = Torque x Angular velocity = T x ω

WD/s = $\rho a V_1 (V_{w1} R_1 + V_{w2} R_2) \times \omega$

= $\rho a V_1 (V_{w1} R_1 \times \omega + V_{w2} R_2 \times \omega)$

As $u_1 = \omega R_1$ and $u_2 = \omega R_2$, we can simplify the above equation as

WD/s = $\rho a V_1 (V_{w1} u_1 + V_{w2} u_2)$

In the above case, always the velocity of whirl at outlet is given by both magnitude and direction as $V_{w2} = (V_{r2} \cos\phi - u_2)$

If the discharge is radial at outlet, then $V_{w2} = 0$ and hence the equation reduces to

WD/s = $\rho a\, u_1 \times V_1 \times V_{w1}$

KE/s = $\frac{1}{2} \times \rho \times a \times V_1^3$

Efficiency of the reaction turbine is given b y

$$\eta = \frac{\text{Workdone/second}}{\text{Kinetic Energy/second}} = \frac{\rho a V_1 (V_{w1} u_1 + V_{w2} u_2)}{\frac{1}{2} \rho a V_1^3}$$

$$\eta = \frac{2(V_{w1} u_1 + V_{w2} u_2)}{V_1^2}$$

Note: The value of the velocity of whirl at outlet is to be substituted as

$V_{w2} = (V_{r2} \cos\phi - u_2)$ along with its sign.

Summary

(i) **Speed ratio** = $\dfrac{u_1}{\sqrt{2gH}}$ where H is the Head on turbine

(ii) **Flow ratio**= $\dfrac{V_{f1}}{\sqrt{2gH}}$ where V_{f1} is the velocit y of flow at inlet

- Discharge flowing through the reaction turbine is given by

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

Where D_1 and D_2 are the diameters of runner at inlet and exit

B_1 *and* B_2 *are* the widths of runner at inlet and exit

V_{f1} *and* V_{f2} *are* the Velocity of flow at inlet and exit

If the thickness (t) of the vane is to be considered, then the area through which flow takes place is given b y ($\pi D_1 - nt$) where n is the number of vanes mounted on the runner

Discharge (Q) flowing through the Kaplan turbine is given by

$$Q = (\pi D_1 - nt)\, B_1 \times V_{f1} = (\pi D_2 - nt)\, B_2 \times V_{f2}$$

*(iv)*The head (H) on the turbine is given by $$H = \frac{p_1}{\rho g} + \frac{V_1^2}{2g}$$

Where p_1 is the pressure at inlet

(v) Work done per second on the runner $= \rho a V_1 (V_{w1} u_1 \pm V_{w2} u_2) = \rho Q\, (V_{w1} u_1 \pm V_{w2} u_2)$

(vi) $u_1 = \dfrac{\pi D_1 N}{60}$

(vii) $u_2 = \dfrac{\pi D_2 N}{60}$

(viii) Work done per unit weight $= \dfrac{\text{Work done per second}}{\text{Weight of water striking per second}}$

$$= \frac{\rho Q \left(V_{w1} u_1 \pm V_{w2} u_2\right)}{\rho Q g} = \frac{1}{g}\left(V_{w1} u_1 \pm V_{w2} u_2\right)$$

If the discharge at the exit is radial, then $V_{w2} = 0$ and hence

Work done per unit weight $= \dfrac{1}{g}\left(V_{w1} u_1\right)$

(ix) Hydraulic efficiency = $R.P. = \rho Q \left(V_{w1} u_1 \pm V_{w2} u_2\right) = 1 \left(V_{w1} u_1 \pm V_{w2} u_2\right)$

$$W.P. \qquad \rho g Q H \qquad g H$$

If the discharge at the exit is radial, then $V_{w2} = 0$ and hence

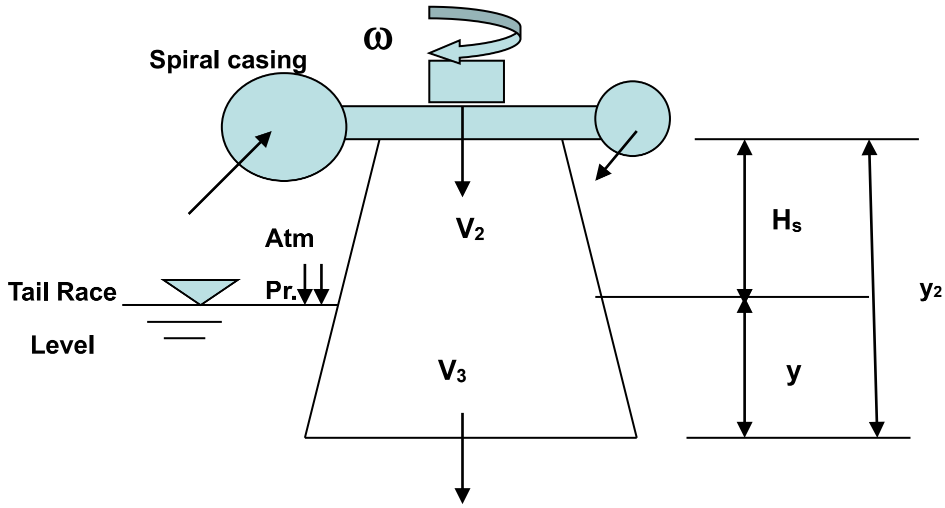
$$\text{Hydraulic efficiency} = \frac{1}{g H}\left(V_{w1} u_1\right)$$

7.5 Draft Tube: Types, efficiency of a Draft Tube:

7.5.1 Draft Tubes: The water after working on the turbine, imparts its energy to the vanes and runner, thereby reducing its pressure less than that of atmospheric pressure (Vacuum). As the water flows from higher pressure to lower pressure, it cannot come out of the turbine and hence a divergent tube is connected to the end of the turbine. A ***Draft tube*** is a divergent tube one end of which is connected to the outletof the turbine and other end is immersed well below the tailrace (Water level). The major function of the draft tube is to increase the pressure from the inlet to outlet of the draft tube (Pressure at draft tube inlet–ve, Pressure at draft tube outlet = Atmospheric or zero). The other function is to safely discharge the water that has worked on the turbine to tailrace. Generally draft tube is made of concrete material and it forms an integral part amongst the component of a reaction turbine and serves followingpurpose;

(i) It allows the turbine to install over the floor level (ii) It recovers the available kinetic energy at the exit of the runner

Fig. 7.4 Draft Tube



Energy at Exit of the Runner or Entry of draft Tube = Energy at the exit of the draft tube

$$\frac{p_2}{\gamma_w} + \frac{V^2{}_2}{2g} + y_2 = \frac{p_3}{\gamma_w} + \frac{V^2{}_3}{2g} + 0 + h_f$$

$$\frac{p_3}{\gamma_w} = \frac{p_a}{\gamma_w} + y$$

$$\frac{p_2}{\gamma_w} = \frac{p_a}{\gamma_w} + (y - y_2) - \left(\frac{(v_2^2 - v_3^2)}{2g} - h_f\right)$$

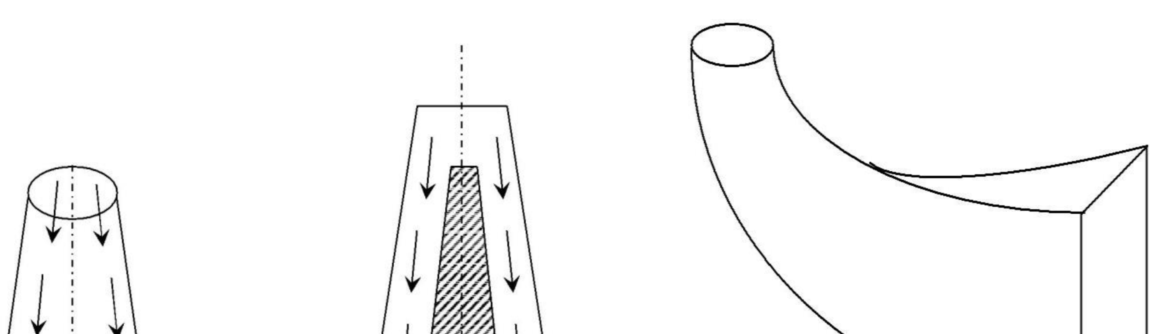
$$\frac{p_2}{\gamma_w} = \frac{p_a}{\gamma_w} - H_s - \left(\frac{v_2^2 - v_3^2}{2g} - h_f\right)$$

Efficeincy of a Draft tube

$$\eta_d = \frac{\text{Net gain in Pressure Head}}{\text{Velocity Head at Entrance of Draft Tube}} = \frac{\left(\frac{v_2^2 - v_3^2}{2g} - h_f\right)}{\frac{v_2^2}{2g}}$$

7.5.2 Types of draft tube: Depending on the shape and alignment, draft tubes are classified as follows:

- Elbow
- Moody’s spreading draft tube
- Elbow type with a rectangular opening (commonly used).
- Conical



Vertical divergent draft tube **Moody draft tube or hydra cone** **Elbow draft tube**

Fig. 7.6

Vertical divergent draft tubeThe draft tube has the shape of a frustum of a cone. This is generall y provided for low specific speed. The cone angle is not to exceed 8°. For greater value of the cone angle it is seen that the flowing body of water may not touch the sides of the draft tube (Leaving the boundary). This will lead to the eddy formation bringing down the efficiency of the draft tube.

Moody's draft tube or hydraulic one:This is a bell mouthed draft tube or aconical tube with a solid conical central core the whirl of discharged water is very much reduced in this arrangement.

Elbow draft tube:This draft tube affords to discharge the waterhorizontally to the tail race.

Elbow draft tube with circular inlet and rectangular outlet:This is afurther improvement of the simple elbow draft tube. In all the types mentioned above, the outlet of the draft tube should be situated below the tail water level.

7.6 Introduction to Cavitation in Turbines:

Cavitation occurs when a **high-velocity, flow of water** (or any other fluid) suffers an **abrupt change in direction or velocity**. This change in direction or velocity in flowingwater causes a **zone of low pressure to occur at the surface of the**pipe or open channel that isimmediately downstream from the direction or velocity change. This low pressure zonemay allow pockets (or cavities) of vapor to form.

When leaving the low pressure zone, **these pockets of vapor collapse**. The collapse of these vapors causes a localized high-energy impact on the pipe material or impeller or runner. This localized high-energy impact leads to erosion of material, vibration and noise in the Turbo-machinery.

Cavitation is not desirable for several reasons. First, it causes noise (as the cavitationbubbles collapse when they migrate into regions of higher pressure). Second, it can lead toinefficiencies and reduction of heat transfer in pumps and turbines (turbo machines). Finally,the collapse of these cavitation bubbles causes pitting and corrosion of blades and othersurfaces nearby. The left figure below shows a cavitating propeller in a water tunnel, and theright figure shows cavitation damage on a blade.

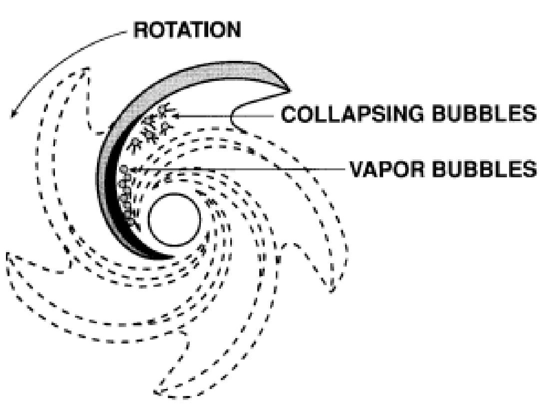


Fig. 7.7 Cavitation Bubble and Damaged Impeller of a Pump

Prevention of Cavitation: The effects of cavitation can be reduced by,

(i) Setting the turbine near the tail race level

(ii) Making the runner blades/impeller using superior materials (stainless steel, Nickel steel) and also with highly polished surface finish

(iii) Spraying thin layers of erosion resistance metals or special paints where cavitation is most likely to occur (places of high velocity).

7.7 Problems

Q.1 A Kaplan turbine working under a head of 25 m develops 16,000 kW shaft power. The outer diameter of the runner is 4 m and hub diameter is 2 m. The guide blade angle is 35°. The hydraulic and overall efficiency are 90% and 85% respectively. If the velocity of whirl is zero at outlet, determine runner vane angles at inlet and outlet and speed of turbine.

