MODULE: FOUR

OPEN CHANNEL FLOW

Introduction:

An open-channel may be defined as any geometrical section through which liquid flows with free surface as a result of gravity (Fig.2.1). Pressure is constant along the channel. The form of free surface is mainly governed by inertial and gravitational forces (Neglecting surface tension).

Importance: The Knowledge of open-channel hydraulics is essential to the world. It finds application in the solution of problems related to several aspects of development of surface water resources. The engineer may be required to solve problems like design and construction of hydraulic structures, dispersion of pollutants, overland flow and sediment transport, in rivers requires the use of Principles of Open-channel flows.

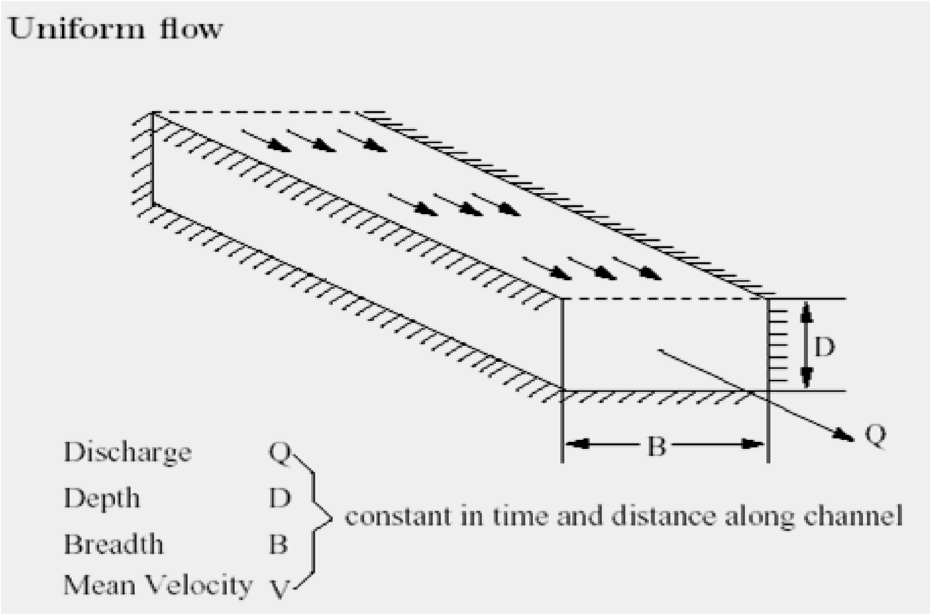


Fig.2.1 Uniform Flow in Open-Channel

The natural drainage of water through numerous creek and river systems is a complex example of Open – channel flow. Other occurrences of open-channel flow are in irrigation canals, sewer lines that flow partially storm drains, street, gutters, the flow of small rivulets and sheets of water across fields of parking lots (Fig 2.2)

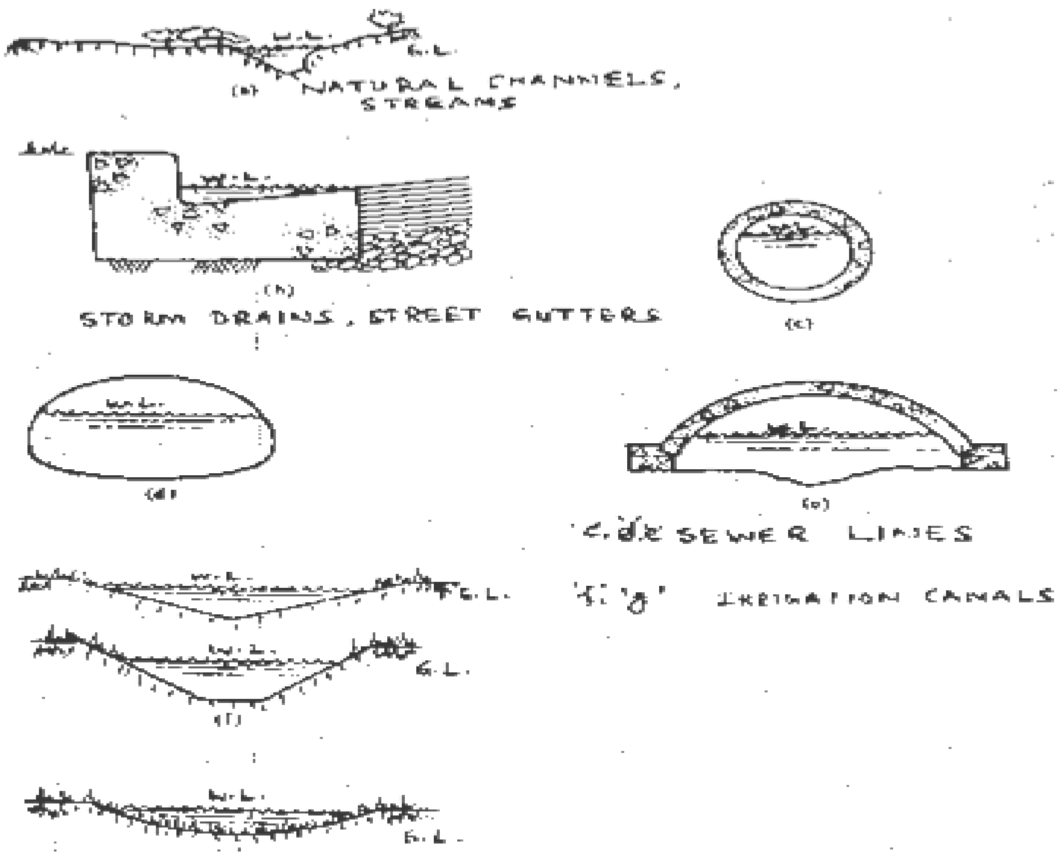


Fig. 2.2 Examples of Open-Channel Flow

COMPARISON BETWEEN PIPE FLOW AND OPEN-CHANNEL FLOW: The Character description and complexity of open-channel flow geometry is quite variable, as compared to pipe flow (Fig. 2.3). Because of complexities like these, most of the open-channel flow results are based on correlations obtained from model and full scale experiments. The formulae defining open channel flow behaviour are mainly empirical in nature as compared to pipe (pressure) flow.

The comparison between pipe flow and open-Channel flow is given in Table 2.1 below:

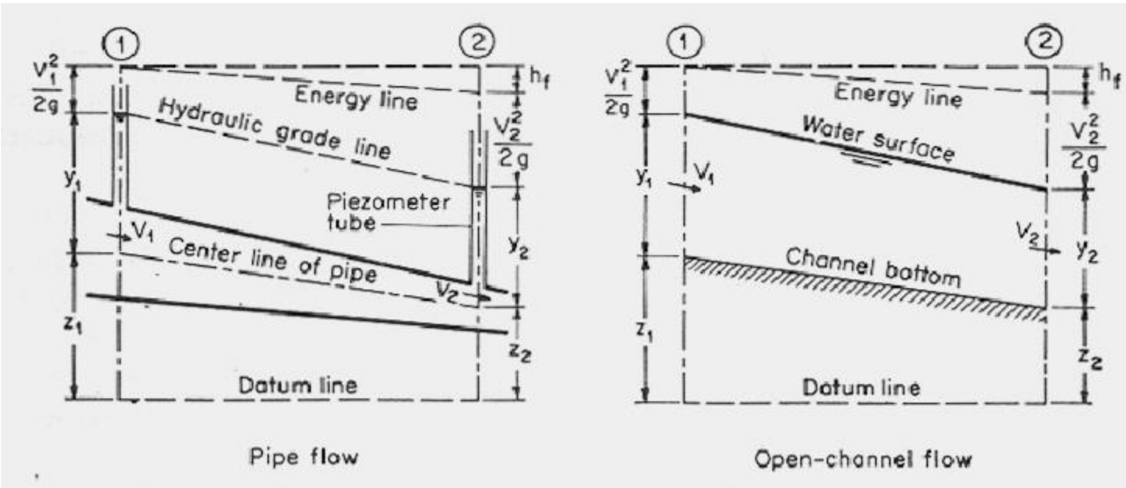
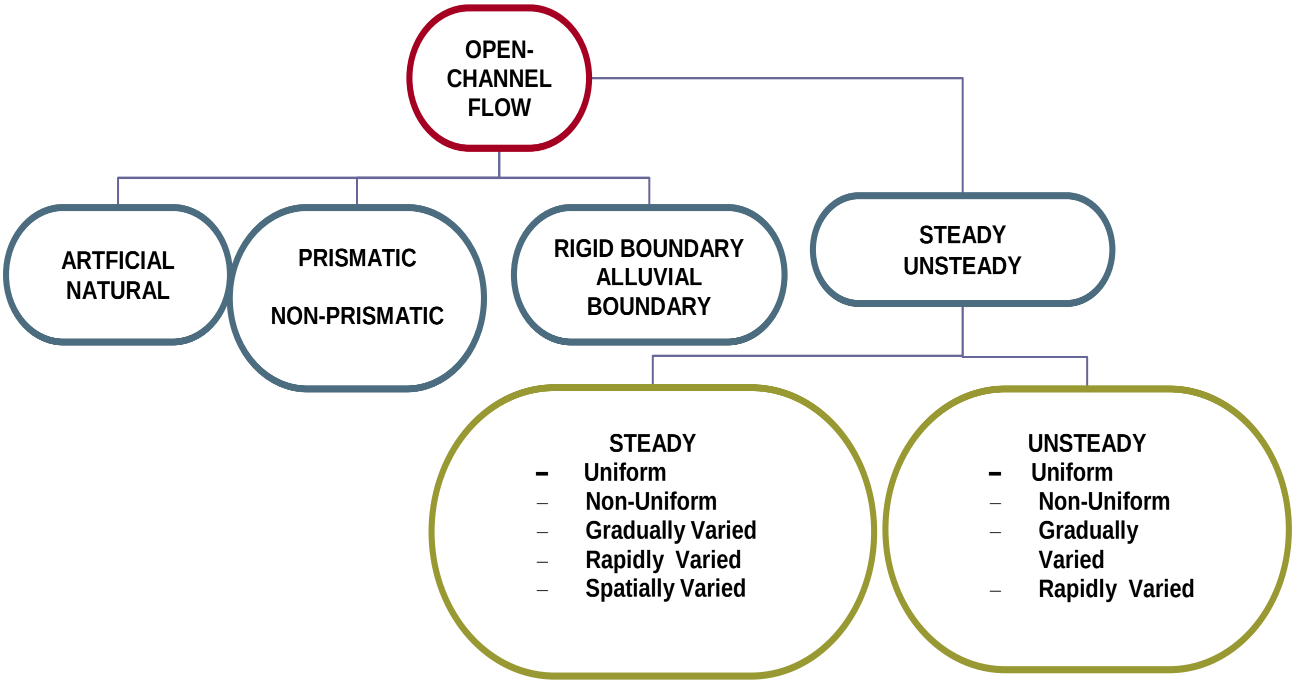


Fig. 2.3 Comparison between pipe flow and open – channel flow

Sl. No	Description	Pipe flow	Open – channel flow
1	Identification	Absence of a free surface	It has free surface open to atmosphere
2	Hydraulic gradient line (H.G.L)	It is defined as the sum of pressure head and datum head i. e $\left(\frac{p}{\gamma}+Z\right)$	The free surface of water itself represents the Hydraulic gradient line (HGL)
3	Analytical Solution	Solution of pipe flow problems are normally simple and well defined	Solution of open-channel flow is more complicated compared to pipe flow. In general the treatment of open-channel flow is somewhat more empirical compared to pipe flow
4	Variation of roughness configuration and cross-sectional shape	Normally roughness and cross sectional shape do not vary in a wider range.	Both roughness as well as cross sectional shape varies over a wide range

5	Some example	Flow of water, oil, gas In penstocks, industrial pipes, pipelines of water distribution	Flow in canals, laboratory flumes, natural channels, rivers, partly filled sewer lines, gutters etc(Fig. 1.2)

Classification of Open – Channel flow



<u>Steady or unsteady flow (Time as the criterion):</u> Flow in an open channel is said to be steady if the depth(y) discharge (Q) and mean velocity (V) at any section do not change with time. If these quantities change with time, the flow is said to be unsteady.

Mathematically for steady flow,

$$\frac{\partial y}{\partial t}=0, \frac{\partial Q}{\partial t}=0, \frac{\partial v}{\partial t}=0 \quad \textit{for steady flow at a given section}$$

<u>Uniform and non-uniform flow</u> (Space as the criterion): Open –channel flow is said to be uniform if the depth(y), discharge (Q) and mean velocity (V) remains the same at every section of the channel (Fig. 2.1). If these quantities change along the length of the channel then, the flow is said to be non-uniform or varied flow. Mathematically for uniform flow mathematically for uniform flow,

i) Uniform Steady flow	iii) Uniform unsteady flow

	(very rare in practice)
ii) Non – Uniform steady flow a) Gradually varied flow (G.V.F) b) Rapidly varied flow (R.V.F) c) Spatially varied Flow (S.V.F.)	iv) Non-uniform unsteady flow a) Gradually varied flow (G.V.F) b) Rapidly varied flow (R.V.F) c) Spatially varied Flow (S.V.F.)

$$\frac{\partial \mathbf{y}}{\partial \mathbf{s}} = 0, \frac{\partial \mathbf{Q}}{\partial \mathbf{s}} = 0, \frac{\partial \mathbf{v}}{\partial \mathbf{s}} = 0 \quad \textbf{for uniform flow at every section of the channel}$$

The different types of open-channel flow are summarized in Table 2.2

Table – 2.2 Types of Open Channel Flow

The following classifications are made according to change in flow depth with respect to time and space.

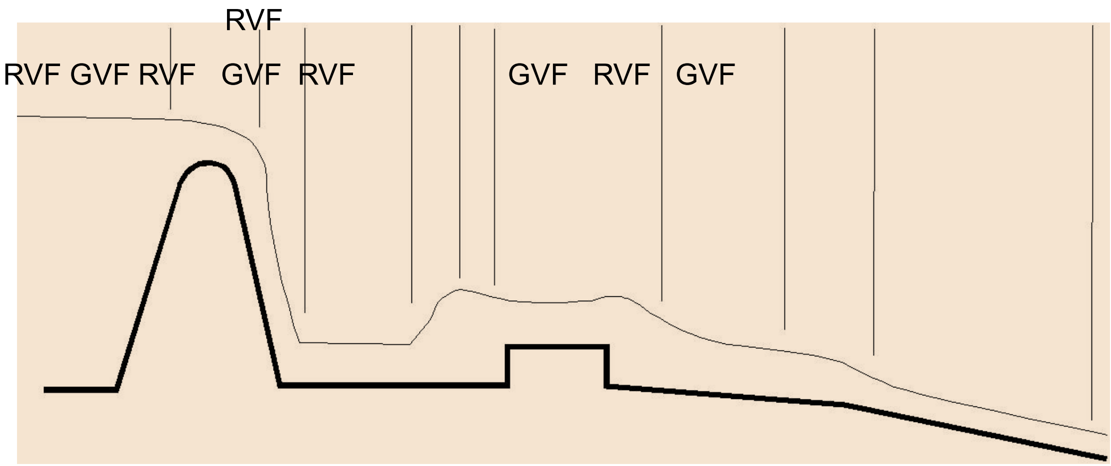


Figure 2.4 Types of flow in open channels

Important Definitions:

- Hydraulic radius "R_h" is defined by, $\mathbf{R_h} = \dfrac{\textbf{Wetted Area}}{\textbf{Wetted Perimeter}} = \dfrac{\mathbf{A}}{\mathbf{P}}$

- Hydraulic Depth (D): It is defined as the ratio of wetted area (A) to Top width of water section (T)

$$\mathbf{D} = \frac{\mathbf{A}}{\mathbf{T}}$$

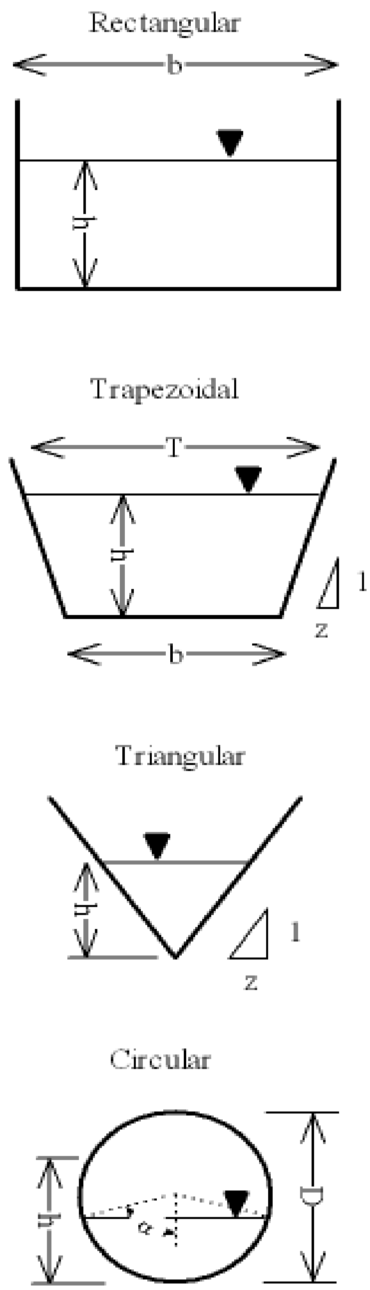
Properties of Open Channels:

Artificial channels: These are channels made by man. They include irrigation canals, navigation canals, spillways, sewers, culverts and drainage ditches. They are usually constructed in a regular cross-section shape throughout – and are thus prismatic channels (they don't widen or get narrower along the channel. In the field they are commonly constructed of concrete, steel or earth and have the surface roughnesses reasonably well defined (although this may change with age – particularly grass lined channels.) Analysis of flow in such well defined channels will give reasonably accurate results.