Solution: Given: $H = 25$ m; $P = 16{,}000$ kW; $D_b = 2$ m; $D_o = 4$ m; $\alpha = 35°$; $\eta_h = 0.9$;

$\boldsymbol{\eta_o = 0.85;\ V_{w2} = 0;\ \theta = ?\ \phi = ?\ N = ?}$

$$\eta_0 = \frac{P}{\rho g Q H};\ 0.85 = \frac{16000 \times 10^3}{1000 \times 10 \times Q \times 25}$$

$Q = 75.29\ \text{m}^3/\text{s}$

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f1} \Rightarrow \frac{\pi}{4}\left(4^2 - 2^2\right) \times V_{f1} = 75.29$$

$V_{f1} = 7.99$ m/s

From inlet velocity triangle,

$$\tan\alpha = \frac{V_{f1}}{V_{w1}}$$

$$V_{w1} = \frac{7.99}{\tan 35} = 11.41\text{m/s}$$

From Hydraulic efficiency

$$\eta_h = \frac{V_{w1} u_1}{g H}$$

$$0.9 = \frac{11.41 \times u_1}{10 \times 25}$$

$u_1 = 19.72$ m/s

$$\tan\theta = \frac{V_{f1}}{u_1 - V_{w1}} = \frac{7.99}{19.72 - 11.41} = 0.9614$$

$\theta = 43.88°$ (Ans)

For Kaplan turbine, $u_1 = u_2 = 19.72$ m/s and $V_{f1} = V_{f2} = 7.99$ m/s

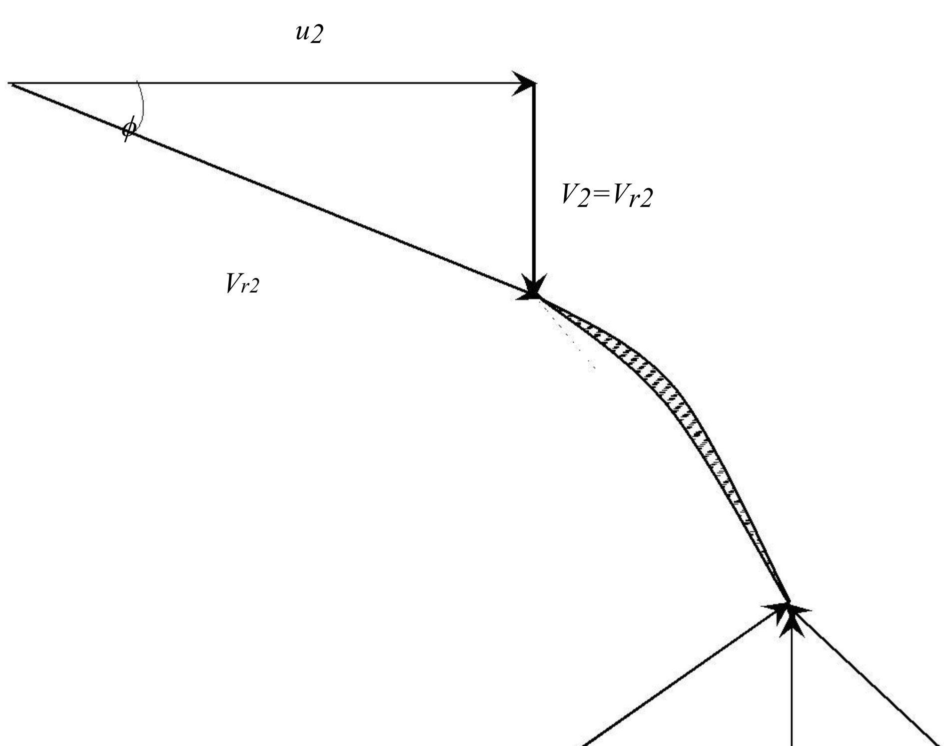
From outlet velocity triangle

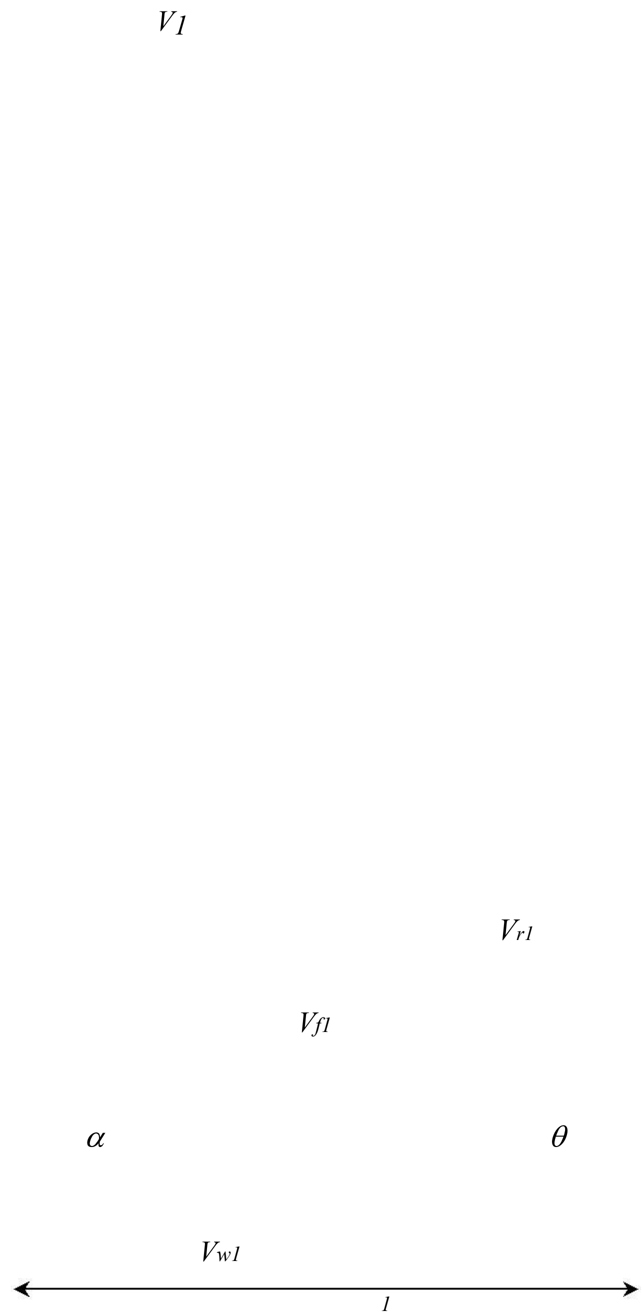
$$\tan\phi = \frac{V_{f2}}{u_2} = \frac{7.99}{19.72} = 0.4052$$

$\phi = 22.06°$ (Ans)

$$u_1 = u_2 = \frac{\pi D_o N}{60} = \frac{\pi \times 4 \times N}{60} = 19.72 \text{ m/s}$$

$N = 94.16$ rpm (Ans)





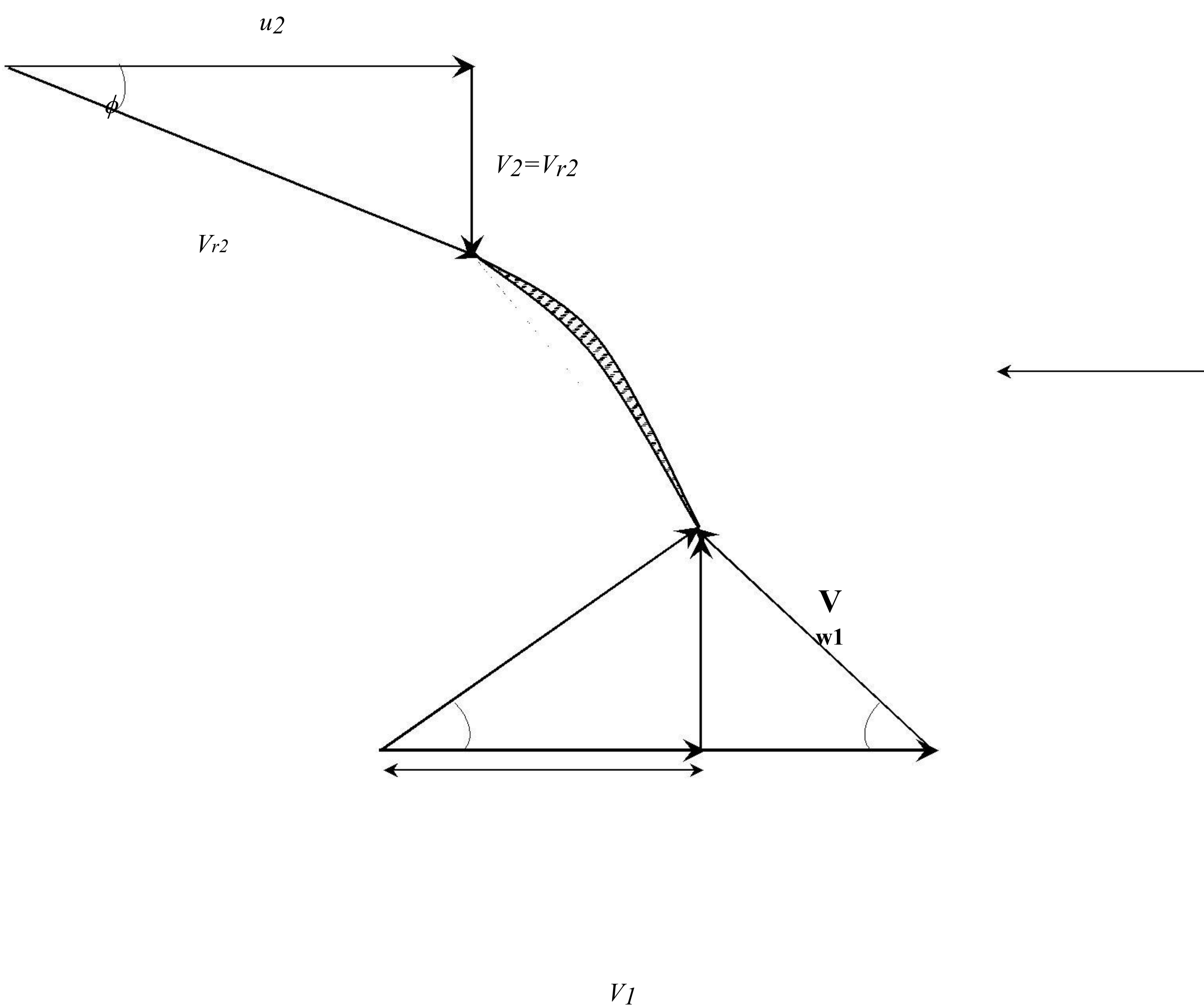
V_1
V_{r1}
V_{f1}
α
θ
V_{w1}
l

Q.2 A Kaplan turbine works under a head of 22 m and runs at 150 rpm. The diameters of the runner and the boss are 4.5 m and 12 m respectively. The flow ratio is 0.43. The inlet vane angle at the extreme edge of the runner is $163^{\circ}19'$. If the turbine discharges radially at outlet, determine the discharge, the hydraulic efficiency, the guide blade angle at the extreme edge of the runner and the outlet vane angle at the extreme edge of the manner.

Solution:

$H = 22$ m; $N = 150$ rpm; $D_o = 4.5$ m; $D_b = 2$ m; $\theta = 163^{\circ}19'$; $V_{\omega 2} = 0$;

$V_2 = V_{f2} = V_{f1}$; $Q = ?$; $\eta_h = ?$; $\alpha = ?$; $\phi = ?$; $\dfrac{V_{f1}}{\sqrt{2gH}} = 0.43$,



V_{r1}

$$u_1 = u_2 = 35.34 = \frac{\pi D_0 N}{60} = \frac{\pi \times 4.5 \times N}{60}$$

N = 150 RPM

$V_{f1} = 0.43\sqrt{2 \times 10 \times 22} = 9.02$ m/s

$$\tan(180 - \theta) = \frac{V_{f1}}{(u_1 - V_{w1})}$$

$$\tan(180 - 163°19') = \frac{9.02}{(35.34 - V_{w1})} = \mathbf{0.2997}$$

***V** **w1**= 5. 24 m/s*

Hydraulic efficiency is given b y

$$\eta_h = \frac{V_{w1} u_1}{g H} = \frac{5.24 \times 35.34}{10 \times 22} = 84.17\%$$

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{9.02}{5.24} = 1.72$$

α = 59. 85°(Ans)

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{9.02}{35.34} = \mathbf{0.2552}$$

ϕ = 14. 32 °(Ans)

Q.3 A Kaplan turbine is to be designed to develop 7,350 kW. The net available head is 5.5 m. Assume that the speed ratio as 0. 68. The overall efficiency as 60%. The diameter of the boss is ⅓rd of the diameter of the runner. Find the diameter of the runner, its speed and its specific speed.

Solution:

$P = 7350$ kW, $H = 5.5$ m

$$\frac{V_{f1}}{\sqrt{2gH}} = 0.68 \quad \text{and hence} \quad V_{f1} = 0.68\sqrt{2 \times 10 \times 5.5} = 7.13 \text{ m/s}$$

$$\frac{u_1}{\sqrt{2gH}} = 2.09 \quad \text{and hence} \quad u_1 = 2.2\sqrt{2 \times 10 \times 5.5} = 23.07 \text{ m/s}$$

$$\eta_0 = \frac{P}{\rho g Q H}; \quad 0.6 = \frac{7350 \times 10^3}{1000 \times 10 \times Q \times 5.5}$$

$Q = 222.72\ \text{m}^3/\text{s}$

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right)\times V_{f1} \Rightarrow \frac{\pi}{4}\left(D_o^2 - \left[\frac{D_o}{3}\right]^2\right)\times 7.13 = 222.72$$

D_o = 6. 69 m (Ans)

$$u = 23.07 = \frac{\pi D_0 N}{60} = \frac{\pi \times 6.69 \times N}{60}$$

N =65. 86 rpm (Ans)

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{65.86\sqrt{7350}}{5.5^{5/4}} = \textbf{670.37 rpm}\ \text{(Ans)}$$

Q4. The following data refers to the runner of a Kaplan Turbine which yields 8850 kW at the turbine shaft: Net available head 5.5m, speed ratio 2.1, flow ratio 0.67 and overall efficiency 85%. Presuming that hub diameter of the wheel is 0.35 times the outside diameter. Calculate the runner diameter and its rotational speed.

Solution: Given: P = 8850 kW = 8850 × 1000 W, H = 5.5m, C_u = 2.1, ϕ = 0.67, η_0= 85% = 0.85

$$\frac{Hub\ Diameter}{Outer\ Diameter} = \frac{D_b}{D_0} = 0.35$$

Peripheral Velocity $U = C_u\sqrt{2gH} = 2.1\times\sqrt{2\times 9.81\times 5.5} = 21.81 m/s$

Flow Velocity $V_f = \phi\sqrt{2gH} = 0.67\times\sqrt{2\times 9.81\times 5.5} = 6.96 m/s$

Power available at the shaft $\quad P = \gamma_w \times Q \times H \times \eta_o$

$8850\times1000 = 9810 \times Q\times5.5 \times 0.85$

Flow rate **$Q = 193\ m^3/s$**

The discharge (Q) through a Kaplan Turbine is given by,

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right)\times\ flow\ Velocity$$

$$Q = \frac{\pi}{4}\left(D_o^2 - (0.35D_o)^2\right)\times V_f$$

$$Q = 193 = \frac{\pi}{4}\left(D_o^2 - (0.35D_o)^2\right)\times 6.96$$

Outside Diameter $D_o = 6.343$ m

Runner Speed $N = \frac{60\times U}{\pi\times D_o} = \frac{60\times 21.81}{\pi\times 6.343} = 65.7 RPM$, **N = 66 N**

Q.5 For a Kaplan turbine with a runner diameter 4m, the discharge is $60 m^3/s$ and the hydraulic and mechanical efficiencies are stated to be 90% and 94% respectively. The diameter of boss is one third times the runner diameter and the speed ratio is 2.0. Assuming the discharge is free and there is no swirl at outlet, calculate the net available head on the turbine, the power developed and specific speed

Solution: Given: Q= 60m^3/s, C_u= 2.0,η_0= 90% = 0.90, η_m= 94%= 0.94,Outer Diameter D_o = 4m

$$\frac{\textit{Hub Diameter}}{\textit{Outer Diameter}} = \frac{D_b}{D_0} = \frac{1}{3}$$

The discharge (Q) through a Kaplan Turbine is given by,

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right)\times \textit{flow Velocity}$$

$$Q = \frac{\pi}{4}\left(D_o^2 - \left(\frac{D_o}{3}\right)^2\right)\times V_f$$

$$Q = 60 = \frac{\pi}{4}\left(4^2 - (0.35\times 4)^2\right)\times V_f$$

V_f = 5.44 m/s

Let 'H' represent the net available head on the Kaplan Turbine. Then

$$H - \frac{V_2^2}{2g} = H_g = \eta_h \times H$$

$$H - \frac{V_2^2}{2g} = 0.9H$$

$$0.1H = \frac{V_2^2}{2g} \quad \cdots Eq.(1)$$

In the absence of swirl at outlet $V_2 = V_{f2}$ = 5.44m/s, substituting the value of V_2 in Eq(1)

$$0.1H = \frac{V_2^2}{2g} \quad \cdots Eq.(1)$$

$$0.1H = \frac{5.44^2}{2\times 9.81}$$

$$H = 15.1m$$

Power available at the shaft P = $\gamma_w \times Q \times H \times \eta_o$

Where Overall Efficiency $\eta_o = \eta_h \times \eta_m$ = 0.9×0.94 = 0.846

P = 9810×60×15.1 × 0.846, P **= 7519.1 kW**

Peripheral Velocity $U = \dfrac{\pi D_o N}{60} = \dfrac{\pi \times 4 \times N}{60} = C_u \sqrt{2gH} = 2.0 \times \sqrt{2 \times 9.81 \times 15.1}$

$\therefore$ Runner Speed N = 164.36 $\equiv$ 165 RPM

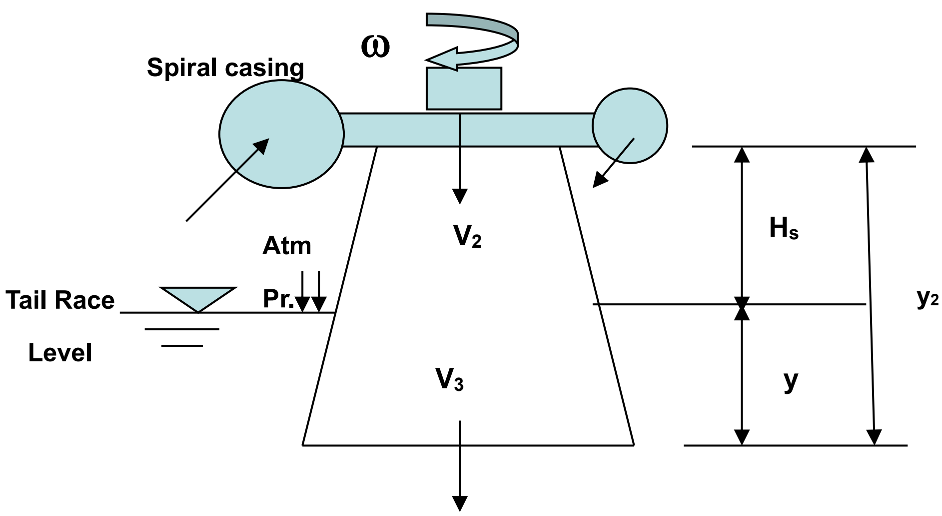
Specific Speed $N_s = \dfrac{N\sqrt{P_{kw}}}{H^{5/4}} = \dfrac{165\sqrt{7519.1}}{15.1^{5/4}} = 458$ RPM

Ns = 458 RPM

Q.6 A Kaplan turbine developing 3250kW under a head of 6m has a draft tube with inlet diameter 2.8m and is placed 1.5 m above the tail water. If the vacuum gauge connected to inlet of draft tube reads 5m of water, determine the efficiency of turbine. Assume the draft tube efficiency as 76% and the atmospheric pressure is 10.3m of water

Solution: Given Inlet pressure (at Draft tube) = 5m, Vacuum or (10.3-5.0) =5.3m abs, D_1 = 2.8m

Atmospheric pressure=10.3m of water, H_s= 1.5m above tail water level, η_{draft} =76% =0.76, H= 6m



Energy at Exit of the Runner or Entry of draft Tube = Energy at the exit of the draft tube

$$\frac{p_2}{\gamma_w} = \frac{p_{atm}}{\gamma_w} - H_s - \left(\frac{\left(V_2^2 - V_3^2\right)}{2 \times g} - H_f\right)$$

Neglecting friction loss H_f in the draft tube H_f = 0

$$\frac{p_2}{\gamma_w} = \frac{p_{atm}}{\gamma_w} - H_s - \left(\frac{(V_2^2 - V_3^2)}{2 \times g}\right); \; 5.3 = 10.3 - 1.5 - \left(\frac{(V_2^2 - V_3^2)}{2 \times g}\right)$$

$$\left(\frac{(V_2^2 - V_3^2)}{2 \times g}\right) = 3.5m$$

The Efficiency of draft tube is given by:

$$\eta_{draft} = \frac{(V_2^2 - V_3^2)/2g}{V_2^2/2g}$$

$$0.76 = \frac{3.5}{V_2^2/2g}$$

$$V_2 = 9.51 m/s$$

Discharge through Turbine Q = $\frac{\pi}{4} \times (2.8)^2 \times 9.51$ = **$58.558 m^3/s$**

Power available at the turbine Shaft,

P = $\gamma_w \times Q \times H \times \eta_o$

$3250 \times 1000 = 9810 \times 58.558 \times 6 \times \eta_o$

η_o = 0.943 = 94.3%

Q.7. A Kaplan turbine develops 2250kW under a net head of 5.5m and with overall efficiency 87%. The draft tube has a diameter of 2.8m at its inlet and has an efficiency of 78%. In order to avoid cavitation, the pressure head at the entry to the draft tube must not drop more than 4.5m below atmosphere. Calculate the maximum height at which the runner may be set above the tail race level

Solution: Given P = 2250kW, H =5.5m, η_0= 87% = 0.87, η_{draft} =78% = 0.78, D_1 = 2.8m

$$\frac{p_{atm} - p_2}{\gamma_w} = 4.5m$$

Power available at the turbine Shaft, P = $\gamma_w \times Q \times H \times \eta_o$

$2250 \times 1000 = 9810 \times Q \times 5.5 \times 0.87$

Flow rate Q = 47.93m^3/s

From continuity equation $V_2 = \frac{Q}{A} = \frac{47.93}{\frac{\pi}{4}(2.8)^2} = 7.79 m/s$

$$\frac{p_2}{\gamma_w} = \frac{p_{atm}}{\gamma_w} - H_s - \left(\frac{(V_2^2 - V_3^2)}{2 \times g} - H_f\right)$$
$$\left(\frac{(V_2^2 - V_3^2)}{2 \times g} - H_f\right) = 4.5 - H_s$$

Draft Tube Efficiency is given by,

$$\eta_{draft} = \frac{\left[(V_2^2 - V_3^2)/2g - H_f\right]}{V_2^2/2g}$$
$$0.78 = \frac{4.5 - H_s}{7.79^2/2g}$$
$$H_s = 2.1m$$

Maximum height (H_s) at which the Runner may be set above the tail race level = 2.1m

UNIT-8: CENTRIFUGAL PUMPS

8.0 Introduction:

Pumps have many uses in water and wastewater plants. There are many types of pumps for a variety of uses. Some of these may be river pumps, chemical feeder pumps, high service or pumps that pump water to the distribution system, booster pumps in stations in the system, lagoon pumps for wash-water disposal, and lab sample pumps, just to name a few. Mainly all water pumps may be classified into two general categories; centrifugal pumps and displacement pumps.

A centrifugal pump is also known as a Rotodynamic pump or dynamic pressure pump. It work on the principle of centrifugal force. In this type of pump the liquid is subjected towhirling motion by the rotating impeller which is made of a number of backward curvedvanes. The liquid enters this impeller at its center or the eye and gets discharged into thecasing enclosing the outer edge of the impeller. The rise in the pressure head at anypoint/outlet of the impeller is Proportional to the square of the tangential velocity of theliquid at that point (i.e. $\boldsymbol{\alpha}\,\frac{\boldsymbol{u}^2}{2\boldsymbol{g}}$). Hence at the outlet of the impeller where the radius ismore the rise In pressure head will be more and the liquid will be discharged at the outletwith a high pressure head. Due to this high pressure head, the liquid can be lifted to ahigher level. Generally centrifugal pumps are made of the radial flow type only. Butthere are also axial flow or propeller pumps which are particularly adopted for low heads.