Natural channels: Natural channels can be very different. They are neither regular nor prismatic and their materials of construction can vary widely (although they are mainly of earth this can possess many different properties.) The surface roughness will often change with time distance and even elevation.

Consequently it becomes more difficult to accurately analyse and obtain satisfactory results for natural channels than is does for manmade ones. The situation may be further complicated if the boundary is not fixed i.e. erosion and deposition of sediments.

2.2 Geometric properties of Rectangular, Triangular, Trapezoidal and Circular



Channels: **F**or analysis various geometric properties of the channel cross-sections are required. For artificial channels these can usually be defined using simple algebraic equations given *'y'* the depth of flow. The commonly needed geometric properties are shown in the figure below and defined as:

Depth (*y*) – The **vertical** distance from the lowest point of the channel section to the free Surface

Stage (*z*) – the vertical distance from the free surface to an arbitrary datum

Area (*A*) – the cross-sectional area of flow, normal to the direction of flow

Wetted perimeter (*P*) – the length of the wetted surface measured normal to the direction of flow.

Surface width (T) – width of the channel section at the free surface

Hydraulic radius (*R*) – the ratio of area to wetted perimeter (*A/P*)

Hydraulic mean depth (D_m) – the ratio of area to surface width (*A/T*)

Rectangular Channel:

Area 'A' = h × b,

Hydraulic Radius $R_h = \dfrac{A}{P} = \dfrac{bh}{b + 2h}$

a) Trapezoidal Channel :

Area ‘A’ = (b+zh)h

Hydraulic Radius $R_h = \frac{A}{P} = \frac{(b+hz)h}{\left(b+2h\sqrt{1+z^2}\right)}$

b) Triangular Channel:

Area ‘A’ = $\frac{1}{2}zh^2$

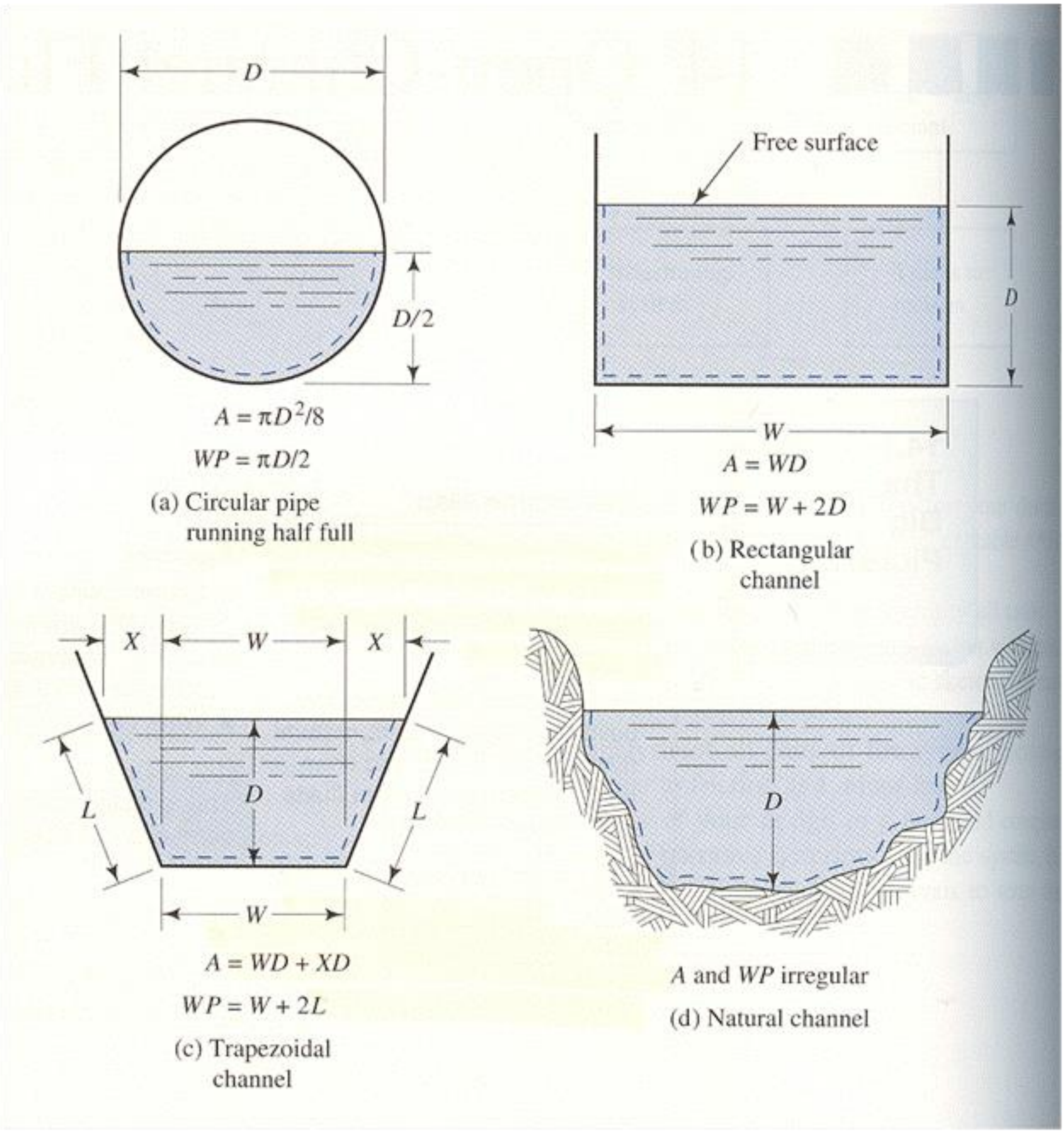
Hydraulic Radius $R_h = \frac{A}{P} = \frac{\frac{1}{2}zh^2}{2h\sqrt{1+z^2}} = \frac{1}{4}\frac{zh}{\left(\sqrt{1+z^2}\right)}$

c) Circular Channel

Area ‘A’ = $\frac{D^2}{4}\left(\alpha - \frac{Sin(2\alpha)}{2}\right)$ where $\alpha = \cos^{-1}(1-(h/r))$

P = αD

Hydraulic Radius $R_h = \frac{A}{P}$



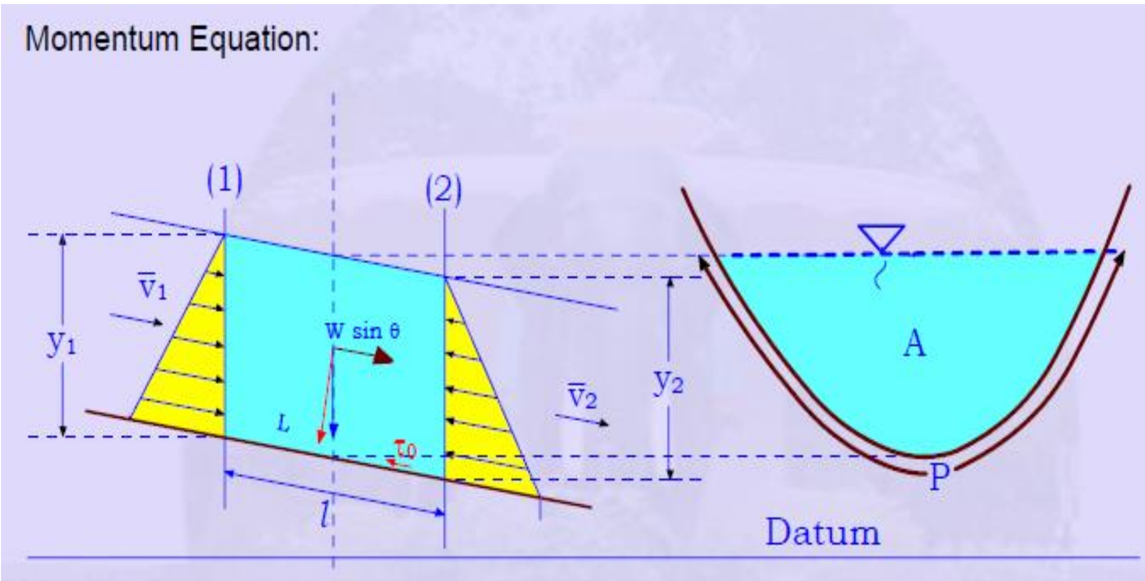
D
D/2
$A = \pi D^2/8$
$WP = \pi D/2$
(a) Circular pipe running half full
Free surface
D
W
$A = WD$
$WP = W + 2D$
(b) Rectangular channel
X
W
X
L
D
L
W
$A = WD + XD$
$WP = W + 2L$
(c) Trapezoidal channel
D
A and WP irregular
(d) Natural channel

2.3 Chezy's equation, Manning's equation:

2.3.1 Chezy's Equation: The mean velocity of a turbulent uniform open channel flow is obtained using the following concept.

Gravitational Force = Shear Force

The uniform flow equations are in the following format $V = CR^x S^y$ in which x and y are components, and vary depending on uniform formula (Fig.2.5).



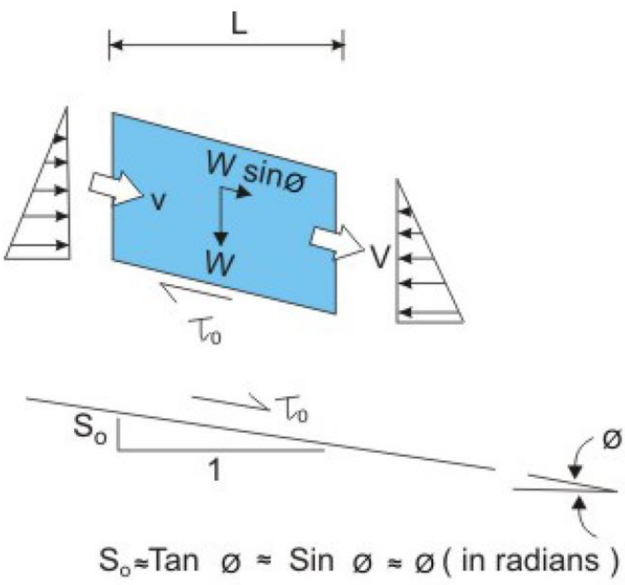


Fig. 2.5Shear stress in an Open Channel

Driving force: gravity

Resisting force: skin friction

Balance τ_0 PL = ρg A L sinϕ

[Bed slope ϕ< 0:5°]

sinϕ ~S_0 (slope)], $\tau_0 = \rho g A S_0 = P = \rho g R_h S_0 = \gamma R_h S_0$

The shear stress $\boxed{\tau_0 = \gamma \; R_h \; S_0}$ Eq...2.1

The Reynolds number in an open channel is given by $R_e = \frac{R_h V}{v_{nu}}$, $R_h = \frac{A}{P}$

If Re > 1000 the flow in open-channel flow is Turbulent. Assuming a state of rough turbulent flow, as is the case for natural rivers and channels, one may write channels, one may write $\tau_0 \; \alpha \; V^2$.For rough turbulent channel flow

$$\tau_0 = KV^2$$

$$V = \sqrt{\frac{\tau_0}{k}}$$

$$V = \sqrt{\frac{\gamma R_h S_0}{k}} = C\sqrt{R_h S_0}$$

Chezy's equation $$\boxed{V = C\sqrt{R_h S_0}}$$ Eq…2.2

The Eq. 2.2 is known as Chezy's equation after the French hydraulic engineer. Antoine Chezy who first proposed the formula around 1768 while designing a canal for Paris water supply. The constant '***C'***in equation 2.2***actually varies depending on Reynolds number and boundary roughness.***

2.3.2 Manning's Equation:

In 1889, Robert Manning's, an Irish engineer proposed another formula for the evaluation of the Chezy coefficient (C), which was later simplified to:

$$C = \frac{R_h^{1/6}}{n}$$ Eq…2.3

From Equation 2.3 the Manning equation may be written as:

Manning's Equation $$V = \frac{1}{n} R_h^{2/3} S_0^{1/2}$$ Eq…2.4

Where Manning's 'n' is a coefficient known as Kutter's 'n', and is dependent solely on the boundary roughness. The Manning equation has the great benefits that it is simple, accurate and now due to it long extensive practical use, there exists a wealth of publicly available values of n for a very wide range of channels.

Type of channel boundary surface	Value of n
Very smooth surface such as glass, plastic or brass	0.010
Very smooth concrete and planned timber	0.011
Smooth concrete	0.012
Ordinary concrete lining	0.013
Glazed brick work	0.014
Vitrified clay	0.014
Brick surface lined with cement mortar	0.015
Cement concrete finish	0.015
Unfinished cement surface	0.017
Earth channel in best condition	0.017
Neatly excavation earth canals in good condition	0.017
Straight unlined earth canals in good condition	0.020
Rubble masonry	0.020
Corrugated metal surface	0.020
River and earth channels in fair condition	0.025
Earth channel with gravel bottom	0.025
Earth channel with dense weed	0.035
Mountain stream with rock beds and rivers with variables section & some vegetation along banks	0.045

Conveyance (K)

Channel conveyance, *K,* is a measure of the carrying capacity of a channel. The *K* is really an

Agglomeration of several terms in the Chezy's or Manning's equation:

$$Q = AC\sqrt{R_h S_0} \qquad \text{Eq....2.5}$$

$$Q = K\sqrt{S_0}$$

The value of $K = AC\sqrt{R_h}$ Eq....2.6

Use of conveyance 'K' may be made when calculating the discharge and stage in compound channels. It is also used for calculating the energy and momentum coefficients in compound channels.

2.4 Problems – on Chezy's equation, Manning's equations:

Solved Problems:

Example: 1

A concrete lined trapezoidal channel with uniform flow has a normal depth is 2m. The base width is 5m and the side slopes are equal at 1:2. Manning's '*n*'can be taken as 0.015 And the bed slope S_0 = 0.001. Calculate Discharge (Q), Mean velocity (V) and Reynolds number (Re). Given ρ = 1000 kg/m^3

and Viscosity 'μ' = 1.14 X 10^{-3}N-s/m^2

Solution: The section properties

$$A = (5+2y)y = 18m^2$$
$$P = 5 + 2y\sqrt{1+2^2} = 13.94m$$

Using Manning's Equation to get the discharge

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_o^{1/2} = \frac{1}{0.015}\frac{18^{5/3}}{13.94^{2/3}}0.001^{1/2}$$
$$= 45m^3/s$$

The mean velocity (V) can be obtained from continuity equation: Q = AV

$$V = \frac{Q}{A} = \frac{45}{18} = 2.5m/s$$

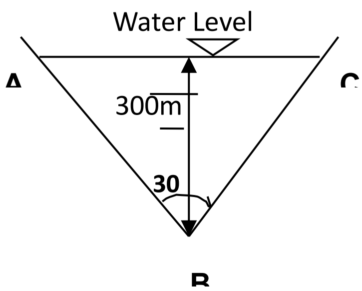
And the Reynolds number (Re =$\rho u R_h/\mu$, R_h = A/P)

$$\text{Re}_{\text{channel}} = \frac{\rho u R}{\mu} = \frac{\rho u A}{\mu P} = \frac{10^3 \times 2.5 \times 18}{1.14 \times 10^{-3} \times 13.94} = 2.83 \times 10^6$$

Reynolds number is very large hence it is in the turbulent zone

Example: 2

A triangular gutter whose sides include an angle of 60^0 conveys water at a uniform depth of 300 mm. If the bed gradient is 1 in 150 find the discharge. Take Chezy's constant C= 55 m $^{1/2}$/s.



Ans: AB =CB=AC

=0.30 Sec 30^0

=0.3464m

Area of cross section of flow area

$$A = \frac{1}{2}(0.3464)\ 0.30$$

$$=0.05196\ m^2.$$

Wettedperimeter

$$P= 2 \times 0.3464\ m = 0.6928\ m$$

Hydraulic mean depth $R_h = \frac{A}{P} = \frac{0.05196}{0.6928} = 0.075m$

Discharge $Q = AC\sqrt{R_h S_0}$

$$= 0.05196 x 55 \sqrt{\frac{0.075}{150}} m^3/s$$

$$Q = 0.0639 m^3/s = 63.90 \text{ liters/sec}$$

Example: 3

A triangular channel with an apex angle of 75^0 carries a flow of $1.2m^3/s$ at a depth of 0.80 m. If the bed slope is 0.009, find the roughness coefficient of the channel.

Ans: y_0 = Depth of flow = 0.80m

Area $A = \frac{1}{2} x 0.80 x 2 x 0.8 \tan \frac{75}{2}$ $= 0.491m^2$

Wetted perimeter $P=2x0.8x \sec 37.5^0 = 2.0168m$

$R_h = A/P = 0.243$ m.

$$n = \frac{AR_h^{2/3}S_0^{1/2}}{Q} = \frac{(0.491)x(0.243)^{2/3}X(0.009)^{1/2}}{1.20} = 0.0151.$$

Example: 4

A trapezoidal section has side slopes of 1 vertical to 1 horizontal and has to convey a dischargeof $14 m^3/s$. The bed slope of the channel is 1 in 1000. Chezy's constant is 45 if the channel is unlined and is 70 if the channel is lined with concrete. Calculate the cost per meter length of providing the lining with and without lining and state which arrangement is economical? Given the cost per square metre of lining = Rs 'x'.

Ans: Given side slope's' = 1, Bed Slope $S_0 = \frac{1}{1000}$

For the trapezoidal channel of best section following criteria has to be satisfied

$$\frac{b+2sy}{2} = y\sqrt{s^2+1} \qquad \frac{b+2x1xy}{2} = y\sqrt{1^2+1}$$

$b+2d=2\sqrt{2}\,y$, $b = (2\sqrt{2}-2)\,y$ $b=0.8284y$

$A = d\,(b + s\,y) = y\,(0.8284y+1y)$

$A = 1.8284y^2$

The cost per square metre for the lined channel = Rsx

Cost per cum of excavation = Rs3x

<u>Case (i) Unlined channel: Cost of Unlined channel</u>

Discharge $Q = AC\sqrt{R_h S_0}$ =14 (given discharge)

$$\therefore 1.8284\, y^2 \times 45 \sqrt{\frac{y}{2}}\sqrt{\frac{1}{1000}} = 14$$

$$y^{5/2} = \frac{14\sqrt{2000}}{1.8284 x 45}$$

$y = 2.252m$

$A=1.8284\,y^2 = 1.8284 \times 2.252^2 = 9.9727\ m^2$.

Cost of excavation for 1 metre length of unlined channel = 9.2727 x 1 x 3x

= Rs27.8181 x

<u>Case (ii) Unlined channel: Cost of lined channel</u>

Discharge $Q = AC\sqrt{R_h S_0}$ =14

$$1.8284\mathbf{d}^2 \mathbf{x} 70 \sqrt{\frac{d}{2}}\sqrt{\frac{1}{1000}} = 14$$

$$\mathbf{d}^{5/2} = \frac{14\sqrt{2000}}{1.8284\mathbf{x}70}$$ d = 1.887 m

$A= 1.8284\ d^2 = 1.8284 \times 1.887^2 = 6.5105\ m^2$

Wetted perimeter,

$$P = b + 2y\sqrt{s^2 + 1}$$

$$P = 0.8284y + 2y\sqrt{1^2 + 1}$$

$$\mathbf{P} = 0.8284\mathbf{x}1.887 + 2\mathbf{x}1.887\sqrt{2}$$

P = 1.5632 +5.3372

P= 6.9004m

For 1m-length of the channel the cost of excavation and lining

Cost of excavation = 6.5105 x 1 x 3x = Rs19.5315x

Cost of lining = 6.9004 x 1 x x = Rs6.9001x

Total cost of excavation and lining for one metre length = **Rs 26.4316 x**

But the cost per metre length of the unlined channel = Rs27.818x

Hence the lined canal will be preferable as against the unlined channel for construction

Example: 5

Design a **rectangular** channel in formed unfinished concrete for the give data:

(i) Q = 5.75 m^3/s (ii) Slope S = 1.2% (iii) Depth = ½ of the width of the channel (iv) n = 0.012

Solution: The equation $AR_h^{2/3} = \frac{nQ}{\sqrt{S_0}}$ Eq(1)

In Eq (1) RHS is known ∴ RHS = $0.012*5.75/(0.012)^{1/2} = 0.63$

Given y = b/2

Express Area and the hydraulic radius in terms of 'y'.

$A = by = 2y^2$

P = b+ 2y = 4y

$R_h = A/P = y/2$

Therefore, $LHS = AR^{2/3} = 2y^2 * \left(\frac{y}{2}\right)^{2/3}$ = RHS = 0.63

$$y = (0.5)^{3/8} = 0.771m$$
$$B = 2 \times y = 2 \times 0.771 = 1.542m$$

Example: 6

Find the diameter of a circular pipe which is laid at a slope of 1 in 7500 and carries a discharge of 1000 litres/s when flowing half full. Given the value of Manning's n = 0.018

Solution: Slope of Pipe $S_0 = \frac{1}{7500}$

Discharge = 1000 litres/s = 0.1m^3/s

Manning's n = 0.018

Let the diameter of pipe = D

Depth of flow'd' = $\frac{D}{2}$

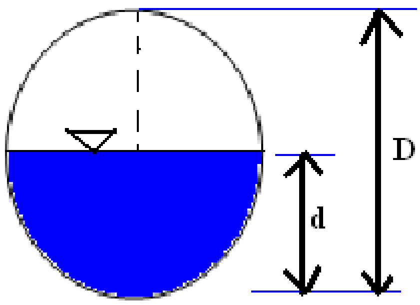


Fig. Circular Sewer Pipe

$$A = \frac{1}{2}\left(\frac{\pi}{4} \times D^2\right)$$

$$P = \frac{\pi D}{2}$$

Hydraulic mean depth $R_h = \frac{A}{P} = \frac{\pi\left(\frac{D^2}{8}\right)}{\pi\left(\frac{D}{2}\right)} = \frac{D}{4}$

Using Manning's formula for velocity and writing equation for discharge

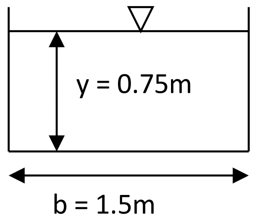
$$Q = A \times \frac{1}{n}(R_h)^{2/3}(S_0)^{1/2}$$

$0.1 = \pi\frac{D^2}{8} \times \frac{1}{0.018}\left(\frac{D}{4}\right)^{2/3}\left(\frac{1}{7500}\right)^{1/2}$ **on solving D = 2.371m**

Example: 7

Find the gradient necessary for a rectangular flume 1.5m wide and 0.75m deep to deliver 3.5m^3/s of water when running full. Take C = 72.

Solution:



$A = by = 1.5 \times 0.75 = 1.125 \text{ m}^2$

$P = b + 2y = 1.5 + 2 \times 0.75 = 3 \text{ m}$

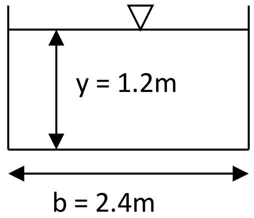
Hence, $R = \dfrac{A}{P} = \dfrac{1.125}{3} = 0.375 \text{ m}$

$$Q = AC\sqrt{RS_0} \quad \Rightarrow \quad S_0 = \frac{1}{R}\left(\frac{Q}{AC}\right)^2 = \frac{1}{0.375}\left(\frac{3.5}{1.125 \times 72}\right)^2 = 1/206.5$$

Example: 8

A rectangular open channel has a bottom width of 2.4m and a surface roughness corresponding to Manning n = 0.015. If the slope of the bed is 0.001 and the depth of flow is 1.2m, what is the discharge under conditions of uniform steady flow?

Solution:



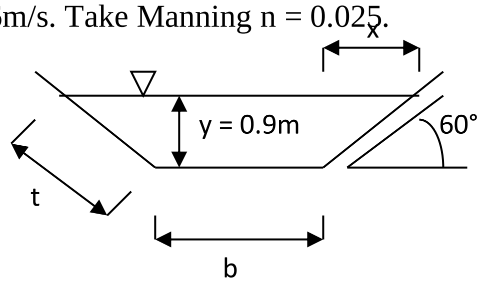
$A = by = 2.4 \times 1.2 = 2.88 \text{ m}^2$

$P = b + 2y = 2.4 + 2 \times 1.2 = 4.8 \text{ m}$

Hence, $R = \dfrac{A}{P} = \dfrac{2.88}{4.8} = 0.6 \text{ m}$

$$Q = \frac{A}{n} R^{2/3} S_0^{1/2} = \frac{2.88}{0.015} \times 0.6^{2/3} \times 0.001^{1/2} = 4.32 \text{ m}^3/\text{s}$$

3. A channel of trapezoidal section, with side slopes at 60° to the horizontal base of the channel, conveys water at a depth of 0.9m. Find the width of the base and the gradient of the bed to discharge 1.7m^3/s with a mean velocity of flow of 0.6m/s. Take Manning n = 0.025.



Solution: $Q = Av \quad \Rightarrow \quad A = \dfrac{Q}{v} = \dfrac{1.7}{0.6} = 2.833$

From Figure $\tan 60° = 0.9/x$

$\therefore x = 0.9/\tan 60°$ and thus

$A = 0.9[b + (b + 2 \times x)]/2 = 2.833$

$\therefore b + x = 2.833/0.9 \quad \Rightarrow \quad b = 3.148 - 0.9/\tan 60° = 2.628\text{m}$

From the diagram, $\sin 60° = 0.9/t \quad \Rightarrow \quad t = 0.9/\sin 60°$

and thus $P = b + 2t = 2.628 + 2 \times 0.9/\sin 60° = 4.706\text{m}$

$$R = A/P = 2.833/4.706 = 0.602m$$

$$Q = \frac{A}{n} R^{2/3} S_0^{1/2} \quad \Rightarrow \quad S_0 = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025 \times 1.7}{2.833 \times 0.602^{2/3}}\right)^2 = 1/2259$$

Example: 9

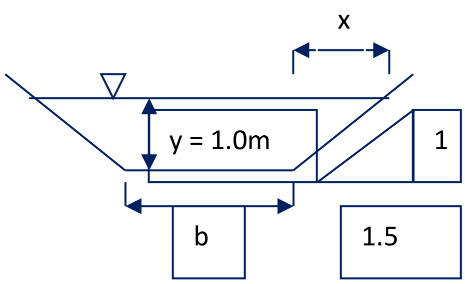
A canal has a bottom width of 4m and sides with a slope of 1 vertical to 1.5 horizontal. The depth of water is 1.0m when the discharge is 4 m^3/s.

(a) Calculate the slope of the channel bed using the Manning formula withn = 0.022.

(b) Calculate the discharge in m^3/s when the depth of flow is 1.2m.

Solution:

(a) $$A = \frac{1.0}{2}[b + (b + 2 \times 1.5 \times y)] = 4 + 1.5 \times 1.0 = 5.5m^2$$



$$P = b + 2y\sqrt{1^2 + 1.5^2} = 4 + 2 \times 1.0\sqrt{3.25} = 7.6m$$

$$R = \frac{A}{P} = \frac{5.5}{7.6} = 0.724m$$

$$Q = \frac{A}{n} R^{2/3} S_0^{1/2} \quad \Rightarrow \quad S_0 = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.022 \times 4}{5.5 \times 0.724^{2/3}}\right)^2 = 1/2540$$

(b) $$A = 1.2(4 + 1.5 \times 1.2) = 4 + 1.5 \times 1.2 = 6.96m^2$$

$$P = 4 + 2 \times 1.2\sqrt{3.25} = 8.327m$$

$$R = \frac{A}{P} = \frac{6.96}{8.327} = 0.836m$$

$$Q = \frac{A}{n} R^{2/3} S_0^{1/2} = \frac{6.96}{0.022} 0.836^{2/3} \times \frac{1}{2540^{1/2}} = 5.57\, m^3/s$$

2.5Most Economical Open Channels - Rectangular, Triangular, Trapezoidal and Circular channels: The most efficient cross section may be defined as that which offers least resistance to flow and thus passes the maximum discharge for a given slope(S_o), Area (A), and roughness and Manning's N (N) referring to manning s formula

$$V = \frac{1}{n} R_h^{2/3} \sqrt{S}_e$$

Where, V = Mean Velocity of flow, $R_h = \frac{A}{P}$ = Hydraulic radius of the channel

A = Area of flow P – Wetted perimeter for the channel

S = Bed slope of the channel

N = Manning's roughness coefficient.

To obtain **maximum discharge (Q),** **hydraulic radius ($R_h = \frac{A}{P}$) should be maximum**. **For a given area 'A' (A = Constant) the wetted perimeter (P) should be at least.**

2.5.1 CONCEPT OF MOST ECONOMICAL SECTION

The Best hydraulic section: The conveyance 'K' of a channel section increases with the increase in the hydraulic radius or with decrease in the wetted perimeter. The channel section having the least wetted perimeter for a given area has. The maximum conveyance, such a section is known as the best hydraulic section.

The semicircle has the least perimeter among all sections with the same area hence it is the most hydraulically efficient of all sections. The geometric elements of six best hydraulic sections are listed in Table-2.3 but these sections may not always be practical owing to difficulties in construction and in use of material.

Table – 2.3 Types of Geometric elements used in Open-channel Flow

Sl. No	Cross Section	Area	Wetted Perimeter	Hydraulic Radius	Top Width	Hydraulic depth	Section factor
		A	P	R=A/P	T	D=A/T	Z = A D
1	Semicircle	$(\Pi/2)\ y^2$	Πy	(1/2)Y	2Y	$(\Pi/4)Y$	$(\Pi/4)y^{2.5}$
2	Rectangle Half of a square	$2\ y^2$	4Y	1/2Y	2Y	Y	$2y^{2.5}$
3	Triangle half	Y^2	2 2Y	(1/4) 2 Y	2Y	(1/2)Y	$(\ 2/2)\ Y^{2.5}$

	Of a square						
4	Trapezoid Half of a hexagon	3 y^2	2 3 Y	(1/2)Y	4/3 3Y	(3/4)Y	(3/2) $Y^{2.5}$
5	Parabola	(4/3) 2y^2	8/30 2 Y	(1/2)Y	2 2Y	(2/3)Y	(8/9) 3 $Y^{2.5}$

Note-2: The Principle of the best hydraulic section applies only to the design of non-erodible channels. For erodible channels, the principle of active force must be used to determine an efficient section.