In this type of pump, the fluid is fed to the center of the rotating impeller (eye of the impeller) and is thrown outward by centrifugal action. As a result of high speed of rotation the liquid acquires a high kinetic energy and the pressure difference between the suction and delivery sides arises from the conversion of kinetic energy into pressure energy.

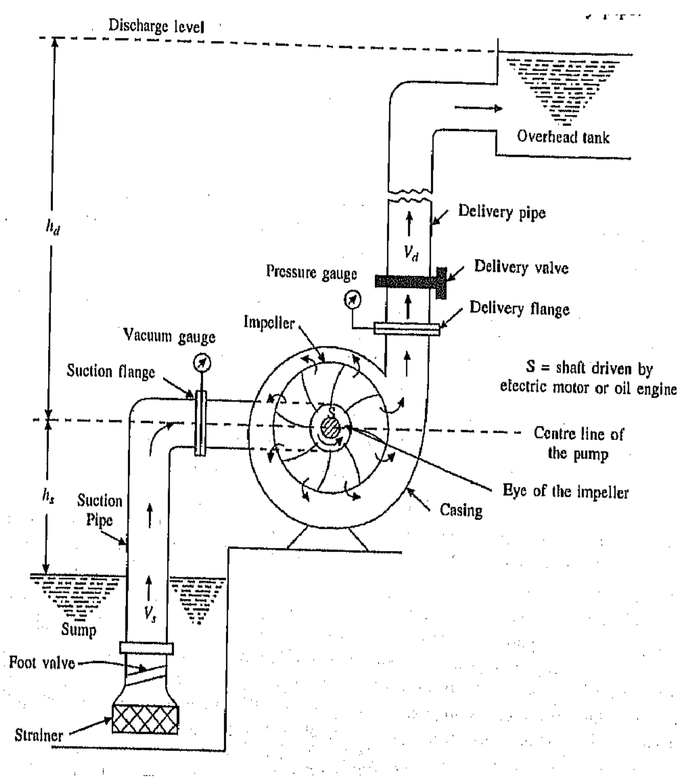
The impeller consists of a series of curved vanes so shaped that the flow within the pump is as smooth as possible. The greater the number of vanes on the impeller, the greater is the control over the direction of motion of liquid and hence smaller is the losses due to turbulence and circulation between the vanes.

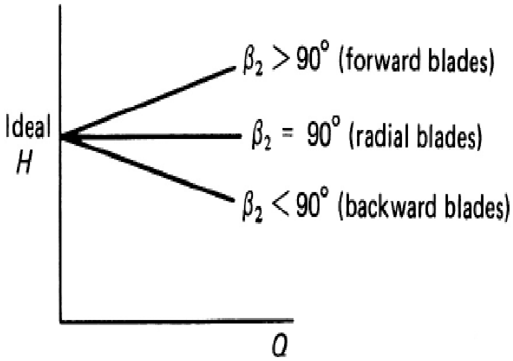
The liquid enters the casing of the pump, normally in an axial direction, and is picked up by the vanes of the impeller. In the simple type of centrifugal pump, the liquid discharges into a volute, a chamber of gradually increasing cross-section with a tangential outlet

Components of Centrifugal Pump:

(i) Impeller (ii) Casing (iii) Suction Pipe (iv) delivery Pipe

It consists of a suction pipe (Fig.8.1) an impeller, a casing and a delivery pipe. The suction pipe dips deep in the fluid to be pumped and is fitted with a strainer and foot valve at the inlet. The strainer keeps out solid impurities from entering the pump system and the foot valve allows flow of fluid only in one direction. It closes automatically if fluid tries to flow back to the sump. Suction pipe is connected to the casing at the centre of the impeller which is known as eye. Impeller is mounted on a shaft, one end of which comes out of the casing where it is connected to the driving unit. Fluid collected all around the periphery of the impeller is led to the delivery pipe by means of an air-tight casing which is spiral shaped of increasing area of cross-section in the direction of flow. This converts kinetic energy of fluid into pressure energy and directs whole





Classification of Pump Based on Fig. 8.1 Centrifugal Pump Type of Blade

ofthe fluid into the delivery pipe. A control valve is fitted in the delivery pipe to regulate flow. A priming funnel and an air hole are provided at the top of the casing. Priming funnel and an air for pouring in the liquid at the time of starting and air hole is opened to allow the air inside to escape during priming.**Foot valve with strainer** the foot valve is a non-return valve which permits the flow ofthe liquid from the sump towards the pump. In other words the foot valve opens only inthe upward direction.The strainer is a mesh surrounding the valve; it prevents the entry of debris and silt intothe pump.

Advantages of centrifugal pumps:-

a) Its initial cost is low
b) Efficiency is high
c) Discharge is uniform and continuous
d) Installation and maintenance is easy
e) It can run at high speeds, without the risk of separation of flow Classification of centrifugal pumps

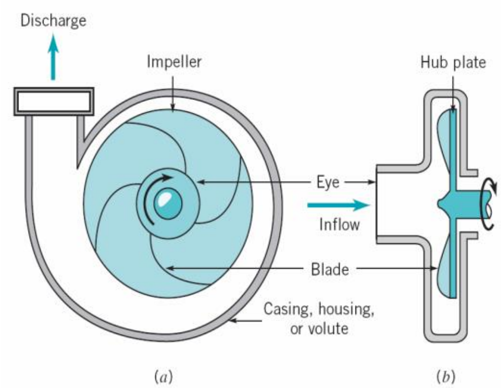
8.2 Centrifugal pumps may be classified into the following types:

- According to Entry of Liquid into Pump:

CENTRIFUGAL PUMPS		
RADIAL FLOW	**MIXED FLOW**	**AXIAL FLOW**

Radial Flow-A centrifugal pump in which the pressure is developed wholly by centrifugal force

Mixed Flow - a centrifugal pump in which the pressure is developed partly by centrifugal force and partly by the lift of the vanes of the impeller on the liquid.



Axial Flow - a centrifugal pump in which the pressure is developed by the propelling or lifting action of the vanes of the impeller on the liquid

- According to casing design

a) Volute pump b) diffuser or turbine pump

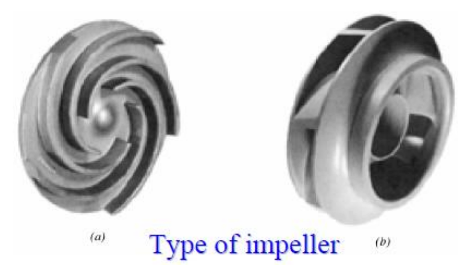
- According to number of impellers

a) Single stage pump b) multistage or multi impeller pump

- According to number of entrances to the impeller:

a) Single suction pump

b) Double suction pump



Type of impeller

(*a*) Open impeller, (*b*) enclosed or shrouded i

- According to disposition of shaft

a) Vertical shaft pump

b) Horizontal shaft pum

-According to liquid handled

a) Semi open impeller: Semi-open impeller has a plate only on back side. The design is adopted to industrial pump problems which required a rugged pump to handle liquids containing fibrous material such as paper pulp, sugar molasses and sewage water etc. in open impeller, no shroud or plate is provided on either side i.e., the vanes are open on both sides. Such pumps are used where the pump has a very rough duty to perform i.e. to handle abrasive liquids etc.

b) Closed and Open impeller pump: Closed, and open impellers in the closed impellers the vanes are covered with side plates (shrouds) on both sides. The back shroud is mounted into shaft and front shroud is coupled to the former by the vanes. The arrangement provides a smooth passage for the liquid; wear is reduced to minimum. This ensures full capacity operation with high efficiency for a prolonged running period.

- According to specific speed

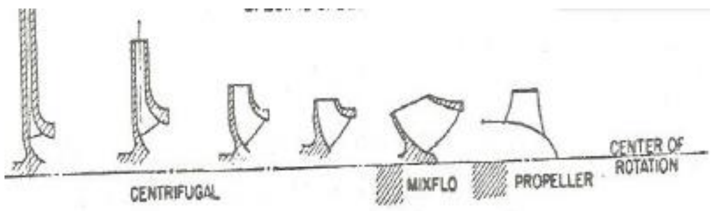
a) Low specific speed or radial flow impeller pump

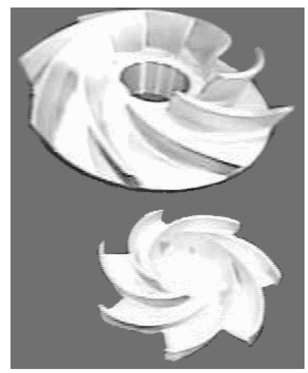
b) Shrouded impeller

c) Medium specific speed or mixed flow impeller pump

c) High specific speed or axial flow type or propeller pump

SN	Pumps	$Ns = \frac{N\sqrt{Q}}{H^{3/4}}$ rpm
1	Centrifugal pump Slow – Fast speed	12 – 95
2	Mixed flow pump	95 – 210
3	Axial flow pump Propeller pump	172 -320





- According to head (H)

- Low head if H<15m
- Medium head if 15<H<40m
- High head if H>40m

Centrifugal pumps are made in a wide range of materials, and in many cases the impeller and casing are covered with resistant material. Thus stainless steel, nickel, rubber, polypropylene, stoneware, and carbon are all used.

8.3 Priming, methods of Priming:

Priming is the process of filling of suction pipe, impeller casing and delivery pipe upto delivery valve by the liquid from outside source before starting the pump is known as priming.The air is removed and that portion is filled with the liquid to be pumped.

If priming is not done the pump cannot deliver the liquid due to the fact that the head generated by the Impeller will be in terms of meters of air which will be very small (because specific weight of air is very much smaller than that of water).

Priming of a centrifugal pump can be done by any one of the following methods:

i) Priming with suction/vacuum pump.

ii) Priming with a jet pump.

iii) Priming with separator.

iv) Automatic or self priming: A **"self-priming"** centrifugal pump overcomes the problem of air binding by mixing air with water to create a fluid with pumping properties much like those of regular water. The pump then gets rid of the air and moves water only, just like a standard centrifugal pump.

8.4 Heads and Efficiencies:

Suction head (hs): it is the vertical distance between the liquid level in the sump and the centre line of the pump. It is expressed as meters.

Delivery head (hd): It is the vertical distance between the centre line of the pump and the liquid level in the overhead tank or the supply point. It is expressed in meters.

Static head (Hs): It is the vertical difference between the liquid levels In the overhead tank and the sump, when the pump is not working. It is expressed as meters. Therefore, **H_S= (hs + hd)**

Friction head (hf): It is the sum of the head loss due to the friction in the suction and delivery pipes. The friction loss in both the pipes is calculated using the Darcy's equation, $h_f = \frac{flV^2}{2gD}$

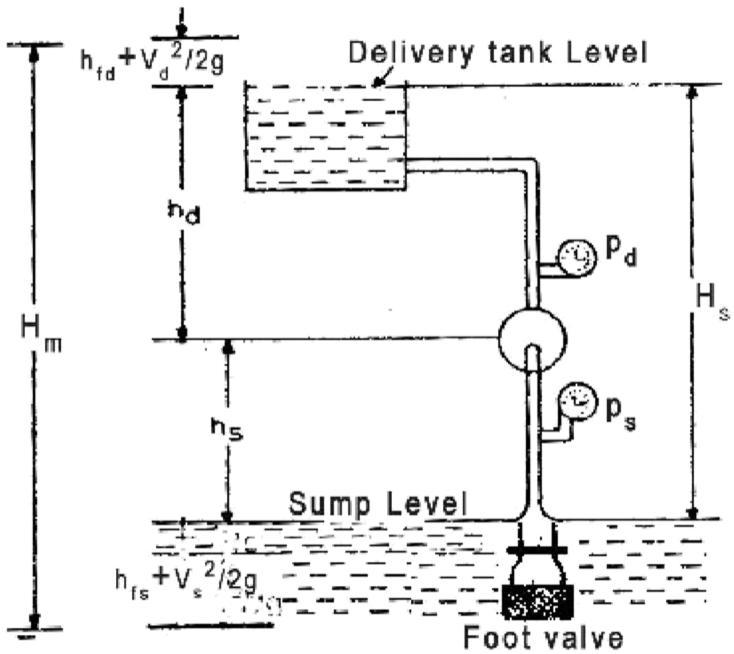
Total head (H): Total external Head 'H' against which the pump has to work.It is the sum of the static head H_s, friction head (h_f) and the velocity head in the delivery pipe $\frac{V_d^2}{2g}$. Where, V_d = Velocity in the delivery pipe.

$$H = \left(h_s + h_d + h_f + \frac{V_d^2}{2g} \right)$$ **... Eq. 8.1**

Manometric head (H_m): Manometric head refers to the difference between the total energy at inlet to and at exit from the pump. This head is slightly less than the head generated by the impeller due to some losses in the pump

$$H_m = \left(H + \frac{V_s^2}{2g} - \frac{V_d^2}{2g} \right)$$ **... Eq. 8.2**

Net positive suction head (NPSH): NPSH represents suction head at the impeller eye; it



Terminolgy of centrufugal pump

represents the head required to make the fluid flow from suction pipe to impeller avoiding cavitation phenomenon during operation of the pump.