A semi-circular cross section has the smallest wetted perimeter and therefore should have highest hydraulic efficiency. But since circular flumes with the exception of metal pipe or wood staves are rarely used, they are either impracticable (for earthen canals) or uneconomical (in concrete line canal).Though it may not always be possible to adhere to such across section (Semi-Circular) from a practical point of view, it should be always close to the theoretical requirements (i.e. 'P' must be minimum). Ina majority of cases, the choice falls on either rectangular or trapezoidal sections or requirement of a most efficient section for such shapes are derived below:

2.5.2 Most economical Rectangular Channel:-

Given: A=by, Where 'b' is the bottom width and 'y' is the depth of flow

The condition is that wetted perimeter should be least

$$R_h = \frac{A}{P} = \frac{A}{b+2y} = \frac{A}{\left(\frac{A}{y}+2y\right)}$$ Eq....2.7

$$P = \left(\frac{A}{y}+2y\right)$$ Eq....2.8

For wetted perimeter to be least, $\frac{dP}{dy}=0$, Differentiating the above equation for'P', we obtain

$$-\frac{A}{y^2}+2=0 \quad or \quad A=2y^2=by: \quad y=\frac{b}{2}$$ Eq....2.9

Thus for a given slope, Area and roughness, **the most efficient hydraulic rectangular section will have a depth (y) of flow equal to one half the width (b) of the channel.**

2.4.3 Most economical Trapezoidal Section:-

Given:

$$A = by + sy^2,$$

Where 'b' is the bottom width and 'y' is the depth of flow and's' is the side slope

The condition is that wetted perimeter should be least

$$P = \left(b + 2y\sqrt{1+s^2}\right) \qquad \text{Eq....2.10}$$

$$b = \frac{A}{y} - sy$$

Therefore,

$$P = \left(\frac{A}{y} - sy + 2y\sqrt{1+s^2}\right)$$

Since area 'A' and side slope's' are constant, For wetted perimeter to be least, $\frac{dP}{dy} = 0$, Differentiating the above equation for 'P', we obtain the following,

$$-\frac{A}{y^2} - s + 2\sqrt{1+s^2} = 0 \quad or \quad \frac{A}{y^2} = 2\sqrt{1+s^2} - s$$

Substituting the value of area A,

$$\frac{by + sy^2}{y^2} = 2\sqrt{1+s^2} - s \qquad \text{Eq....2.11}$$

$$b = 2y\left(\sqrt{1+s^2} - s\right), \quad y = \frac{b}{2\left(\sqrt{1+s^2} - s\right)} = \sqrt{\frac{A}{2\sqrt{1+s^2} - s}}$$

Giving the relationship between the bottom width of the channel and depth of water in the channel

the hydraulic radius 'R_h' $R_h = \frac{A}{P} = \frac{by + sy^2}{b + 2y\sqrt{1+s^2}}$ Eq....2.12

Substituting the value of 'b' from above results in the following important relationship,

$$R_h = \frac{y}{2}$$

In other words, for a trapezoidal section for a given channel and side slope, Area and roughness, to have maximum hydraulic efficiency, the hydraulic radius 'R_h' should be one half the water depth (y).

To determine the side slope (s) for this section the expression for wetted perimeter is expressed in terms of area 'A' (a constant) and a variable's'. The final expression for 'P' is given below:

$$P = 2\sqrt{A}\left(2\sqrt{1+s^2} - s\right)^{1/2}$$

Is then differentiated with respect to side slope 's' and equated to 'zero', which gives, $\frac{2s}{\sqrt{1+s^2}} = 1$, squaring and solving for side slope 's'

$$s = \frac{\sqrt{3}}{3} = \tan 30° \quad or \quad \theta = 30°$$

Solving for bottom width of trapezoidal channel $P = 2y\sqrt{3} = 3b$, $b = \frac{2}{\sqrt{3}}y, \quad A = y^2\sqrt{3}$

Thus for maximum hydraulic efficiency the trapezoidal section should be half hexagon so as to have the least wetted perimeter

<u>2.5.4 Most economical Triangular section:-</u>

For a triangular channel section, If θ is the angle of inclination of each of the sloping sides with the vertical and y is the depth of flow. The following expression for the wetted area 'A' and wetter perimeter 'P' can be written

$A = y^2\tan\theta$ Eq....2.13

$$y = \sqrt{A/\tan\theta}$$

$P = 2y \sec\theta$ Eq....2.14

Substituting the value of 'y' from equation we obtain,

$$A = \frac{2\sqrt{A}}{\sqrt{\tan\theta}}(\sec\theta) \qquad \text{Eq....2.15}$$

Assuming the area 'A' to be constant Eq.13.1.9 can be differentiated with respect to θ and equated to zero for obtaining the condition for minimum P

Thus $$\frac{dP}{d\theta} = 2\sqrt{A}\left[\frac{\sec\theta\tan\theta}{\tan\theta} - \frac{\sec^3\theta}{2(\tan\theta)^{\frac{3}{2}}}\right] = 0$$

Or

$$\sec\theta\ (2\tan^2\theta - \sec^2\theta) = 0$$

Since $\sec\theta \neq 0$

$$2\tan^2\theta - \sec^2\theta = 0$$

$$\sqrt{2}\tan\theta = \sec\theta$$

$\theta = 45^{0}$; or $Z = 1$

Hence a triangular channel section will be most economical when each of its sloping sides makes an angle of 45^0 with the vertical.

The hydraulic radius R of a triangular channel section can be expressed as

$$R = \frac{A}{P} = \frac{y^2\tan\theta}{2y\sec\theta}$$

Substituting the value of θ from equation in the above expression

$$R = \frac{y}{2\sqrt{2}} \qquad \text{Eq....2.16}$$

The most economical triangular section will be half square described on a diagonal and having equal sloping sides. It may however be noted that in all these cases the conditions for the discharge to be maximum would be the same, if instead of area of cross section (A) the perimeter (P) is given.

2.5.5 Most economical Circular channel section:-

For a circular channel section of any radius, as the depth of flow varies the shape of the flow area also varies due to convergence of the boundaries towards the top. As such both the wetted area A as well as wetted perimeter P varies with the depth of flow, and hence in the case of circular channels the condition of area of flow section being constant cannot be applied. Hence in case of circular channels two separate conditions may be derived for the maximum discharge and the maximum mean velocity of flow.

<u>Condition for maximum discharge through a circular channel section:-</u>

For a circular channel section of radius r if y is the depth of flow, the following expressions for the wetted area A and the wetted perimeter P can be written

$$A = \frac{r^2}{2}(\theta - \sin\theta) \qquad \text{Eq....2.17}$$

$$P = r\theta \qquad \text{Eq....2.18}$$

Where 'θ' is the angle subtended at the center by the portion of circular arc in contact with water. In accordance with Chezy's formula, discharge 'Q' passing through the channel may be expressed as

$$Q = AC\sqrt{RS} = CA\left(\sqrt{\frac{A}{P}S}\right) = C\sqrt{\frac{A^3}{P}S} \qquad \text{Eq....2.19}$$

For maximum discharge the depth of flow in the channel is

y= 1.8988r = 0.95 D

Where D is the diameter of the channel. Further hydraulic radius for a circular section is

R = 0.5733r = 0.29 D

That is for maximum discharge in a circular the hydraulic radius is equal to 0.29 times the diameter of the channel.

<u>Condition for maximum discharge through a circular channel section:-</u>

The depth of flow in the channel, for maximum velocity is

y= 1.626r =0.81D

Further hydraulic radius for a circular section is

R = 0.6086r = 0.30 D

That is for maximum mean velocity in a circular the hydraulic radius is equal to 0.30 times the diameter of the channel.

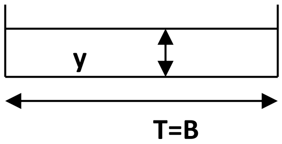
2.6 Problems on Economical Open Channels

Solved Example

Example-1

Find the depth of flow in the most efficient rectangular section carrying a discharge of 0.25m^3/sec on a slope of in 2500, Given Manning's. Constant n= 0.015.

Solution: Using manning formula the discharge 'Q' can be given by



$$Q = by\frac{1}{n}R_h^{2/3}\sqrt{S_0}$$

Substituting for most efficient rectangular section in the above equation

b = 2y, R_h = y/2

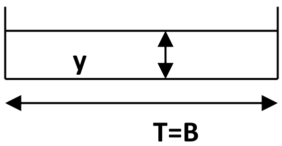
$$Q = (2y)y\frac{1}{n}\left(\frac{h}{2}\right)^{2/3}\sqrt{S_e} = 2h^{\frac{8}{3}}\left(\frac{1}{0.015}\right)\left(\frac{1}{2^{\frac{2}{3}}}\right)\sqrt{\frac{1}{2500}} = 0.25$$

Solving for 'h' **h = 0.84m, B = 1.682m**

Example-2

A rectangular channel carries water at the rate of 5.0 m^3/s when the slope of the channel is 0.25 % find the most economical dimensions of the channel if the manning's n =0.020

Solution: Given Q = 5.0 m^3/s, S0 = $\frac{0.25}{100}$; n = 0.02



For best economical rectangular channel

A = By (i)

P = B+2y (ii)

Condition for most economic channel is B=2y and R= $\frac{y}{2}$

$$Q = (2y)y\frac{1}{n}\left(\frac{h}{2}\right)^{2/3}\sqrt{S_0} = 2y^{\frac{8}{3}}\left(\frac{1}{0.02}\right)\left(\frac{1}{2^{\frac{2}{3}}}\right)\sqrt{\frac{0.25}{100}} = 5.0$$

On solving **h = 2.122 m B= 2 h = 4.244 m.**

Example-3

A triangular channel section of 20m^2 area, what is the apex angle and depth for the condition of maximum discharge.

Solution: when the channel carries the maximum discharge it will be most economical or best hydraulic section. For such a channel $R_h = \frac{y}{2\sqrt{2}}$ and side slopes are $\theta = 45^0$ with the vertical

Area 'A' is given by $A = \frac{1}{2}(2y \times \tan\theta) \times y = y^2 \tan\theta = y^2 \tan 45^o = y^2$

Or $y = \sqrt{A} = \sqrt{20} = 4.47m$

Example-4

Determine the bed width and discharge of the most economical trapezoidal channel with side slopes of 1V:2H and bed slope of 1m per km and depth of flow equal to 1.25m. Roughness coefficient of channel=0.024.

Solution:

Given: side slope s = 2, Bed slope $S_0 = \frac{1}{1000}$, y = 1.25m, n = 0.024

The condition for most economical trapezoidal channel

$$A = y^2\left(2\sqrt{1+s^2} - s\right) = 1.25^2\left(2\sqrt{1+2^2} - 2\right) = 3.863m^2$$

But the bed width is given by $B = \left(\frac{A}{y} - s \times y\right) = \left(\frac{3.863}{1.25} - 2 \times 1.25\right) = 0.59m$

The best hydraulic radius $R_h = \frac{y}{2} = \frac{1.25}{2} = 0.625m$

The maximum discharge for a given depth and side slope for a trapezoidal channel is given by

$$Q_{\max} = A \times \frac{1}{n}(R_h)^{2/3}(S_0)^{1/2} = 3.863 \times \frac{1}{0.024}(0.625)^{2/3}\left(\frac{1}{1000}\right)^{1/2} = 3.71m^3/s$$

UNIT-3: NON-UNIFORM FLOW IN OPEN CHANNELS

3.1 Introduction

3.2 Specific energy, Specific energy diagram

3.3 Critical depth, Conditions for Critical flow- Theory & problems.

3.4 Hydraulic jump in a Horizontal Rectangular Channel (Theory and problems)

3.5 Dynamic equation for Non-Uniform flow in an Open channel

3.6 Classification of Surface profiles

3.1 Introduction: Uniform Flow:

Uniform open channel flow has constant properties along the open channel, such as depth (y) and velocity (V). Also, the head loss equals the change in elevation. The energy grade line Sf, water surface slope Sw and channel bed slope S_0 are all **Parallel, i.e. $S_f = S_w = S_o$.**

The normal depth (y_n) is defined as the depth of flow at which a given discharge flows as uniform flow (y_n) in a given channel.

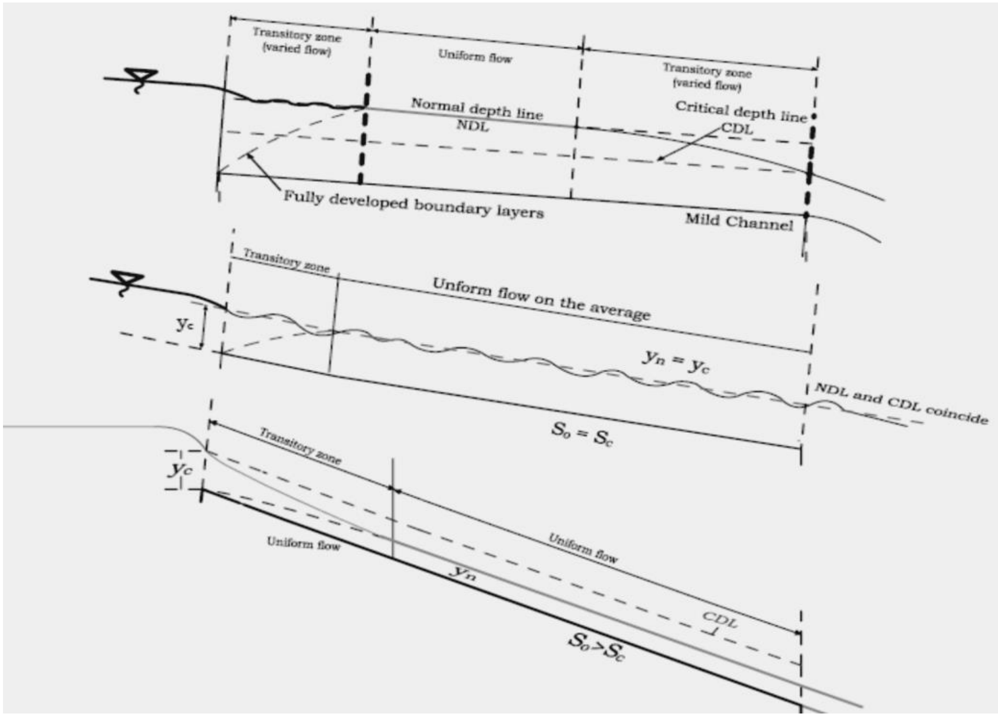


Fig 3.1 Development of Uniform Flow in a long Channel

Computation of Normal Depth and Velocity Determination:

$$V = \frac{1}{n} R_h^{2/3} \sqrt{S_e} \qquad \text{Eq…3.1}$$

$$Q = AV = A\frac{1}{n}R_h^{2/3}\sqrt{S_e} = K\sqrt{S_e}$$ **Eq…3.2**

'K' is known as 'Conveyance of the Channel'.

Gradually and Rapidly Varied Flow:

If flow is gradually varied, the depth h and other flow factors vary smoothly from section to another. In rapidly varied flow, depth h and other flow factors change abruptly over a very short distance. Discontinuity of flow is also possible. Rapidly varied flow can be seen e.g. at a weir, at a change in channel width, at a hydraulic jump or in a hydraulic drop.

3.2 Specific energy, Specific energy diagram

The specific energy in a channel section is defined as the energy per unit weight of water measured ***with respect to channel bottom.***The specific energy E in a channel section or energy head measured with respect to the bottom of the channel at the section is $E = y + \alpha\frac{V^2}{2g}$

Eq…3.3

Where 'α' is a coefficient that takes into account the actual velocity distribution in the Particular channel section, whose average velocity is 'V'. The coefficient 'α' can vary from a minimum of 1.05 - for a very uniform distribution- to 1.20 for a highly uneven distribution. Nevertheless in a preliminary approach it can be used α = 1, Eq.1 becomes, $E = y + \frac{V^2}{2g}$

A channel section with a water area A and a discharge Q, will have a specificEnergy

$$E = y + \frac{Q^2}{2gA^2}$$ Eq…..3.4

Above equation shows that given a discharge Q, the specific energy at a given section, is a function of the depth of the flow only. When the depth of flow y is plotted, for a certain discharge Q, against the specific energy E, a specific energy curve, with two limiting boundaries, like the one represented in figure is obtained. The lower limit, AC, is asymptotic to the horizontal axis and the upper, AB, to the line E=y. The vertex point A on the specific energy curve represents the depth y at which the discharge Q can be delivered through the section at a minimum energy. For every point over the axis E, greater than A, there are two possible water depths. At the smaller depth the discharge is delivered at a higher velocity ñ and hence at a higher specific energy of flow known as super-critical flow. At the larger depth the discharge is delivered at a smaller velocity but also with a higher specific energy, a flow known as sub critical flow. **In the critical state the specific energy is a minimum, and its value can therefore be computed by equating the first derivative of the specific energy (equation 3.4) with respect to is zero**.

$$\frac{dE}{dy} = -\frac{Q^2}{gA^3}\frac{dA}{dy} + 1 = 0 \qquad \text{Eq.3.5}$$

The differential water area near the free surface, dA/dy = T, where T is the top width of the channel section.

By definition $D = \frac{A}{T}$

The parameter D is known as the 'hydraulic depth' of the section, and it plays a big role in the studying the flow of water in a channel. Substituting in equation (3.5) $\frac{dA}{dy} = \frac{A}{T}$ replaced by D:

$$\frac{V}{\sqrt{gD}} = 1 = F_r$$

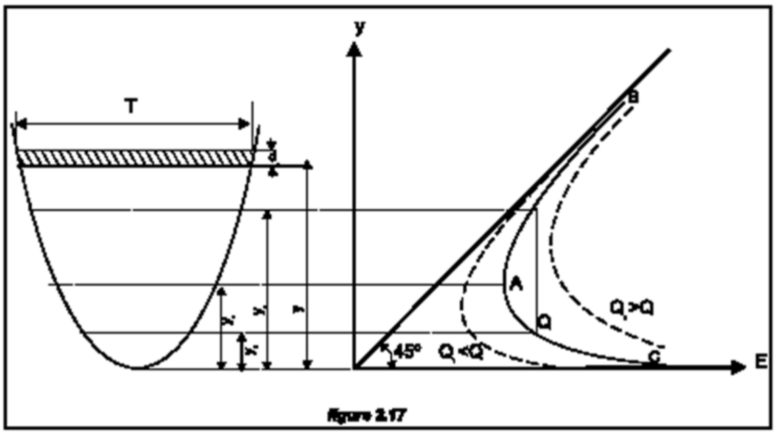


Fig. Specific Energy Diagram

The quantity $\frac{\mathbf{V}}{\sqrt{\mathbf{gD}}} = 1 = \mathbf{F_r}$ is dimensionless and known as the Froude's number

3.2.1 Flow over a raised hump - Application of the Bernoulli equation:

Steady uniform flow is interrupted by a raised bed level as shown Fig.3.2. If the upstream depth and discharge are known we can use equation 3.4 and the continuity equation to give the velocity and depth of flow over the raised hump.

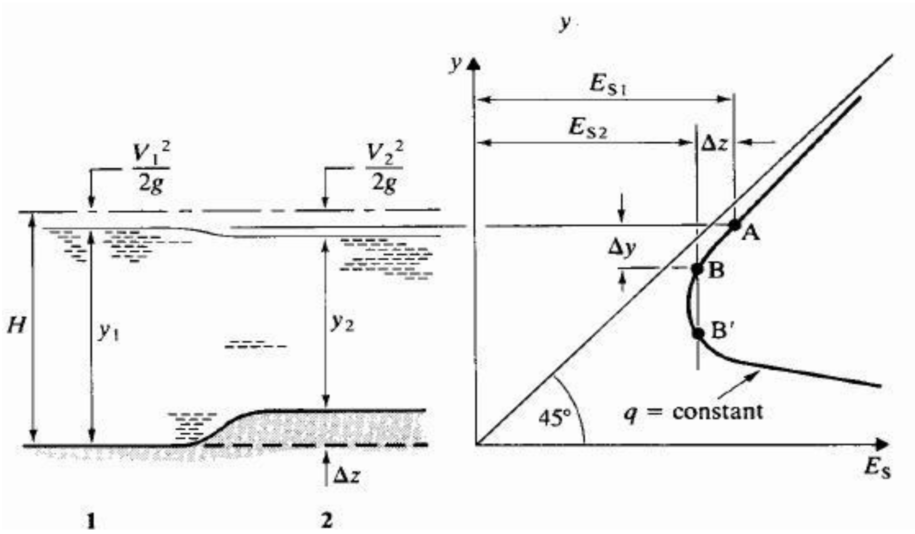


Figure 3.2 of the uniform flow interrupted by a raised Hump

Apply the Bernoulli equation between sections 1 and 2. (Assume a horizontal rectangular channel z1 = z2 and take α = 1)

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z$$

Equation 3.6

Use the continuity equation

$$V_1A_1 = V_2A_2 = Q$$
$$V_1y_1 = V_2y_2 = \frac{Q}{B} = q$$

Where '*q'* is the flow *per unit width*.

Substitute this into the Bernoulli equation to give:

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} + \Delta z$$

Rearranging: $$2gy_2^3 + y_2^2\left(2g\Delta z - 2gy_1 - \frac{q^2}{y_1^2}\right) + q^2 = 0$$

Thus we have a cubic with the only unknown being the downstream depth, y_2. There are three solutions to this - only one is correct for this situation. We must find out more about the flow before we can decide which it is.

Solved Example of the raised bed Hump:

A rectangular channel with a flat bed and width 5m and maximum depth 2m has a discharge of 10m^3 /s. The normal depth is 1.25 m. What is the depth of flow in a section in which the bed rises 0.2m over a distance 1m? Assume frictional losses are negligible.

$$E_{s1} = E_{s2} + \Delta z$$

$$E_{s1} = 1.25 + \frac{\left(\frac{10}{1.25\times 5}\right)^2}{2g} = 1.38$$

$$E_{s2} = y_2 + \frac{\left(\frac{10}{5\times y_2}\right)^2}{2g} = y_2 + \frac{0.2039}{y_2^2}$$

$$1.38 = y_2 + \frac{0.2039}{y_2^2} + 0.2$$

$$1.18 = y_2 + \frac{0.2039}{y_2^2} = E_{s2}$$

Again this can be solved by a trial and error method:

y_2	E_{s2}
0.9	1.15
1.0	1.2
0.96	1.18

i.e. the depth of the raised section is 0.96m or the water level (stage) is 1.16m a drop of 9cm when the bed has raised 20cm.

3.2.2 Critical, Sub-critical and super critical flow:

The specific energy change with depth was plotted above for a constant discharge *Q,* it is also possible to plot a graph with the specific energy fixed and see how *Q* changes with depth. These two forms are plotted side by side below

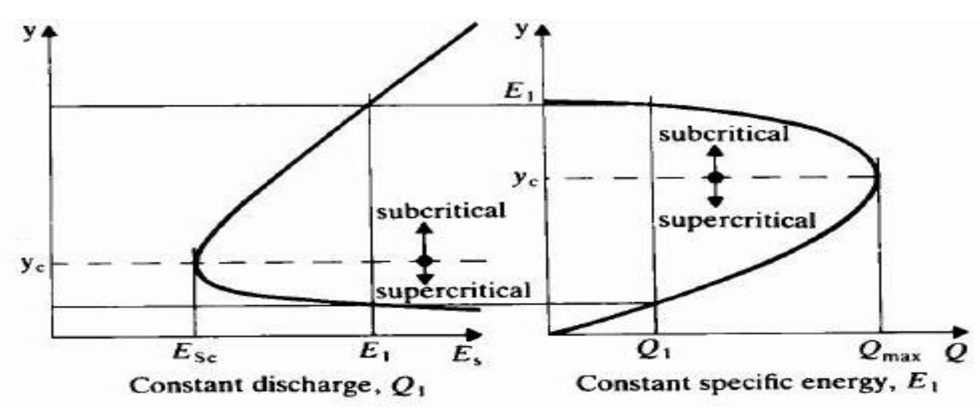


Figure 3.3 of variation of Specific Energy and Discharge with depth.

From these graphs (Fig.3.3) we can identify several important features of rapidly varied flow.

For a fixed discharge:

1. The specific energy is a minimum, *Esc* , at depth *yc* , This depth is known as *critical depth*.
2. For all other values of *E s* there are two possible depths. These are called alternate depths. For *subcritical* flow y >yc

   Supercritical flow y <yc

For a fixed Specific energy

1. The discharge is a maximum at critical depth, yc
2. For all other discharges there are two possible depths of flow for a particular Es

i.e. There is a sub-critical depth and a super-critical depth with the same *Es*

An equation for critical depth can be obtained by setting the differential of *E* to zero:

$$E_s = y + \frac{\alpha (Q/A)^2}{2g}$$

$$\frac{dE_s}{dy} = 0 = 1 + \frac{\alpha Q^2}{2g}\frac{d}{dA}\left(\frac{1}{A^2}\right)\frac{dA}{dy}$$

Since $\delta A = B\delta y$, in the limit $dA/dy = B$ and

$$0 = 1 - \frac{\alpha Q^2}{2g} B_c 2A_c^{-3}$$

$$\frac{\alpha Q^2 B_c}{g A_c^3} = 1$$

Equation 3.7

For a rectangular channel $Q = qb$, $B = b$ and $A = by$, and taking a = 1 this equation becomes

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

as $V_c\, y_c = q$

$$V_c = \sqrt{g y_c}$$

Equation 3.8

Substituting this in to the specific energy equation

$$E_{sc} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$y_c = \frac{2}{3} E_{sc}$$

Equation 3.9

3.2.3 The Froude number:

The Froude number (Fr) is defined for channels as**the ratio of Inertia force to gravity force:**

$$Fr = \frac{V}{\sqrt{g D_m}}$$

Equation 3.10

Its physical significance is the ratio of inertial forces to gravitational forces squared

$$Fr^2 = \frac{\text{inertial force}}{\text{gravitational force}}$$

It can also be interpreted as the ratio of water velocity to wave velocity

$$Fr = \frac{\text{water velocity}}{\text{wave velocity}}$$

This is an extremely useful non-dimensional number in open-channel hydraulics. Its value determines the regime of flow – sub, super or critical, and the direction in which disturbances travel

- Fr< 1 sub-critical water velocity > wave velocity upstream levels **affected** by downstream controls

- Fr = 1 critical

- Fr> 1 super-critical water velocity < wave velocity upstream levels **not affected** by downstream controls

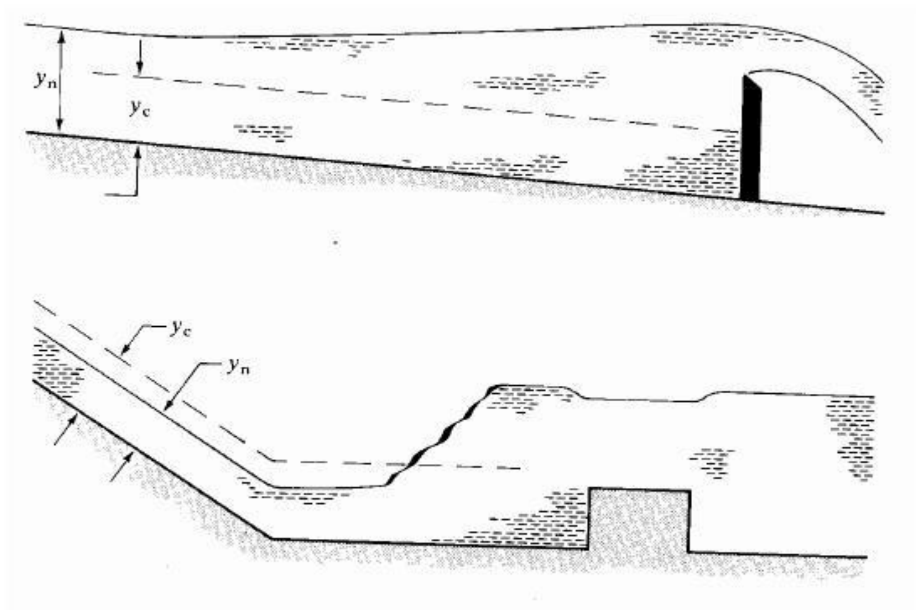
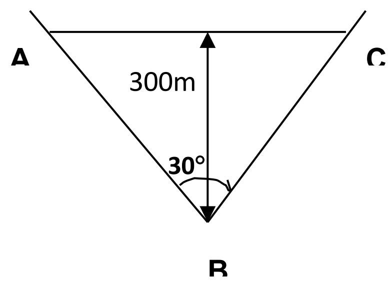


Figure 3.4 Sub-critical and super critical flow and transmission of disturbances

Solved Examples:

Q1.A triangular gutter whose sides include an angle of 60^0 conveys water at a uniform depth of 300 mm. If the bed gradient is 1 in 150 find the discharge. Take Chezy's constant C= 55 $m^{1/2}/s$.



Ans: AB =CB=AC

=0.30 Sec 30^0

=0.3464m

Area of cross section of flow area

$$A = \frac{1}{2}(0.3464)\ 0.30$$

$=0.05196\ m^2$.

Wettedperimeter

P= 2 x 0.3464 m = 0.6928 m

Hydraulic mean depth $R_h = \frac{A}{P} = \frac{0.05196}{0.6928} = 0.075m$

Discharge $Q = AC\sqrt{R_h S_0}$

$$= 0.05196 x 55\sqrt{\frac{0.075}{150}} m^3/s$$

Q $=0.0639m^3/s$=63.90 liters/sec

Q2. A trapezoidal section has side slopes of 1 vertical to 1 horizontal and has to convey a discharge of $14m^3/s$. The bed slope of the channel is 1 in 1000. Chezy's constant is 45 if the channel is unlined and is 70 if the channel is lined with concrete. The cost per metre length of providing the channel and state which arrangement is economical. Take cost per square metre of lining = x.

Ans: Given s = 1, $S_0 = \frac{1}{1000}$

For the trapezoidal channel of best section following criteria has to be satisfied

$$\frac{b+2sy}{2} = y\sqrt{s^2+1}$$

$$\frac{b+2x1xy}{2} = y\sqrt{1^2+1}$$

$b+2d=2\sqrt{2}\,y$, $b = (2\sqrt{2}-2)\,y$

b=0.8284y

A = d (b + s y) =y (0.8284y+1y)

A = $1.8284y^2$

Let the cost per square metre for the lined channel = x

Cost per cum of excavation = 3x

<u>Case (i) Unlined channel: Cost of Unlined channel</u>

Discharge $Q = AC\sqrt{R_h S_0}$ =14 (given discharge)

$\therefore 1.8284\, y^2 \times 45 \sqrt{\frac{y}{2}}\sqrt{\frac{1}{1000}} = 14$

$$y^{5/2} = \frac{14\sqrt{2000}}{1.8284 \times 45}$$

y = 2.252m

$A=1.8284\ y^2 = 1.8284 \times 2.252^2 = 9.9727\ m^2$.

Cost of excavation for 1 metre length

= 9.2727 x 1 x 3x

= 27.8181 x

Case (ii) Unlined channel: Cost of lined channel

Discharge $Q = AC\sqrt{R_h S_0}$ =14

$$1.8284 d^2 x 70 \sqrt{\frac{d}{2}}\sqrt{\frac{1}{1000}} = 14$$

$$d^{5/2} = \frac{14\sqrt{2000}}{1.8284 x 70}$$ d = 1.887 m

$A= 1.8284\ d^2 = 1.8284 \times 1.887^2 = 6.5105\ m^2$

Wetted perimeter,

$$P = b + 2y\sqrt{s^2 + 1}$$

$$P = 0.8284y + 2y\sqrt{1^2 + 1}$$

$$P = 0.8284 x 1.887 + 2 x 1.887\sqrt{2}$$

P = 1.5632 +5.3372

P= 6.9004m

For 1m-length of the channel the cost of excavation and lining

Cost of excavation	= 6.5105 x 1 x 3x	=19.5315x
Cost of lining	= 6.9004 x 1 x x	= 6.9001x

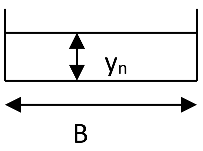
Total cost per metre length of lined canal = 26.4316 x

But the cost per metre length of the unlined channel = 27.818x

Hence the lined channel is more economical.

Q.3 Find the normal depth in a wide rectangular channel for the given data:

Gives: n = 0.015, S_0 = 0.001, Q = 2.5 m^3/s, B =10m



$$Q = \frac{1}{n} A R_n^{2/3} S_0^{1/2},$$

For a wide channel: $R_h = y_n$; $S_e = S_0 = 0.001$

$$2.5 = \frac{1}{0.015} 10 y_n y_n^{2/3} (0.001)^{1/2}$$

$$y_n^{5/3} = \frac{2.5 x 0.015}{(0.001)^{1/2}} x \frac{1}{10}$$

y_n = 0.278m

Q4. A discharge of 16.0 m^3/s flows with a depth of 2.0 m in a rectangular channel 4.0 m wide. At a downstream section the width is reduced to 3.5 m and the channel bed is raised by (a) ΔZ = 0.20 m and (b) ΔZ = 0.35 m.

Ans: Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively.