EFFICIENCIES OF A CENTRIFUGAL PUMP:

Manometric efficiency -The ratio of the manometric head to the head imported by the impeller to the water is known as manometric efficiency. Mathematically, it is written as

$$\eta_{manometric} = \frac{Measured\ Head}{Theoretical\ Head} = \frac{H}{H_{ideal}} = \frac{H}{\left(\frac{V_{u2} \times u_2}{g}\right)} = \frac{gH}{V_{u2} \times u_2}$$ **Eq.8.3**

Mechanical efficiency—Mechanical losses in a pump represent the degradation of energy due to mechanical friction in packing, glands and bearings etc. “it is defined as the ratio of the energy transferred to water by the rotor to the mechanical energy delivered at the shaft coupling”.

$$\eta_{mechanical} = \frac{Rotor\ or\ Impeller\ power}{Shaft\ Power} = \frac{\gamma_w (Q+q) H_i}{Shaft\ power}$$ Where H_i – Ideal Head generated by pump

$\eta_{mechanical}$ ranges between 95% to 98%

Volumetric efficiency—Let Q denote the actual discharge at the pump outlet and. q represent the internal leakage then the total or theoretical volume flowing through the impeller is (Q + q).

'Volumetric efficiency is defined as the ratio of actual to theoretical discharge'.

$$\eta_v = \frac{Q}{Q+q}$$ With medium and large size pump units' η_v – ranges from 0.96-0.98

Hydraulic efficiency—Hydraulic losses due to fluid separation and energy consumed by friction in the flow.'Hydraulic efficiency is defined as the ratio of actual head to the theoretical or ideal head'. $\eta_{hyd} = \frac{H}{H_i}$

Overall Efficiency -The ratio of power output of the pump to the power output to the pump is known as overall efficiency $\boldsymbol{\eta_o = \eta_h \times \eta_m}$

Applying Bernoulli's equation at (0) and (1) (Fig.8.2)

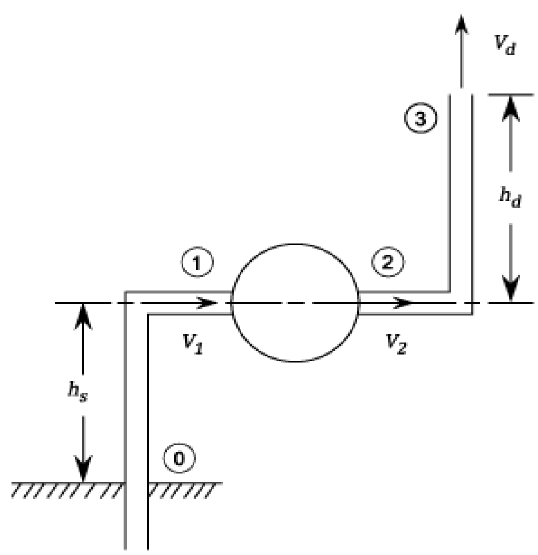
$$0 = \frac{p_1}{w} + \frac{V_1^2}{2g} + h_s + h_{fs}$$

Therefore the reading of the pressure gauge at the suction flange is:

$$\frac{p_1}{w} = -\left(\frac{V_1^2}{2g} + h_s + h_{fs}\right)$$

Apply Bernoulli's equation at (2) and (3)

$$\frac{p_2}{w} + \frac{V_2^2}{2g} = \frac{V_d^2}{2g} + h_d + h_{fd}$$



Therefore the pressure gauge reading at the delivery flange is:

$$\frac{p_2}{w} = \frac{V_d^2}{2g} + h_d + h_{fd} - \frac{V_2^2}{2g}$$

Fig. 8.2

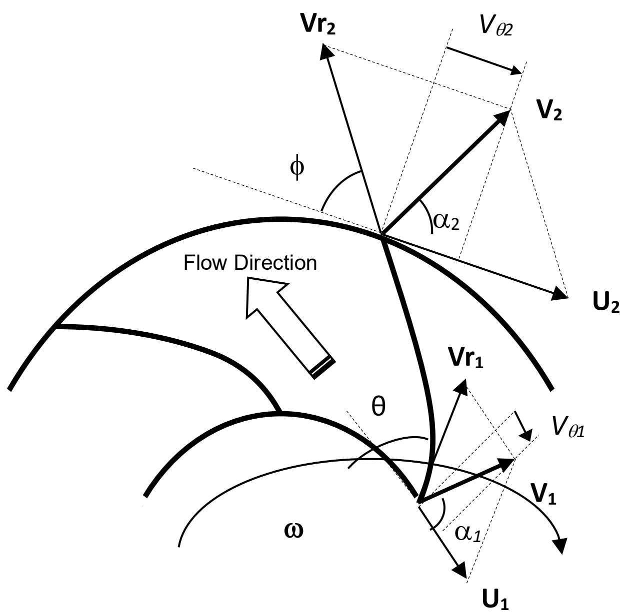
Thus the reading of the pressure gauge across the pump flanges equals:

$$\frac{p_2 - p_1}{w} = (h_d + h_s) \ + (h_{fd} + H_{fs}) + \frac{V_1^2 - V_2^2}{2g} \ + \frac{V_d^2}{2g}$$

$\frac{p_2 - p_1}{w}$ =(Actual lift) +(Friction head lost in pipe) + (Difference of velocity head across pump) + (the velocity head at discharge)

8.5 Equation for Work Done and Velocity Triangles:

Work done by the centrifugal pump on water: The work is done by the impeller on the water. The inlet and out let velocity diagrams is shown in **Fig. 8.3**



Subscript:

1 - inlet

Fig. 8.3 Velocity Triangles at Inlet and Outlet

Let

N = Speed of the impeller in RPM

D_1 = Diameter of Impeller at inlet

U_1 = Tangential Velocity of impeller at inlet = $\frac{\pi D_1 N}{60}$

D_2 = Diameter of impeller at outlet

U_2 = Tangential Velocity of impeller at inlet = $\frac{\pi D_2 N}{60}$

V_1 = Absolute velocity of water at inlet

W_1 = Relative velocity of water at inlet

α_1 = Angle made by absolute velocity (V_1) at inlet with the direction of motion of vane

θ = Angle made by relative velocity (V_{r1}) at inlet with the direction of motion of vane, and

V_2, V_{r2}, β and ϕ are the corresponding values at outlet

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle α=90˚and Vw1 = 0

A centrifugal pump is the reverse of a radially inward flow reaction turbine. The Work done by the water on the runner per second per unit weight of the water striking per second is given by equation W.D. = $\frac{1}{g}\left[V_{w1}u_1 - V_{w2}u_2\right]$

Therefore work done by the impeller on the water per second per unit weight of water striking per second = - [Work Done in case of Turbine]

$$\text{W.D.} = -\left[\frac{1}{g}\left(V_{w1}u_1 - V_{w2}u_2\right)\right] = \left[\frac{1}{g}\left(V_{w2}u_2 - V_{w1}u_1\right)\right] = \frac{1}{g}\left(V_{w2}u_2\right)$$

$$\left(\because V_{w1} = 0 \ \textit{Here}\right)$$

Theoretical Head or Euler Head H_i = $\left[\frac{1}{g}\left(V_{w2}u_2 - V_{w1}u_1\right)\right]$ **Eq.8.4**

Work done by impeller on water per second W.D. = $\frac{W}{g}\left(V_{w2}u_2\right)$ **Eq.8.5**

W = Weight of water = ρ × g × Q

Q = Area × Velocity = $\pi \times D_1 \times B_1 \times V_{f1} = \pi \times D_2 \times B_2 \times V_{f2}$

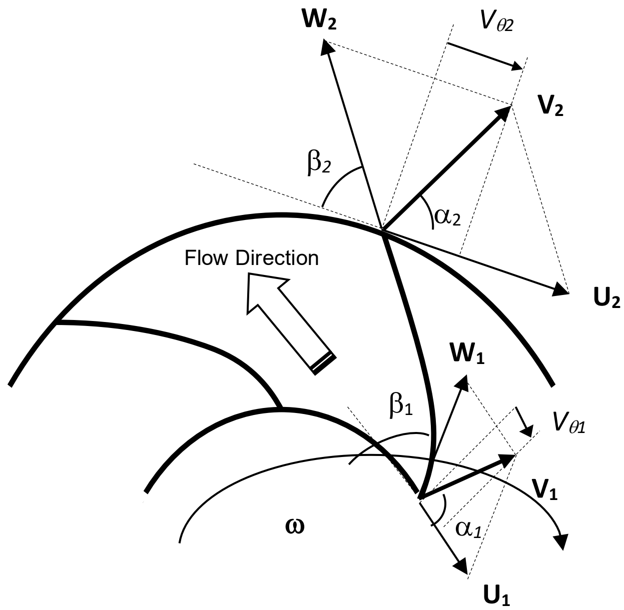
Where, B_1 and B_2 are width of impeller at inlet and outlet

V_{f1} and V_{f2} are the velocities of flow at inlet and outlet

Velocity Triangles: To construct a velocity triangle:

(a) Draw U tangent to the rotor (b) Draw W tangent to the blade surface (c)Draw V

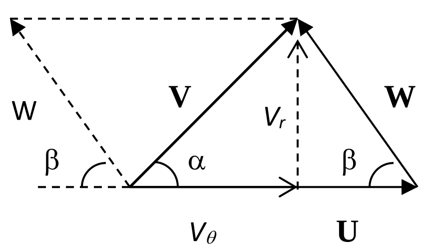
Centrifugal Pump:



Subscript:

1 - inlet

Velocity Triangle:



U: circumferential speed of impeller ($U = r\omega$)

W: velocity tangent to blade surface

V: absolute velocity ($\vec{V} = \vec{U} + \vec{W}$)

V_r: radial component of V

V_θ: circumferential component of V

β: Blade angle

Euler Turbo machine Equation:

- Shaft torque: $T_{shaft} = \rho Q(r_2 V_{\theta 2} - r_1 V_{\theta 1})$
- Brake horsepower: $bhp = \rho Q(U_2 V_{\theta 2} - U_1 V_{\theta 1})$

Note:

- Euler's equation is valid for both pump and turbine
- bhp is the power required to drive shaft of pump (bhp> 0)

  or the power required to deliver to shaft of turbine (bhp< 0)

8.5 Minimum starting speed (centrifugal Pump):

When a centrifugal pump is started, Will start delivering liquid only if the pressure rise in the impeller is more than or equal to the manometric head (H_{mano}). In other words, there will be no flow of liquid until the speed of the pump is such that the required centrifugal head caused by the centrifugal force or rotating water when the impeller is rotating, but there is no flow i.e flow will commence only if $\frac{u_2^2 - u_1^2}{2g} \geq H_m$

For minimum speed, we must have $\frac{u_2^2 - u_1^2}{2g} = H_m$ …Eq. 8.6

But from equation $\eta_{mano} = \frac{gH_m}{V_{w2} u_2}$

$$H_m = \eta_{mano} \times \frac{V_{w2} u_2}{g}$$

Substituting the value of 'H_m' in equation, Eq. 8.6

$$\frac{u_2^2 - u_1^2}{2g} = \eta_{mano} \times \frac{V_{w2} u_2}{g}$$

Since $U_1 = \frac{\pi D_1 N}{60}$ and $U_2 = \frac{\pi D_2 N}{60}$

Substituting the values of U_1 and U_2

$$\frac{\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2}{2g} = \eta_{mano} \times \frac{V_{w2} u_2}{g}$$

$$\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_{mano} \times V_{w2} \frac{\pi D_2}{60}$$

$$N_{\min} = \frac{120 \times \eta_{man} \times V_{w2} \times D_2}{\pi\left(D_2^2 - D_1^2\right)}$$

Minimum speed for starting a Centrifugal Pump:

$$N_{\min} = \frac{120 \times \eta_{man} \times V_{w2} \times D_2}{\pi\left(D_2^2 - D_1^2\right)} \quad \text{Eq...8.7}$$

8.7 Multistage Centrifugal Pumps (Pumps in Series and Pumps in parallel)

8.7.1 PUMP IN SERIES - MULTI - STAGE PUMPS:

The head produced by a centrifugal pump depends on the rim speed of the impeller. To increase the rim speed, either rotative speed or the diameter of the impeller or both must be increased. Increasing either of these has the effect of increasing the stress in the impeller material. For this reason it is usually not possible to produce very high head with one impeller. **Normally a pump with a single impeller can be used to deliver the required discharge against a maximum head of about 100 m.** But if the liquid is required to be delivered against a still larger head then **it can be done by using two or more pumps in series.** The pumps in series may be used in boiler feeding system to satisfy high head demand, in multi stations interposed in the water mains, etc. The higher heads may also be produced by using multistage pumps.