At the upstream section, $v_1 = \frac{16}{4x2} = 2.0m/s$

F1 = Froude number $= \frac{v_1}{\sqrt{gy1}} = \frac{2.0}{\sqrt{9.81x20}} = 0.452$

The upstream flow is sub-critical and the transition will cause a drop in the water surface elevation.

$$\frac{v_1^2}{2g} = 0.204m$$

The upstream-specific energy $E_1 = y_1 + \frac{v_1^2}{2g}$ = 2.0 +0.204 =2.204m

q_2 = discharge intensity at the downstream section $= \frac{Q}{B_2} = \frac{16.0}{3.5} = 4.571m^3/s/m$

'y_{c2}'= critical depth corresponding to 'q_2'

$$y_{c2} = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(4.571)^2}{9.81}} = 1.87m$$

$$E_{c2} = \frac{3}{2} y_{c2} = 1.930m$$

(a) When ΔZ = 0.20 m

E_2 = available specific energy at section- 2

= $E_1 - \Delta Z$ = 2.204 -0.20 = 2.004 m> Ec_2

Hence the depth $y_2 > y_{c2}$ and the upstream depth will remain unchanged at y_1.

$$y_2 + \frac{V_2^2}{2g} + \Delta Z = E_1$$

$$y_2 + \frac{(4.571)^2}{2x9.81y_2^{\,2}} + 2.204 - 0.20$$

$$y_2 + \frac{1.065}{y_2^{\,2}} = 2.004$$

Solving by trial and error, y_2= 1.575 m.

Hence when Δ Z = 0.20 m, y_1 = 200 m and y_2 = 1.575 m

(b) When Δ Z = 0.35 m.

E_2 = available specific energy at section-2

= 2.204 – 0.350 = 1.854 m $< E_{c2}$

Hence the contraction will be working under choked conditions. The upstream depth must rise to create a higher total head. The depth of flow at section 2 will be critical with $y_2 = y_{c2}$ = 1.287 m.

If the new upstream depth is y'_1

$$y_1^{'} + \frac{Q^2}{2gB_1^{\,2}y_1^{'2}} = E_{c2} + \Delta Z = 1.930 + 0.350$$

$$y_1^{'} + \frac{(16)^2}{2x9.81x(4.0)^2\,y_1^{'2}} = 2.28$$

i.e. $y_1^{'} + \frac{0.8155}{y_1^{'2}} = 2.280$

By trial- and -error, y'_1 = 2.94m.

The upstream depth will therefore rise by 0.094 m due to the choked condition at the constriction.
Hence, When Δ Z = 0.35 m.y'_1 = 2.094m and $y_2 = y_{c2}$=1.287 m.

Q5. A triangular channel with an apex angle of 75^0 carries a flow of $1.2m^3/s$ at a depth of 0.80 m. If the bed slope is 0.009, find the roughness coefficient of the channel.

Ans: y_0 = normal depth = 0.80m

Area $$A = \frac{1}{2} x 0.80 x 2 x 0.8 \tan \frac{75}{2}$$ = 0.491m^2

Wetted perimeter P=2×0.8× sec 37.5^0 = 2.0168m

R_h = A/P = 0.243 m.

$$n = \frac{AR_h^{2/3} S_0^{1/2}}{Q} = \frac{(0.491) x (0.243)^{2/3} X (0.009)^{1/2}}{1.20}$$ =0.0151.

3.3 Critical depth, Conditions for Critical flow- Theory & problems.

Critical depth (y_c): It is defined as a depth of flow at which specific energy is minimum.The critical flow has been defined as that flow at which the specific energy is the minimum for a given discharge, utilizing this definition; a general criterion may be arrived at as follows:

$$E = y + \frac{(q.b)^2}{2g(b.y)^2} = y + \frac{q^2}{2gy^2} = f(y, q)$$ Eq.(3.11)

For minimum specific energy, differentiating the above expression for 'E' with respect to 'y' and equating $\frac{dE}{dy} = 0$

$$\frac{dE}{dy} = 1 - \frac{Q^2}{2g} x \frac{2}{A^3} x \frac{dA}{dy}$$

If 'T' = top width of the channel, then d A = T. dy and $\frac{dA}{dY} = T$, and using the condition of minimum specific energy i.e. $\frac{dE}{dy} = 0$, yields,

$$\frac{dE}{dy} = 1 - \frac{Q^2}{g} x \frac{T}{A^3} = 0$$

$$\frac{Q^2}{g} x \frac{T}{A^3} = 1 \quad \text{or} \quad \frac{Q^2}{g} = \frac{A^3}{T}$$

represents the criterion for critical flow in the channel. The equation may be re-arranged to yield other important relationships,

$$\frac{Q^2}{gA^2} = \frac{A}{T} \quad \text{or} \quad \frac{v^2}{g} = D$$

$$\frac{v^2}{2g} = \frac{D}{2} \qquad \text{Eq.(3.12)}$$

the above equation states that in critical flow, the 'velocity head '$\frac{V^2}{2g}$'is equal to half the hydraulic depth '$\frac{D}{2}$'

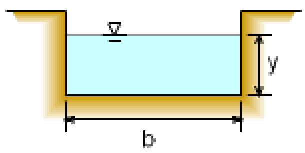
From which, $\frac{v}{\sqrt{gD}} = 1 = F_r$ Eq.(3.13)

The above equation shows that for critical flow the Froude's number is unity.

9.1.1 Critical flow in a Rectangular channel

The below equation may be used to obtain critical depth in any channel. In case of a rectangular channel of width 'b', carrying a discharge 'Q' under critical conditions we obtain, Q = q. b, A = b y, T = b

$$\frac{Q^2}{g} = \frac{A^3}{T}$$



Substituting for 'Q', 'A' and 'T' for a rectangular channel,

$$\frac{qb^2}{g} = \frac{(by)^3}{b}$$

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

The above equation relates the critical depth 'y_c' and the discharge per unit width 'q' and shows that the critical depth is independent of the channel slope.

From Eq.16 the critical velocity $\mathbf{v_c = \sqrt{gD},}$ **where** $\mathbf{D = \frac{A}{T} = \frac{b.y_c}{b} = y_c}$

We know that for a given discharge flowing at the critical depth, the specific energy is the minimum, thus

$$\textbf{Specific Energy } \mathbf{E_{min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2} = \frac{3}{2} y_c}$$

The critical slope is the channel slope at which uniform critical flow is obtained in the channel, and is evaluated either using the Manning's or the Chezy's formula. It is noted that in uniform flow the slope of the energy line, the water surface slope and the slope of the channel bottom are equal (S_e=S_w=S_0). Using Manning's equation, we obtain,

$$\mathbf{v_c = \frac{1}{n} R_c^{2/3} S_c^{1/2}}$$

Where, R_c = Hydraulic radius at the critical depth, S_c – Critical slope

In case the Chezy's formula is used, then,

$$\mathbf{v_c = C\sqrt{R_c S_c}}$$

In which C = Chezy's constant.

Q.6: Find y_c for a rectangular channel for the given data,

Q= 1 m^3/s, b = 2 m.,

Solution: The discharge per unit width 'q'

$$q = \frac{1}{2} = 0.5 \text{ m}^2/\text{s}$$

At critical condition, **Fr** = 1

$$F_r = \frac{V}{\sqrt{g y_c}} = 1$$

$$\frac{V}{\sqrt{gy_c}} = 1$$

$y_c = y_m = y$ (for a rectangular channel) and using continuity equation we have,

$$V = \frac{Q}{A} = \frac{Q}{bxy} = \frac{q}{y}$$

Given: $Q = 1\ m^3/s$, $b = 2$ m., $q = 1/2 = 0.5\ m^2/s$

$$y_c = (0.5/9.81^{1/2})^{2/3} = 0.294 \text{ m}$$

$$y_c = 0.294 \text{ m}$$

3.4 Hydraulic jump in a Horizontal Rectangular Channel:

Analogous to a normal shock in compressible flow, a hydraulic jump provides a mechanism by which an incompressible flow, once having accelerated to the supercritical regime, can return to sub critical flow. This is illustrated by the following figure. (Fig. 3.5 and Fig. 3.6)

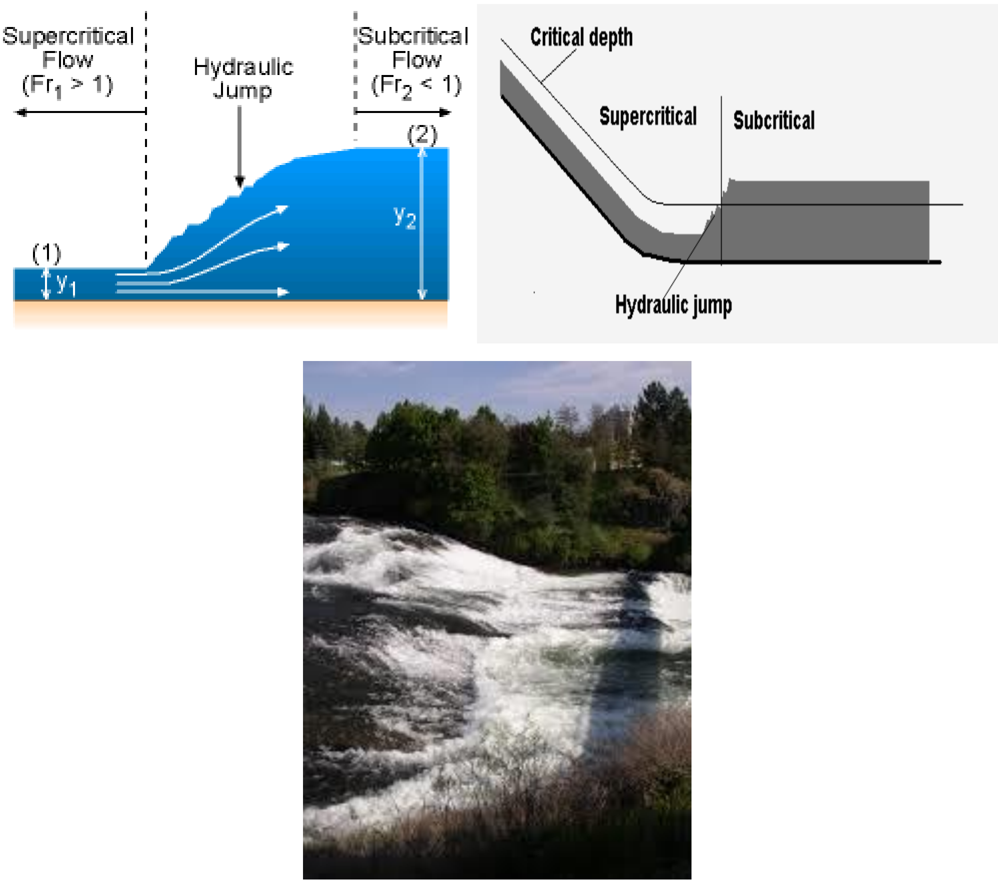


Fig. 3.5 Hydraulic jump

The critical depth is an important parameter in open-channel flow and is used to determine the local flow regime.

$$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3} \qquad \text{Eq...3.14}$$

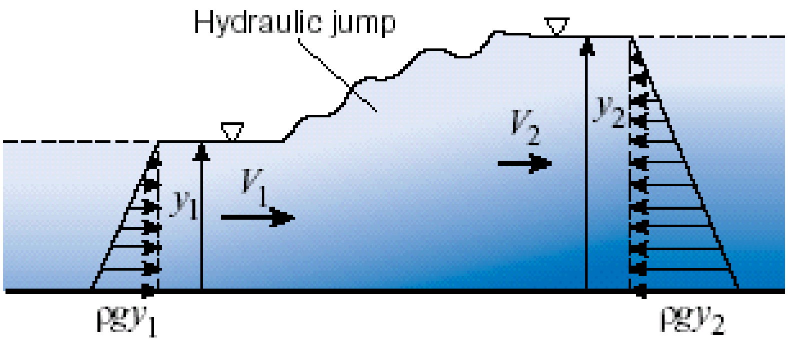


Fig.3.6 Control volume for hydraulic jump

Specific Force:

In case of a horizontal channel, $\theta = 0$ and hence the weight component W Sinθ, vanishes from the momentum equation, if, the length L involved between the sections is small, the resistance force R_f may be ignored and then becomes

$$\frac{\gamma Q}{g}(V_2 - V_1) = P_1 - P_2 \qquad \text{Eq...3.15}$$

The hydrostatic forces P_1 and P_2 may be evaluated and $P_1 = \gamma Z_1 A_1$ and $P_2 = \gamma Z_2 A_2$ where Z_1 and Z_2 are the distance of centroids of flow areas, A_1 and A_2 from their respective water surfaces. From the continuity equation,

$Q = V_1 A_1 = V_2 A_2$

Substituting for V_1 and V_2 in the momentum equation, one gets.

$$\frac{\gamma Q^2}{g}\left(\frac{1}{A_2} - \frac{1}{A_1}\right) = \gamma Z_1 A_1 - \gamma Z_2 A_2 \qquad \text{Eq...3.16}$$

$$\boxed{\text{Specific force } F = \frac{Q^2}{gA} + AZ}$$ **Eq...3.17**

The first term of the above equation $\frac{Q^2}{gA}$ is the momentum of flow passing the channel section per unit time per specific weight 'γ' of the liquid.

The second term (A Z) is the hydrostatic force per specific weight of the liquid. Both the terms being basically the forces per specific weight, their sum denoted by **'F'** is generally known as the **specific force**.

For a rectangular channel with varying width 'b' having depth y_1, y_2 and velocity V_1, V_2 the expression for continuity equation becomes

$Q = V_1 b y_1 = V_2 b y_2$ introducing the discharge per unit width $q = \frac{Q}{b}$ in Eq.3.17

$$\frac{\gamma Q}{g}(V_2 - V_1) = P_1 - P_2$$

$$\frac{\gamma (bq)}{g}\left(\frac{q}{y_2} - \frac{q}{y_1}\right) = \frac{1}{2}\gamma b y_1^2 - \frac{1}{2}\gamma b y_2^2$$

On simplification

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

The above equation relates the conjugate depth (or sequent depth) depths y_1 and y_2 with the discharge per unit width 'q' for a given discharge 'Q'. For a rectangular channel the quadratic equation in y_2.

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 \pm \sqrt{1 + \frac{8q^2}{y_1^3}}\right]$$

$$\boxed{y_2 = \frac{y_1}{2}\left[-1 \pm \sqrt{1 + 8F_{r1}^2}\right]}$$ Eq...3.18

(i) For $F_{r1}<1$, $\frac{y_2}{y_1}<1$. This condition for a falling liquid surface (hydraulic Drop) in sub-critical flow

(ii) For $F_{r1}=1$, $\frac{y_2}{y_1}=1$. This condition is critical

(iii) For $F_{r1}>1$, $\frac{y_2}{y_1}>1$. This condition is hydraulic jump

3.4.1 Theoretical jump relations:

By means of a one-dimensional control-volume analysis, one can obtain analytical expressions for the downstream velocity, depth, and specific energy in terms of the upstream velocity, depth, and specific energy, as follows. Consider the fixed control volume shown in Fig.3.6 With the approximation of steady flow in the control volume, the conservation-of-mass equation for flow before and after the hydraulic jump can be written as

$$\rho b y_1 V_1 = \rho b y_2 V_2 \qquad \text{Eq...3.19}$$

and the momentum-balance equation for steady flow through the control volume requires that the sum of the forces on the fluid be equal to the rate of change in momentum of the fluid. The horizontal component of this vector equation can be simplified to

$$\left(\frac{\rho g y_1}{2}\right) y_1 b - \left(\frac{\rho g y_2}{2}\right) y_2 b = -(\rho b y_1 V_1) V_1 + (\rho b y_2 V_2) V_2$$

In order to express the downstream parameters in terms of the inlet parameters to the hydraulic jump, one can simplify and introduce the upstream Froude number,

$$F_{r1} = \frac{V_1}{\sqrt{g y_1}}$$

It can then be shown that

$$F_{r1}^2 = \frac{1}{2} \frac{y_2}{y_1} \left(\frac{y_2}{y_1} + 1\right)$$

Which is a quadratic equation for the dimensionless depth ratio $\frac{y_2}{y_1}$ in terms of the upstream Froude number F_{r1}. This quadratic equation has two solutions for the depth ratio $\frac{y_2}{y_1}$, but one of these roots is negative and has no physical meaning, while the other is,

$$\frac{y_2}{y_1} = \frac{\sqrt{1+8F_{r1}^2}-1}{2} \qquad \text{Eq...3.20}$$

The theoretical relationship between the depth ratio $\frac{y_2}{y_1}$ and the upstream Froude number Fr_1 given in Eq. 3.20 is that of a hyperbola, as illustrated in Fig. 3.22. Mechanical energy is dissipated in the jump, and therefore a simple equation for

Conservation of energy is not valid. By definition, the head loss $h_L$is given by the difference in mechanical energy *E* before and after the jump:

$$h_L = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right) \qquad \text{Eq...3.21}$$

This expression for head loss can be made dimensionless by considering the ratio $\frac{h_L}{y_1}$, which can be simplified by using the equations for conservation of ass and momentum balance, to one that is dependent solely on depth ratio (y_2/y_1),

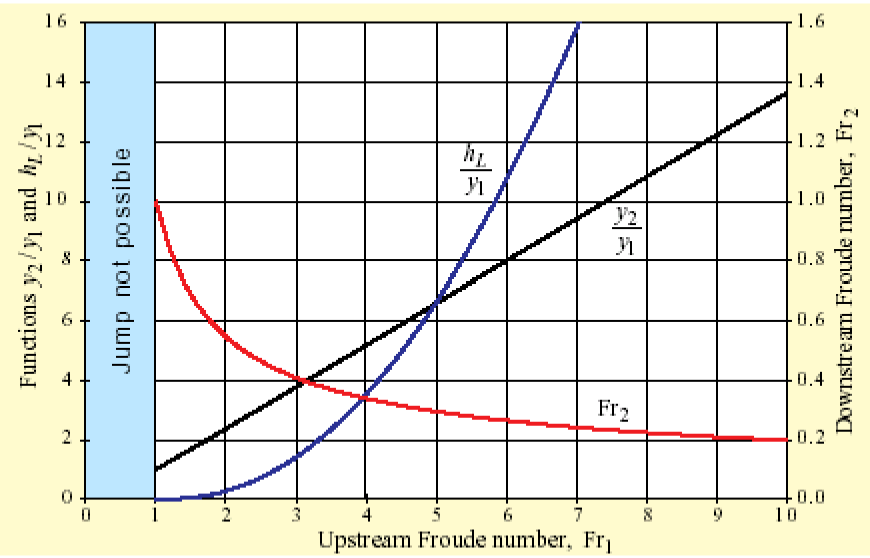


Fig.3.22 Effect of Depth Ratio and Froude No. on Hydraulic jump

and thus on the upstream Froude number:

$$\frac{h_L}{y_1} = \frac{1}{4}\frac{y_1}{y_2}\left(\frac{y_2}{y_1} - 1\right)^3 = f(F_{r1}) \qquad \text{Eq...3.22}$$

A theoretical plot of $\frac{h_L}{y_1}$ versus F_{r1} is also given in Fig. 3. In addition, the downstream Froude number F_{r2} given by,

$$F_{r2} = \frac{V_2}{\sqrt{gy_2}} = \frac{V_1(y_1 / y_2)}{\sqrt{gy_1}\sqrt{y_2 / y_1}} = F_{r1}\left(\frac{y_2}{y_1}\right)^{-3/2} \qquad \text{Eq...3.23}$$

is also plotted on the same figure. Note that, if a jump is formed, then the downstream flow is always sub-critical (that is, $F_{r2}<1$), while the upstream flow is always supercritical (that is, $F_{r1}>1$).

3.4.2 Energy dissipation:

The energy-dissipating effectiveness of a hydraulic jump can be classified in terms of the upstream Froude number as follows (Round and Garg 1986):

For $Fr_1^2 = 1$ **to** 3 : **Standing wave.** There is only a slight difference in conjugate depths *y*1 and *y*2. Near $F_{r1}^2 = 3$ a series of small rollers develops.

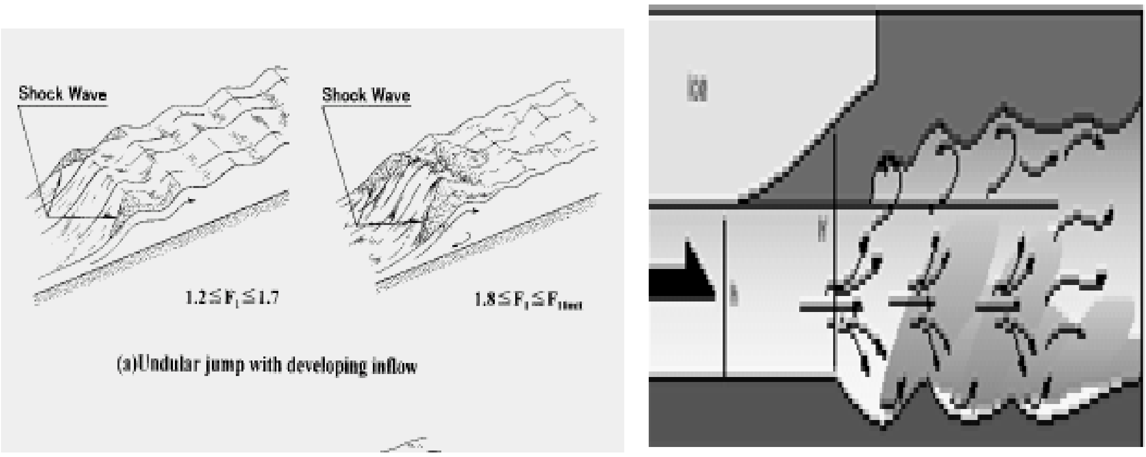


Fig.3.7Undular Hydraulic Jump

For $Fr_1^2 = 3$ to 6 : **Pre-jump condition.** The water surface is quite smooth, the velocity is reasonably uniform, and the energy loss is low. No baffles are required if the proper length of pool is specified.

For $Fr_1^2 = 6$ to 20 : **Transition region**. An oscillating action of the jump exists. Each oscillation of the jump produces a large wave of irregular period that can travel downstream for miles and damage earth banks and rip-rap. It is recommended to avoid this range of Froude numbers in the design of stilling basins.

For $Fr_1^2 = 20$ to 80 : The best range for dissipating energy effectively. The jump is well balanced and the action is at its best. Energy losses range from 45% to 70%. Baffles and sills may be utilized to reduce the length of the basin.

For Fr_1^2 above 80 : Effective but rough at dissipating energy. Energy losses range from 70% to 85%. Other types of stilling basins may be more economical.

3.4.3 Types of Hydraulic Jump:

- For jumps in which the ratio $\left(\frac{y_2}{y_1}\right)$ is not greater than 2.0 the liquid surface does not rise abruptly and has a number of undulations of gradually diminishing size. Such a jump is called as an <u>undular jump</u>.

- For $\left(\frac{y_2}{y_1}\right) = 2.0$, the upstream Froude number F_{r1} = 3. This fixes the upper limit of F_{r1} for undular jumps. For higher values $\left(\frac{y_2}{y_1}\right) > 2$ and consequently $F_{r1} > 3$, the liquid surface rises fairly abruptly, and the hydraulic jump then is known as a <u>direct jump</u>.

The United States Bureau of Reclamation has classified the jump into the following five categories, depending upon the magnitude of the Froude number of the approaching flow F_{r1}

(i) **Undular jump:** The upstream Froude number F_{r1} ranges from 1 to 1.7 and the liquid surface shows undulations of gradually decreasing size.

(ii) **Weak jump:** The upstream Froude number F_{r1} ranges from 1.7 to 2.5, number of small rollers appear on the surface of the jump, and the downstream liquid surface remains smooth. The energy loss in the jump is low.

(iii) **Oscillating jump:** For upstream Froude number F_{r1} ranging between 2.5 between to 4.5 there is an oscillating jet which enters the jump bottom and oscillates to the surface. Each oscillation produces large wave of irregular period and does extensive damage to the canal bed banks while traveling miles downstream.

(iv) **Steady jump:** This type of jump occurs in the upstream Froude number F_{r1} range of 4.5 to 9.0. The fluctuations in the tail water depth have a very little effect on the position and the action of the jump. The energy dissipation may be in the range of 45% to 70%

(v) **Strong jump:** For Froude number greater than 9.0, the surface downstream of the jump is rough and the energy dissipation may be up to 85%.Figure (12.39) illustrates the types of jump described above.

3.4.4 Elements and Characteristics of a Hydraulic Jump:

The following quantities are generally known as the elements of the hydraulic -jump:

(i) pre-jump depth y_1

(ii) post jump depth y_2

(iii) height of the jump, $H_j = y_2 - y_1$

(iv) length of the jump, $L_j = 5H_j$ approx, and

(v) specific energies before and after the jump(i.e. E_1 and E_2), and

(vi) The Loss of energy (ΔE), in the jump.

With the exception of the jump length L_j all the remaining elements can be determined theoretically with the aid of equations derived earlier. It can be noted that these elements are functions of the depths y_1 and y_2. It, therefore, appears that the conjugate depths constitute the most important elements of the jump. As regards the length of the jump, it has been observed from experimental data that there is a wide variation in the relationship between length and height of jump. It is because of the fact that the downstream end of the jump cannot be precisely demarcated on account of wavy surface which follows a jump. The length of the jump in rectangular channels may be usually of the order of five times its height. The following dimensionless quantities are generally known as the characteristics of the jump:

(i) Relative loss $\left(\frac{\Delta E}{E_1}\right)$ is defined as the ratio energy loss and the specific energy before the jump.

The relative loss, being a ration of energies, is dimensionless,

$$\frac{\Delta E}{E_1} = \frac{1}{8} \frac{\left[\sqrt{1+8F_1^2} - 3\right]^3}{(2+F_1^2)\ [\sqrt{1+8F_1^2} - 1]}$$

Eq...3.24

(ii) Efficiency of the Jump $\left(\frac{E_2}{E_1}\right)$: The ratio of the specific energies after and before the jump is known as the efficiency of the jump. The efficiency may be expressed in terms of the Froude number F_{r1}, thus

$$\left(\frac{E_2}{E_1}\right) = \frac{1 + (8F_{r1}^2 + 1)^{3/2} - 4F_{r1}^2}{8F_{r1}^2 \left(2 + 8F_{r1}^2\right)}$$

Eq...3.25

(i) Relative height of the jump is given by $\frac{H_j}{E_1}$. The height of the jump is defined as difference between the depths after and before the jump, thus

$$\frac{H_j}{E_1} = \frac{(y_2 - y_1)}{\left(y_1 + \frac{V_1^2}{2g}\right)}$$

Eq...3.26

(iv) The loss of energy 'ΔE' in the normal hydraulic jump is equal to the difference in specific energies before (E_1) and after (E_2) the jump and can be shown to be equal to

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

Eq...3.27

The power lost in the jump P_{lost} is given by,

$$P_{lost} = \frac{\gamma Q(y_2 - y_1)^3}{300\, y_1 y_2} (\textit{in metric Horsepower})$$

Eq...3.28

Q6. A spillway discharge a flood flow at a rate of 7.75m^3/s per meter width. At the downstream horizontal apron the depth of flow was found to be 0.50 m. What tail water depth is needed to form a hydraulic jump? If a jump is formed, find its (a) type, (b) length (c) head loss, and (d) energy loss as a percentage of the initial energy.

Ans: q=7.75 m^3/s/ m, and y_1 = 0.50 m

$$v_1 = \frac{7.75}{0.50} = 15.50 \ m/s$$

$$F_1 = \frac{15.50}{\sqrt{9.81x0.50}} = 7.0$$

$$\frac{y_2}{y_1} = \frac{1}{2}(\ 1 + \sqrt{1 + 8x(7)^2} = 9.41$$

y_2 = 4.71m = required tail water depth.

(a) Type: Since F_1 = 7.0, a 'steady' jump will be formed
(b) Since $F_1 > 5.0$ l L_j = 6.1 y_2

L_j = length of the jump = 6.1 x 4.71 = 28.7 m

(c) E_L= head loss = $\frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(4.7 - 0.5)^3}{44.7 \ 0.50} = 7.92 \quad m$

(d) $E_1 = y_1 + \frac{V_1^2}{2g} = 0.5 + \frac{(15.50)^2}{2X9.81} = 12.75m$

$$\frac{E_L}{E_1} = 62.1\%$$

Q7. A rectangular channel carrying a supercritical stream is to be provided with a hydraulic-jump type of energy dissipater. If it is desired to have an energy loss of 5.0 m in the jump when the Intel Froude number is 8.5 determine the sequent depths.

Ans: Given F_1 = 8.5, and E_L = 5.0m

$$\frac{y_2}{y_1} = \frac{1}{2}\left(1+1\sqrt{1+8F_1^2}\right)$$

$$= \frac{1}{2}\left(1+1\sqrt{1+8(8.5)^2}\right) = 11.53$$

$$\frac{E_L}{y_1} = \frac{\frac{y_2}{y_1}\quad 1^{\;3}}{4\;\frac{y_2}{y_1}}$$

$$\frac{5.0}{y_1} = \frac{(11.53\quad 1.0)^3}{4x11.53} = 25.32m$$

y_1=0.98 m and y_2 = 2.277m.

Q.8 A sluice gate in a 3.0 m wide rectangular, horizontal channel releases a discharge of 18.0m^3/s. The gate opening is 0.67 m and the coefficient of contraction can be assumed to be 0.6 Examine the type of hydraulic jump formed when the tail water is (i) 3.60 m (ii) 5.00 m and (iii) 4.09m.

Ans: Let A be the section of vena contract

Y_a = Depth at vena contracta = 0.67 x 0.6 = 0.40m

V_a = 18.0/(3.0 x 0.4) = 15.0 m/s.

F_a = Froude number at vena contracta $= \frac{V}{\sqrt{gy_a}}$

$$F_a = \frac{15.0}{\sqrt{9.81x0.4}} = 7.573$$

If y_2 = Sequent depth required for a jump at vena contract

$$\frac{y_2}{y_a} = \frac{1}{2}(\ \ 1+\sqrt{1+8F_q^{\ 2}})$$

$$= \frac{1}{2}(-1+\sqrt{1+8x(7.573)}^{\,2}) = 10.22$$

y_2 = 10.22 x 10.40 = 4.09 m.

(i) When the tail water depth y_t = 3.60m,

Since y_t<y_2, a free, repelled jump will form.

$$V_t = \frac{18.0}{3.0x3.60} = 1.667m/s$$

$$F_t = \frac{1.667}{\sqrt{9.81x3.60}} = 0.281$$

The depth at the toe of this repelled jump y_1 is given by

$$\frac{y_1}{y_t} = \frac{1}{2}(\ \ 1+\sqrt{1+8Ft^2})$$

$$\frac{y_1}{3.60} = \frac{1}{2}(\ \ 1+\sqrt{1+8x(0.281)^2}) = 0.1387$$

y_1 = 0.50 m.

An M3 curve will extend from section A (y_a=0.40 m) to section 1 (y_1=0.50m).

(ii) When the tail water depth y_t = 5.0 m.Since, $y_t > y_2$, a submerged jump will occur.

(iii) When y_t= 4.09, y_t= y2 and a free jump will occur at section I with $y_1 = y_a$ = 0.40m.

Q9. A rectangular horizontal channel of 3.0 m wide carries a discharge of 10m^3/s. Determine whether hydraulic jump may occur at an initial depth of 0.50 m or not. If jump occurs determine the sequent depth and height, length of the jump?

Ans: The hydraulic jump will form if the critical depth of the channel I more than the initial depth (y_1) for the given discharge.

Given Q = 10 m^3/s, B = 3.0m, y_1 = 0.5m

$$\mathbf{q} = \frac{\mathbf{Q}}{\mathbf{B}} = \frac{10}{3} = 3.3333$$

$$\mathbf{Y_c} = \sqrt[3]{\frac{\mathbf{q}^2}{\mathbf{g}}} = \sqrt[3]{\frac{(3.333)^2}{9.81}} = 1.05\mathbf{m}$$

Since the critical depth $y_c > y_1$, Hence the hydraulic jump will form.