A multi-stage pump consists of two or more identical impellers mounted on the same shaft, and enclosed in the same casing. All the impellers are connected passages to the inlet of the next impeller and so on, till the discharge from the last impeller passes into the delivery pipe. The impellers are surrounded by guide vanes which are generally provided within the connecting passages, and are meant for the recuperation of the velocity energy of the liquid leaving the impeller into pressure energy. According to the number of impellers fitted in the casing a multi-stage pump is designated as two stage, three-stage, etc. Fig. 8.4 shows a three-stage centrifugal pump.

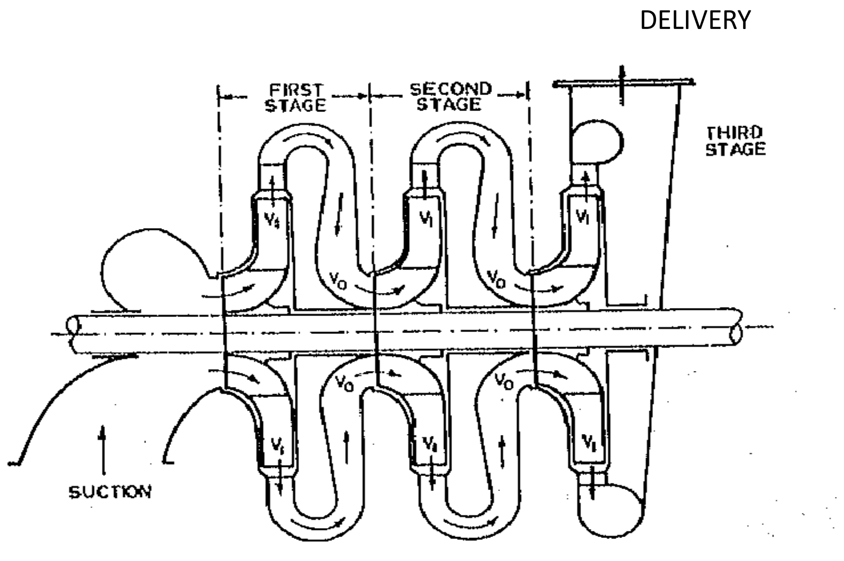


Fig.8.4 Three- stage centrifugal pump

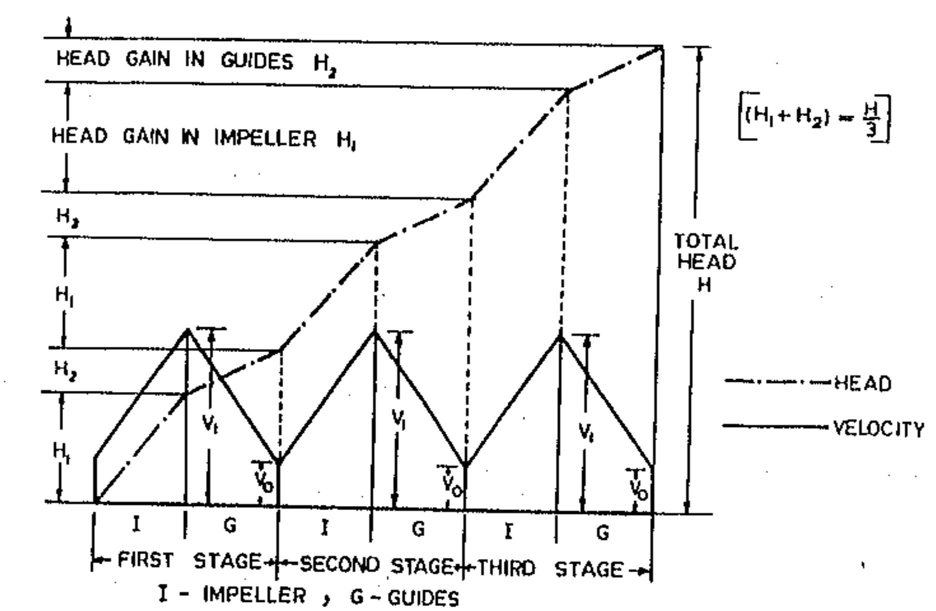


Fig.8.5 Variations of velocity and pressure head in a three – stage pump

Figure 8.5 indicates the variations of the velocity and the pressure head of the liquid as it passes through the pump.As the liquid passes through each impeller the absolute velocity of the velocity decreases again to V_o; but the pressure head continuously increases.

If H_1 and H_2 are the pressure head gained by the liquid in each impeller and the surrounding guide vanes respectively, then the pressure head impressed on the liquid at each stage is $H_m = (H_1+H_2)$. Now, if there are n impellers then sine at each stage the pressure head will be raised by the same amount, the total head H developed by this multi-stage pump will be,

$$H = n\,(H_m) = n\,(H_1 + H_2 + H_3 + H_3) \qquad \text{Eq.(8.8)}$$

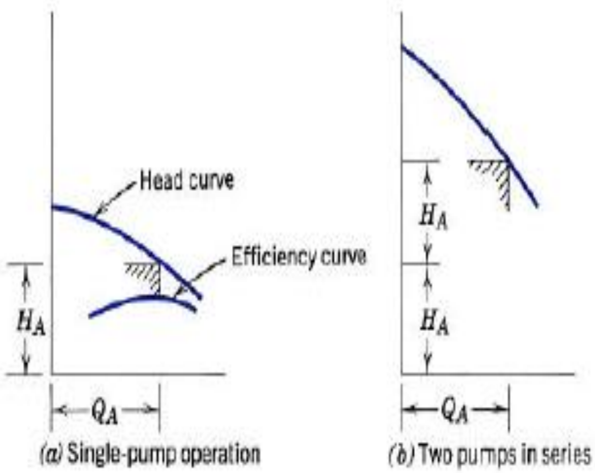
The head loss will be added for individual stage of the pumps

$$H_f = \frac{f_1 L_1 V_1^2}{2gd_1} + \frac{f_2 L_2 V_2^2}{2gd_2} + \frac{f_1 L_3 V_3^2}{2gd_3} \ldots (\textit{For Series pump arrangement})$$

Eq.(8.9)

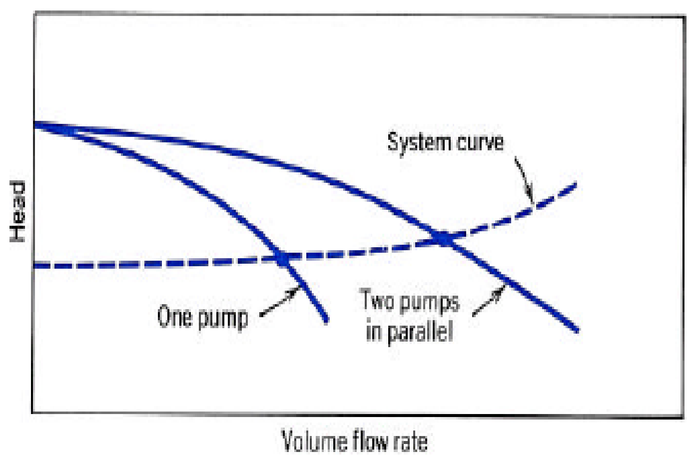
Since the same liquid flows through each impeller, the discharge of a multi-stage pump is same as the discharge passing through each impeller of the series.

The number of stages to be adopted depends on the limitations of speed of the driving motor and also upon the discharge and the total head. If these factors are known, the head per stage can be fixed on the basis of the fact that specific speed Ns per impeller should not be less than about 700, with a provisional maximum limit of head per stage of 160m.



Pumps in Series

$$H_{2s} = 2H_0 - 2AQ^2$$



Pumps in Parallel

$$H_{2p} = H_0 - A\left(\frac{Q}{2}\right)^2$$

Figure 8.6 Operation of pumps in series and parallel

8.7.2PUMPS IN PARALLEL:

The multi-stage pumps or the pumps in series as described earlier are employed for delivering a relatively large head, and then it may not be possible for a single pump to deliver the required discharge. **To deliver a high dischargetwo or more pumps are used in parallel** to lift the liquid from a common sump and deliver it to a common collecting pipe through which it is carried to the required height (Fig.8.7). Since in this case each of the pumps delivers the liquid against the same head, the arrangement is known as pumps in parallel.If Q_1, Q_2, Q_3…………….$Q_n$are the discharging capacities of n pumps arranged in parallel then the total discharged delivered by these pumps will be. In such cases the head loss equation is same

$$H_f = \frac{f_1 L_1 V_1^2}{2gd_1} = \frac{f_2 L_2 V_2^2}{2gd_2} = \frac{f_1 L_3 V_3^2}{2gd_3} \ldots (\textit{For Parallel pump arrangement}) \quad \text{Eq.(8.10)}$$

$Q = (Q_1 + Q_2 + Q_3 \ldots\ldots\ldots\ldots + Q_n)$ Eq.(8.11)

If the discharging capacity of each of the 'n' pumps is same (i.e. $Q_1=Q_2=Q_3$), equal to Q then the total discharge delivered by these pumps will be

$Q = nQ_1$ Eq.(8.12)

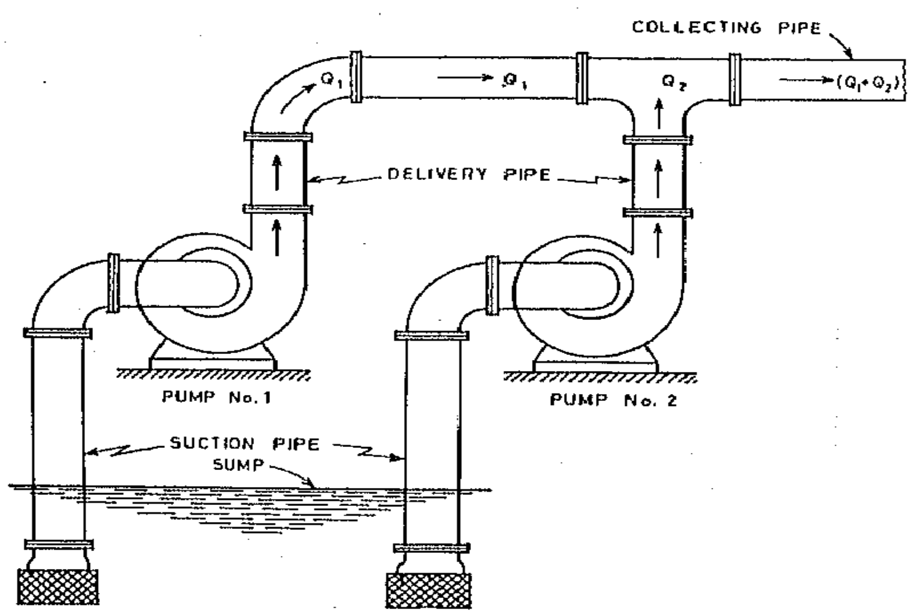


Fig. 8.7 Two Centrifugal pumps arranged in parallel

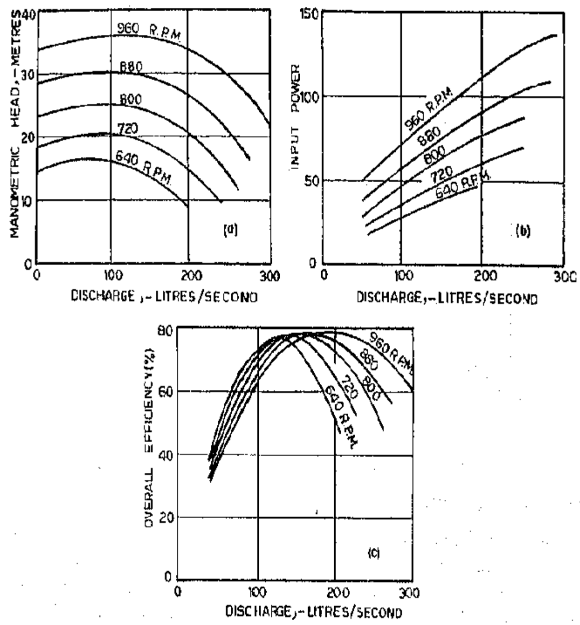
8.8 Characteristic Curves for a Single stage Centrifugal Pumps:

Characteristics curves of centrifugal pump are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump. **These curves are necessary to predict the beahaviour and performance of the pump when the pump is working under flow rate, head and speed.** The following characteristics curves are important for Centrifugal Pumps:

8.8.1 Main Characteristics Curve: Variation of Head, Power and Discharge with respect to speed (Fig. 8.8) $\frac{Q}{D^3 N} = Cons\tan t$ Eq. (8.11)

8.8.2 Operating Characteristics Curves: If the speed is kept constant, the variation of manometric head, power and efficiency with respect to discharge gives the operating characteristics (Fig. 8.9)

8.8.3 Constant Efficiency Curves: For obtaining constant efficiency curves (muschel curves) for a pump, the head versus discharge curves and efficiency versus discharge curves for different speed are used. The points of equal efficiency are plotted and are also called Iso-efficiency curves (Fig. 8.9)



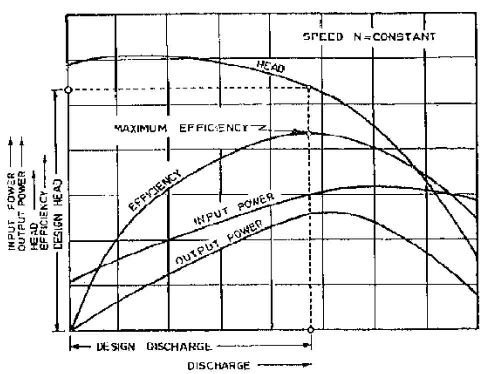


Fig. 24.13 Operating characteristic curves of a centrifugal pump

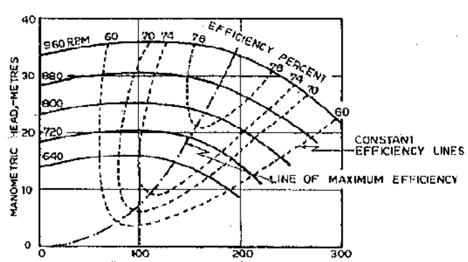


Fig. 8.8 Main Characteristics **Fig. 8.9 Operating Characteristics & Constant efficiency Curves**

System Characteristics and Pump Selection: There is also a uniquerelationship between theactual pump head gainedby the fluid and flowrate,which is governed by thepump design (fig.8.10)

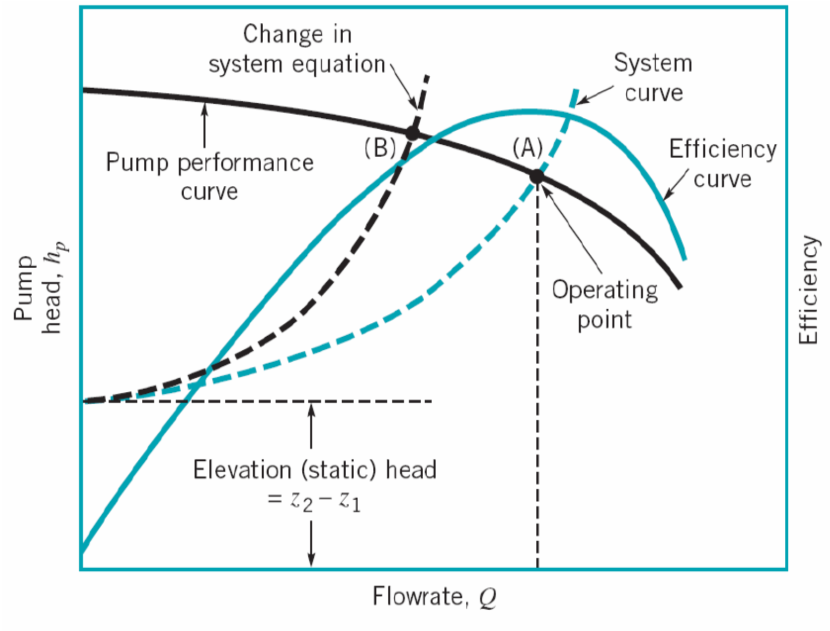


Fig.8.10 System Characteristics for Pump Selection

8.9 Problems:

Q.1. A centrifugal pump running at 800 Rpm is working against a total head of 20.2 m. the external diameter of the impeller is 480mm and outlet width 60mm. If the valve angle at outlet is 40 and manometric efficiency is 70% determine?

a) Absolute velocity of water leaving

b) Flow velocity at outlet

c) Angle made by the absolute velocity at outlet with the direction of motion at outlet.

d) Rate of flow through the pump.

Soln: velocity of valve at outlet $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi x 0.48 x 800}{60} = 20.1 m/s$

$manometric\ efficiency\ n_{mano} = \frac{gHm}{Vw_2 u_2}$, $0.70 = \frac{9.81 x 20.2}{Vw_2\, x 20.1}$, $Vw_2 = 14.08 m/s$

From the outlet velocity triangle $\tan\phi = \frac{Vf_2}{u_2 - Vw_2}$

$\therefore Vf_2 = \tan 40^0\, x(20.1 - 14.08) = 5.05 m/s$

Absolute velocity of water leaving the valve V_2 is given by

$V_2 = \sqrt{Vf_2^2 + Vw_2^2} = \sqrt{5.05^2 + 14.08^2} = 14.96 m/s$

Angle made by the absolute velocity at outlet with the direction of motion is given by

$\tan\beta = \frac{Vf_2}{Vw_2} = \frac{5.05}{14.08} = 0.3586 \qquad \therefore \beta = 19.7^0$

Rate of flow through the pump Q $= \pi D_2 B_2 Vf_2 = \pi x 0.48 x 0.06 x 5.05 = 0.457 m^3/s$

Q2. A centrifugal pump impeller having external and internal diameter 480mm and 240mm respectively is running at 100 Rpm. The rate of flow through the pump is 0.0576 m3/s and velocity of flow is constant and equal to 2.4m/s. the diameter of the section and delivery pipes are 180mm and 120mm respectively and section and delivery heads are 6.2m(abs) and 30.2m(abs) of water respectively. If the power required to drive the pump is 23.3KW and the outlet vane angle is 45 determine

(a) inlet vane angle (b) Overall efficiency (c) manometric efficiency of the pump

Soln: tangential velocity or impeller velocity at inlet

$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi x 0.24 x 1000}{60} = 12.56 m/s$

From the inlet velocity triangle $\tan\phi = \frac{Vf_1}{u_1} = \frac{2.41}{12.56} = 0.191$

$\therefore \theta = 10.8^0 \quad (inlet\ vane\ angle)$

Overall efficiency $n_0 = \frac{\gamma QHm}{P} = \frac{9.81x0.05xHm}{23.3}$ $\therefore n_0 = 0.02387Hm$ (1)

$$but,\ Hm = \left\{\left(Z_2 + \frac{p_2}{r} + \frac{V_2^2}{2g}\right) - \left(Z_1 + \frac{p_1}{r} + \frac{V_1^2}{2g}\right)\right\}$$

$where,\ V_2 = V_d = \frac{4Q}{\pi d_d^2} = \frac{4x0.0567}{\pi x0.12^2} = 5.01m/s$

$where,\ V_2 = V_s = \frac{4Q}{\pi d_s^2} = \frac{4x0.0567}{\pi x0.18^2} = 2.23m/s$

$let\ Z_1 = Z_2$ i.e pump inlet and outlet are at same level.

$= h_s = 6.2m(abs)$ $\frac{p_2}{r} = h_d = 30.2m(abs)$

$$\therefore Hm = \left\{\left(30.2 + \frac{5.01^2}{2x9.81}\right) - \left(6.2 + \frac{2.23^2}{2x9.81}\right)\right\} = 25.03m$$

η_0, overall efficiency of pump= 0.02387x25.03=0.597=59.7%

Velocity of the impeller at outlet $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi x0.48x1000}{60} = 25.13m/s$

From the outlet velocity triangle $\tan\phi = \frac{Vf_2}{u_2 - Vw_2}$, $\tan 45^0 = \frac{2.4}{25.13 - Vw_2}$,

$Vw_2 = 22.73m/s$

Manometric efficiency $n_{mano} = \frac{gHm}{Vw_2 u_2} = \frac{9.81x25.03}{22.73x25.13} = 0.43 = 43\%$

Q.3. It is required to deliver $0.048m^3/s$ of water to a height of 24m through a 150mm diameter and 120m long pipe by a centrifugal pump. If the overall Efficiency of the pump is 75% and co efficient of friction f=0.01 for the pipe line. Find the power required to drive the pump.

Soln: velocity of water pipe $V_s = V_d = V = \frac{4Q}{\pi d^2} = \frac{4x0.048}{\pi x0.15^2} = 2.7m/s$

Overall efficiency $\eta_0 = \dfrac{\gamma Q H m}{P}$ $\quad 0.75 = \dfrac{9.81 x 0.048 x 27.37}{P}$, P=17.2KW

Q4. The following data relate to a centrifugal pump. Diameter of the impeller at inlet & outlet =180mm and 360mm respectively. Width of impeller at inlet and outlet=144mm and 72mm respectively. The rate of flow through the pump=17.28lps. Speed of the impeller = 1500 Rpm. Vane angle at outlet=45 water enters the impeller radially at inlet neglecting losses through the impeller. Find the pressure rise in the impeller.

Soln: velocity of flow at inlet $Vf_1 = \dfrac{Q}{\pi D_1 B_1} = \dfrac{0.01728}{\pi x 0.18 x 0.0144}$

Velocity of flow at outlet $Vf_2 = \dfrac{Q}{\pi D_2 B_2} = \dfrac{0.01728}{\pi x 0.36 x 0.0072} = 2.12 m/s$

Tangential velocity of impeller at outlet $u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi x 0.36 x 1500}{60} = 28.27 m/s$

Pressure rise in the impeller is given by the equation $= \dfrac{1}{2g}\left\{ Vf_1^2 + u_2^2 - Vf_2^2 \cos ec^2 \phi \right\}$

$$= \frac{1}{2 \times 9.81}\left\{ 2.12^2 + 28.27^2 - 2.12^2 \, x \cos ec^2 45^0 \right\}$$

Q.5 A centrifugal pump delivers water at the rate of 1800 lpm, to a height of 20m, Through a 0.1m, dia, 80m. long pipe. Find the power required to drive the pump, if the overall efficiency is 65%, and Darcy's friction factor = 0.02.

Soln. Discharge Q=1800 lpm=0.03 cumecs.

Delivery head hd =20m

Dia of delivery pipe dd=0.1m

Length of delivery pipe $l_d = 80m$

Overall efficiency $\eta_0 = 0.65$ $f = 0.02$

Total head $\mathbf{H} = \mathbf{h_s} + \mathbf{h_d} + \mathbf{h_{fs}} + \mathbf{h_{fd}} + \dfrac{V_d^2}{2g}$

So this prob $h_s = 0$ $h_{fs} = 0$ (details are not given)

$\therefore \mathbf{H} = \mathbf{h_d} + \mathbf{h_{fd}} + \dfrac{\mathbf{Vd^2}}{\mathbf{2g}}$

$$H = \left\{ \mathbf{20} + \frac{\mathbf{8x0.02x80x0.03^2}}{\mathbf{9.81x\pi^2x0.1^5}} + \left(\frac{\mathbf{4x0.03}}{\mathbf{\pi x0.1^2}}\right)^2 x \frac{\mathbf{1}}{\mathbf{9.81x2}} \right\} = 32.65m$$

Output of the pump= $\eta QH = 9.81 x 0.03 x 32.65 = 9.6 kW$

$0.18 = \pi x 0.1 x D_2 \, x2$

But overall efficiency $\eta_0 = \dfrac{Output\ of\ the\ pump}{power\ require\ to\ drive\ the\ pump}$

Power required to drive the pump = 9.6/0.65=14.8kw

Q6. A centrifugal pump is required to deliver 280 ltrs of water per second against a head of 16m, when running at 800rpm. If the blades of the impeller are radial at inlet and velocity of flow is constant and equal to 2m/sec, find the proportions of the pump. Assume overall efficiency as 80% and ratio of breadth to diameter at outlet as 0.1

Soln: the inlet and outlet velocity triangles will be as shown

From continuity equation $Q = \pi D_2 B_2 Vf_2$

$\therefore D_2 = 0.67m$ (*diameter of the impeller at outlet*)

$B_2 = 0.1 \times 0.67 = 0.067m = 6.7cm$ (Width of the impeller at outlet).

$$n_{mano} = \frac{gHm}{Vw_2 u_2} \qquad 0.8 = \frac{9.81x16}{Vw_2 u_2}$$

$$Vw_2 u_2 = 196.2 \quad (i) \qquad but\ u_2 = \frac{\pi D_2 N}{60} = \frac{\pi x 0.67\ x800}{60} = 28.1m/s$$

From eq (i) $Vw_2\ x2.81 = 196.2\ or\ Vw_2 = 6.99m/s$

From the outlet velocity triangle $\tan\phi = \frac{Vf_2}{u_2 - Vw_2} = \left\{\frac{2}{28.1 - 6.99}\right\} = 0.0947$

$\therefore \phi = 5.41^0$ (Blade angle at outlet) $\tan\beta = \frac{Vf_2}{Vw_2} = \frac{2}{6.99} = 0.286\ \therefore \beta = 16^0$

Q7. The following data refer to a centrifugal pump static head = 40m, suction height 5m, dia of suction and delivery pipes = 0.1m, loss of head in suction pipe = 2m, loss of head in delivery pipe = 8m, impeller dia at outlet =0.4m, impeller breadth at outlet 25mm.blades occupy 10% of the outlet area, speed 1200rpm. Exit angle of blade = 1500 with the tangent, Manometric efficiency = 80%, overall efficiency = 70%. Find the power required driving the pump and what pressures will be indicated by the gauges mounted on the suction and delivery sides.

Soln: Outlet vane angle $\therefore \phi = 180 - 150 = 30^0$

Delivery head $h_d = (H_s - h_s) = (40 - 5) = 35m$

Head on the pump H=40+2+8=50m

From the outlet velocity triangle $\tan\phi = \left\{\frac{Vf_2}{u_2 - Vw_2}\right\}$

$$where,\ u_2 = \frac{\pi D_2 N}{60} = \frac{\pi x 0.4x1200}{60} = 25.13m/s$$

Also from the equation $n_{mano} = \frac{gHm}{Vw_2 u_2} \qquad Vw_2 = \frac{9.81x50}{25.13x0.8} = 24.4m/s$

$$so\ \ln : u_1 = \frac{\pi D_1 N}{60} = \frac{\pi x 0.4 x 1200}{60} = 25.13 m/s$$

$$u_2 = \frac{\pi D_1 N}{60} = \frac{\pi x 0.8 x 1200}{60} = 50.26 m/s$$

From the inlet velocity triangle $\tan\theta = \frac{Vf_1}{u_1}$

$$\therefore Vf_1 = Vf_2 = 25.13 \tan 20^0 = 9.15 m/s$$

From the outlet velocity triangle tanΦ=Vf2/(u2-vw2)=9.15/(50.26-V_{w2})

or , Vw2=34.41m/s

Work done/sec=1/g($V_{w2} \times u_2$) =34.41*50.26/9.81=176.3kn-m/s/kn

Q10. The impeller of a centrifugal pump runs at 90 Rpm and has vanes inclined at 120 to the direction of motion at exit. If the manometric head is 20m and manometric efficiency is 75% Vane angles at inlet. Take the velocity of flow as 2.5m/s, throughout and the diameter of the impeller at exits as twice that at inlet.

Soln: From the definition of manometric efficiency $\eta_{mano} = gHm / Vw_2 U_2$

$$Vw_2 U_2 = \frac{9.81 x 20}{0.75} = 261.6\ (i)$$

From the outlet velocity triangle $(U_2 - Vw_2) = \frac{Vf_2}{\tan 60^0} = \frac{2.5}{\tan 60^0} = 1.44 \quad Vw_2 = (u_2 - 1.44)$

Substituting the value Vw_2, $(u_2 - 1.44) u_2 = 261.6 = 16.9 m/s$ $\quad$ *but* $U_2 = \frac{\pi D_2 N}{60}$

$$D_2 = \frac{60 x 16.9}{\pi \times 90} = 3.59 m/s$$

further $u_1 = \frac{u_2}{2} = \frac{16.9}{2} = 8.45 m/s$

From the inlet velocity triangle $\tan\theta = \frac{Vf_1}{u_1} = \frac{2.5}{8.45} = 0.2959$

$\therefore \theta = 16.48^0$ (*Inlet Vane Angle*)

Q.11 A centrifugal pump working in a dock, pumps 1565 litres per second against a mean lift of 6.1m when the impeller rotates at 200 rpm. The impeller diameter rotates at 200 rpm. The impeller diameter is 122cm and the area of outlet periphery is 6450cm2. If the vanes are set back at an angle of 26 ° at the outlet, find (i) Hydraulic efficiency (ii) Power required to drive the pump. If the ratio of external to internal diameter is 2, find the minimum speed to start pumping.

Solution: Peripheral or tangential velocity of Impeller at outlet;

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.22 \times 200}{60} = 12.77 m/s$$

$$Flow \quad Velocity V_{f2} = \frac{Q}{A} = \frac{1565 \times 10^{-3}}{6450 \times 10^{-4}} = 2.43 m/s$$

Whirl component $V_{u2} = u_2 - V_{f2} Cot\beta_2$

$V_{u2} = 12.77 - 2.43 Cot 26° = 7.79 m/s$

Euler Head $H_i = \frac{V_{u2} \times u_2}{g} = \frac{12.77 \times 7.29}{9.81} = 10.14 m$

(i) Neglecting the effect of slip, the ideal head H_i equals the Euler head

Hydraulic Efficiency = $\frac{Actual \ Head}{Ideal \ Head} = \frac{6.1}{10.14} = 0.601 = 60.1\%$

(ii) Power required to drive the pump,

$P = \gamma_w QH = 9810 \times 1.565 \times 6.1$

P = 93651W=93.65kW

(iii) Minimum speed for centrifugal pump to commence is given by,

$\frac{u_2^2 - u_1^2}{2g} = H$; Given $D_2 = 2D_1$; $u_2 = 2 u_1$

$$\therefore \frac{(2\boldsymbol{u}_1)^2 - \boldsymbol{u}_1^2}{2\boldsymbol{g}} = 6.1\boldsymbol{m}$$

;

$$\boldsymbol{u}_1 = \sqrt{\frac{2 \times 9.81 \times 6.1}{3}} = 6.316 \boldsymbol{m/s}; \boldsymbol{u}_2 = 2\boldsymbol{u}_1 = 12.63 \boldsymbol{m/s}$$

Minimum rotational speed N = $\frac{u_2 \times 60}{\pi \times D_2} = \frac{12.63 \times 60}{\pi \times 1.22} = 197.75 \equiv 198 RPM$

N_{min} = 198 RPM

Q.14 A 4-stage centrifugal pump supplying water is to be designed for a total lift of 120m when running at 1450 rpm; its discharge under these conditions is 0.24m³/s. The vanes are set back at an angle of

30° with the tangent to the wheel at outlet, and the impeller is surrounded by guide vanes. The water enters the vane passages in a radial direction, the velocity of flow through the impeller is 0.3 of the outlet peripheral velocity and the losses in the amount to one-third of the velocity head at discharge from the impeller. Find the diameter and width of impeller at outlet. The manometric efficiency and the angle of the guide vanes

Solution: The useful head per stage of centrifugal pump H = $\frac{120}{4} = 30m$

Actual head per stage H_i = H + losses = 30 + $\frac{1}{3} \times \frac{V_2^2}{2g} = 30 + \frac{V_2^2}{6g}$

$H_i = 30 + \frac{V_2^2}{6g}$ ...Eq.1

The actual head equals the energy transfer per unit weight; referred to as Euler head (H_i)

$H_i = \frac{V_{u2} \times u_2}{g}$ Substituting in Eq.1

$H_i = \frac{V_{u2} \times u_2}{g} = 30 + \frac{V_2^2}{6g}$ ... Eq.2

Now given $V_{f2} = 0.3\ u_2$

$V_{u2} = u_2 - V_{f2} Cot\beta_2$

$V_{u2} = u_2 - 0.3 \times u_2\ Cot30° = 0.48u_2$

Now
$$V_2^2 = V_f^2 + V_{u2}^2$$
$$V_2^2 = (0.3u_2)^2 + (0.48u_2)^2 = 0.32u_2^2$$

Substituting values of V_2 and V_{u2} in Eq.2 we get,

$\frac{V_{u2} \times u_2}{g} = 30 + \frac{V_2^2}{6g}$ ... Eq.2

$\frac{0.48u_2 \times u_2}{g} = 30 + \frac{0.32u_2^2}{6g}$ ... Eq.2

On solving u_2 = 26.26 m/s

Since $u_2 = \frac{\pi D_2 N}{60}$; $26.26 = \frac{\pi \times D_2 \times 1450}{60}$

D_2 = 0.346m = 34.6cm

Discharge Q = π×D2×b2×Vf2

0.24=π×0.346×b_2×(0.3×26.26)

b_2= 0.028m = 2.8cm

Actual head per stage = H_i =30 + $\frac{V_2^2}{6g}$ ...Eq.1

H_i = 30+ $\frac{0.32\times u_2^2}{6\times 9.81}$ $=30+$ $\frac{0.32\times 26.26^2}{6\times 9.81}=33.75m$

∴ Manometric efficiency η_{mano}= $\frac{30}{33.75}=0.889=88.9\%$

Now $\tan\alpha_2=\frac{V_{f2}}{V_{u2}}=\frac{0.3\times u_2}{0.48\times u_2}$ =0.625

α_2 =tan^{-1}(0.625) = 32

Q.15 The internal and external diameters of the impeller of a centrifugal pump are respectively 200mm and 400mm. The pump is running at 1000rpm. The vane angles of the impeller a inlet and outlet are 20^0 and 30^0. Water enters rapidly and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.

Soln: internal diameter of the impeller=D_1= 200mm

External diameter of the impeller=D_2= 400mm

Angle at inlet θ = 20^0

Angle at inlet ϕ = 30^0

Water enters radially α=90^0 and V_{w1}=0

Velocity of flow V_{f1}= V_{f2}

Tangential velocity of the impeller at inlet and outlet are

$$U_1=\frac{\pi\times D_1\times N_1}{60}=\frac{\pi\times 0.4\times 1000}{60}=10.46\ m/s$$

From inlet velocity triangle, tanθ= $\tan 20^o=\frac{V_{f1}}{U_1}=\frac{V_{f1}}{10.46}$

V_{f1}= 10.46 tanθ =10.46 tan20^0 = 3.80 m/s

From outlet velocity triangle,

$$U_2 = \frac{\pi \times D_2 \times N_2}{60} = \frac{\pi \times 0.4 \times 1000}{60}$$ = 20.93 m/s

U_2 = 20.93

$$\tan\varphi = \frac{V_{f2}}{(U_2 - V_{w2})}$$

$$\tan 30 = \frac{3.8}{(20.93 - V_{w2})}$$

V_{w2}=14.348 m/s

The work done by the impeller per kg of water per seconds (W)

$$W = \frac{1}{9.81} \times (V_{w2} \times U_2) = \frac{1}{9.81} \times (14.348 \times 20.93) = 30.61\ N - m / N$$