The initial Froude number $\mathbf{F}_1 = \dfrac{\mathbf{Q}}{\mathbf{A}_1\sqrt{\mathbf{g}\mathbf{D}_1}} = \dfrac{10}{(3x0.5)\sqrt{9.81x\left(\dfrac{3x0.5}{3}\right)}} = 3.0$

Where $\mathbf{D} = \dfrac{\mathbf{A}_1}{\mathbf{T}_1} = \dfrac{3x0.5}{3} = 0.5\mathbf{m}$

The post depth y_2 is determined by using the following relationship,

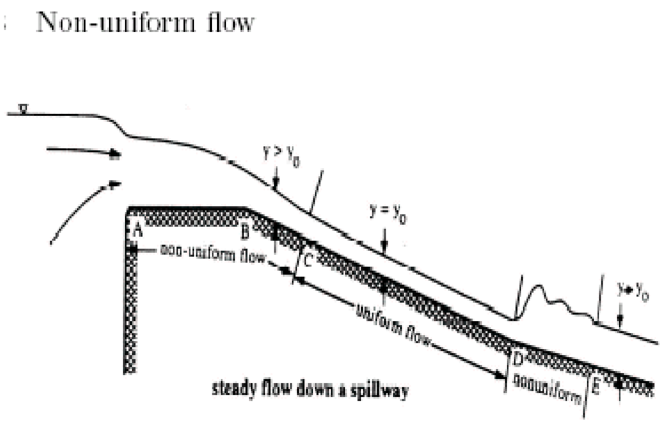
$$\mathbf{y}_2 = \frac{\mathbf{y}_1}{2}\left[-1+\sqrt{1+8\mathbf{F}_1^2}\right]$$

$$\mathbf{y}_2 = \frac{0.5}{2}\left[-1+\sqrt{1+8(3)^2}\right] = 1.9\mathbf{m}$$

Height of the jump = 1.9 – 0.5 = 1.4m

Length of the jump = 5 x 1.4 = 7.0 m

3.5 Dynamic equation for Non-Uniform flow in an Open channel: the steady flow whose depth varies gradually along the length of the channel.



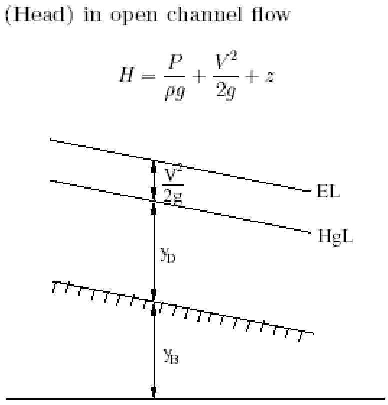


Figure 3.8 Gradually varied flow

The flow in an open channel may be either uniform or non-uniform (also known as varied flow). In uniform flow, the depth of flow remains constant along the length of the channel. This type of flow is possible in prismatic channels of sufficient length. The uniform flow is characterized by parallelism of channel bottom, liquid surface and the energy line. The non-uniform or varied flow is one in which the flow depth changes in the flow direction. The flow is said to be gradually varied when the depth changes gradually over a long distance, whereas in rapidly varied flow, the change in depth takes place in a short distance. The change in depth may sometimes be quite abrupt, as in case of a hydraulic jump. Since this type of non-uniformity is limited to a short distance, a hydraulic jump and a hydraulic drop thus represent local phenomena. The rapidly varied flow is, therefore, a local phenomenon.

The flow in an open-channel is termed as gradually varied flow (GVF) when the depth of flow varies gradually with longitudinal distance. Such flows are encountered both on upstream and downstream sides of control sections. Analysis and computation of gradually varied flow profiles in open-channels are important from the point of view of safe and optimal design and operation of any hydraulic structure.

Gradually varied flow may be caused as a result of one or more of the following factors:

- Change in the shape and size of the channel cross-section
- Change in the channel slope.
- Presence of obstruction, such as a weir , and

- Change in the frictional forces at the boundaries.

3.5.1 Equation of Gradually Varied Flow:

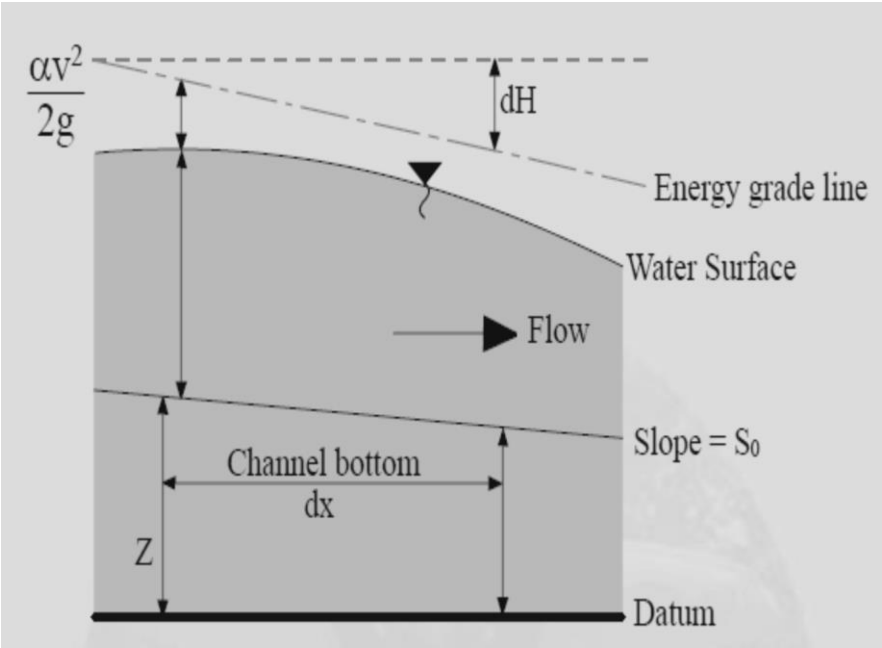
In open channel flow computations, we are often called upon to predict depth of flow at a certain location or to estimate the distance over which backwater effects due to construction of a weir or a spillway would be transmitted upstream. If the depth of flow in a channel is known for a given discharge, the area and the mean velocity of flow can be determined. The position of liquid surface in relation to the channel bottom determines the depth of flow, which in turn is used to define the type of flow.

The following assumptions are necessary for analyzing the gradually varied flow:

a) That the flow is steady.
b) That the pressure distribution over the channel section is hydrostatic, i.e.,
c) streamlines are practically straight and parallel,
d) That the head loss is same ass for uniform flow
e) That the channel slope is small, so that the depth measured vertically is the same as depth measured normal to the channel bottom,
f) That the channel is prismatic.
g) That the kinetic energy correction factor is very close to unity and
h) That the channel roughness does not depend upon the depth of flow, and is constant along the channel length.

Let Z, y and $V^2/2g$ be the datum head, the depth of flow and the velocity head respectively at any section. Then according to the Bernoulli's equation, the total energy of flow per unit weight of liquid above the horizontal datum is

$$H = Z + y + \alpha \frac{V^2}{2g} \qquad \text{Eq...3.28}$$



sign convention

$$\frac{dH}{dx} = -S_f$$

$$\frac{dz}{dx} = -S_0$$

Figure 3.9 Gradually Varied flow

Taking the bottom of the channel as the x- axis and the vertically upwards direction measured from the channel bottom, as the y-axis differentiation of the below equation with respect to x yields.

$$\frac{dH}{dX} = \frac{dZ}{dX} + \frac{dy}{dX} + \frac{d}{dX}\left(\frac{V^2}{2g}\right) \qquad \text{Eq...3.29}$$

Let, S_e = slope of the energy line, and

S_0 = slope of the channel bottom.

Now as the total energy H and the datum head Z both decrease in the direction of flow, the differential terms 'dH /dx' and dZ/dx representing the energy line slope and the slope of the channel bottom both are negative from which the slope of the free surface w. r. to channel bottom. The general di0fferential equation for gradually varied flow is given by,

$$\boxed{\frac{dy}{dx} = \frac{S_o - S_f}{1 - \alpha F_r^2}} \qquad \text{Eq...3.30}$$

Where, $\frac{dy}{dx}$ Represent the slope of the water surface with respect to bottom of the channel S_0 and $S_f$represent bed slope and friction slope of the channel respectively 'α' is kinetic energy correction co efficient 'F_r' is Froude number of flow which is given by,

$$F_r = \frac{V}{\sqrt{gD}}$$ Eq...3.31

The above equation is used to describe the various types of water surface profiles that occur in open-channels. This equation known as the equation of the gradually varied flow or the differential equation of gradually varied flow. It represents change in the depth of flow y with respect to the channel bottom (x – axis) for channel with constant width.

a) If K = conveyance at any depth y; K_0 = conveyance corresponding to normal depth y_o, then

$$Q = AC\sqrt{mi}$$

$$Q = K\sqrt{S_f} \text{ where } k = AC\sqrt{m}$$

$$K = \frac{Q}{\sqrt{S_f}}$$

$$K_0 = \frac{Q}{\sqrt{S_0}} \text{ (uniform flow)}$$ Eq...3.32

$$\frac{S_f}{S_0} = \frac{K_0^2}{K^2}$$ Eq. ..3.33

Similarly, if "Z" is section factor at depth 'y';

Z_c = Section factor at critical depth y_c

$$Z^2 = \frac{A^3}{T} \qquad \left[Z = A\sqrt{A/T}\right]$$

$$Z_C^2 = \frac{A_C^3}{T_C} = \frac{Q^2}{g} \qquad \left[\frac{Q^2}{g} = \frac{A^3}{T}\right]$$

Hence, $$\frac{Q^2T}{gA^3} = \frac{Z_C^2}{Z^2}$$ Eq. ..3.34

Using GVF equation and equation 3.2.3,

$$\frac{dy}{dx} = S_0 \frac{1 - \frac{S_f}{S_0}}{1 - \frac{Q^2T}{gA^3}}$$

$$\frac{dy}{dx} = S_0 \frac{1-\left(\frac{K_0}{K}\right)^2}{1-\left(\frac{Z_C}{Z}\right)^2} \qquad \text{Eq. ..3.35}$$

This equation is helpful in developing direct integration techniques.

(b) If Q_n represents normal discharge at a depth y and Q_c represents the critical discharge at same depth 'y'. $Q_n = K\sqrt{S_0}$ $Q_C = Z\sqrt{g}$

Use these conditions Eq 3.30 can be written as,

$$\frac{dy}{dx} = S_0 \frac{1-\left(\frac{Q}{Q_n}\right)^2}{1-\left(\frac{Q}{Q_C}\right)^2} .$$

c) Another form of Eq.(3.30) is

$$\frac{dE}{dx} = S_0 - S_f \qquad \text{Eq. ..3.36}$$

This Eq.3.36 is called differential - energy equation of GVF. This equation is helpful in developing numerical techniques for GVF profiles.

Q10. Draw the specific energy curve and explain.

Ans: ***The specific energy*** in a channel section is defined as the energy per unit weight of water measured ***with respect to channel bottom.***The specific energy E in a channel section or energy head measured with respect to the bottom of the channel at the section is

$$E = y + \alpha \frac{V^2}{2g}$$

Where 'α' is a coefficient that takes into account the actual velocity distribution in the Particular channel section, whose average velocity is 'V'. The coefficient 'α' can vary from a minimum of 1.05 - for a very uniform distribution- to 1.20 for a highly uneven distribution. Nevertheless in a preliminary approach it can be used α = 1, Eq.1 becomes, $E = y + \frac{V^2}{2g}$

A channel section with a water area A and a discharge Q, will have a specific

Energy $E = y + \frac{\mathbf{Q}^2}{2\mathbf{gA}^2}$

Above equation shows that given a discharge Q, the specific energy at a given section, is a function of the depth of the flow only. When the depth of flow y is plotted, for a certain discharge Q, against the specific energy E, a specific energy curve, with two limiting boundaries, like the one represented in figure is obtained. The lower limit, AC, is asymptotic to the horizontal axis and the upper, AB, to the line E=y. The vertex point A on the specific energy curve represents the depth y at which the discharge Q can be delivered through the section at a minimum energy. For every point over the axis E, greater than A, there are two possible water depths. At the smaller depth the discharge is delivered at a higher velocity ñ and hence at a higher specific energy of flow known as super-critical flow. At the larger depth the discharge is delivered at a smaller velocity but also with a higher specific energy, a flow known as sub critical flow. In the critical state the specific energy is a minimum, and its value can therefore be computed by equating the first derivative of the specific energy (equation 2.36) with respect to is zero.

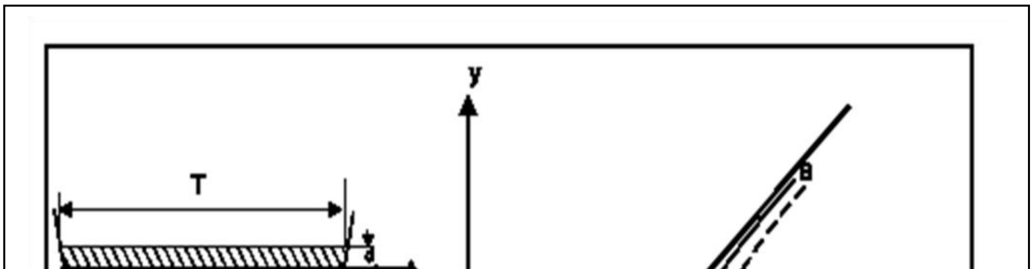
$$\frac{\mathbf{dE}}{\mathbf{dy}} = -\frac{\mathbf{Q}^2}{\mathbf{gA}^3}\frac{\mathbf{dA}}{\mathbf{dy}} + 1 = 0$$

The differential water area near the free surface, dA/dy = T, where T is the top width of the channel section.

By definition $D = \frac{\mathbf{A}}{\mathbf{T}}$

The parameter D is known as the 'hydraulic depth' of the section, and it plays a big role in the studying the flow of water in a channel. Substituting in equation (2.37) $\frac{dA}{dy} = \frac{A}{T}$ replaced by D:

$$\frac{\mathbf{V}}{\sqrt{\mathbf{gD}}} = 1 = \mathbf{F_r}$$



The quantity $\frac{V}{\sqrt{gD}} = 1 = F_r$ is dimensionless and known as the Froude's number

3.6 Classification of Surface profiles- simple Problems.

Normal depth (y_n):

It is defined as the depth of flow at which a given discharge flows as uniform flow in a given channel.

Critical depth (y_c):

It is defined as a depth of flow at which specific energy is minimum. In a given channel y_n and y_c are 2 fixed depths if Q, n and S_0 are fixed. Also, there are 3 possible relations between y_n and y_c.

as i) $y_n > y_c$

ii) $y_n < y_c$

iii) $y_n = y_c$

Further there are 2-cases where y_n does not exist.

i) When the channel bed is horizontal ($S_0 = 0$)

ii) When the channel has an adverse slope (S_0 is negative)

Based on the above the channel is classified into 5-categories.

For each 5-categories lines representing critical depth & normal depth (if exists) can be drawn in longitudinal section. This would divide the whole flow space in to 3-regions.

1

Zone 1: space above both the lines (NDL & CDL)

2

Zone 2: space between upper line and lower line

3

Zone 3: space between lower line and the channel bottom

Zone 1: $y > y_n > y_c$

Zone 2: $y_n > y > y_c$

Zone 3: $y_n > y_c > y$

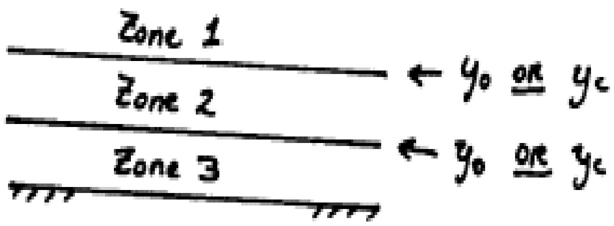


Figure 3.10 Zonal classifications of water surface profiles

Water surface longitudinal profiles can be characterised according to bottom slope S_o. According e.g. to Graf and Altinakar (1998) five different cases can be distinguished:

- M : Channels on mild slope; $S_o < S_C$
- S : Channels on steep slope; $S_o > S_C$
- C : Channels on critical slope; $S_o = S_C$
- H : Channels on horizontal slope $S_o = 0$
- A : Channels on adverse slope $S_o < 0$

Channel beds can be classified as *mild* (M), *steep* (S), *critical* (C), *horizontal* (H) ($S_O = 0$), or *adverse* (A) ($S_O < 0$). To determine the classification of a channel bed, the normal depth y_n is compared with the critical depth y_c.

Mild: $y_n > y_c$

Steep: $y_n < y_c$

Critical: $y_n = y_c$

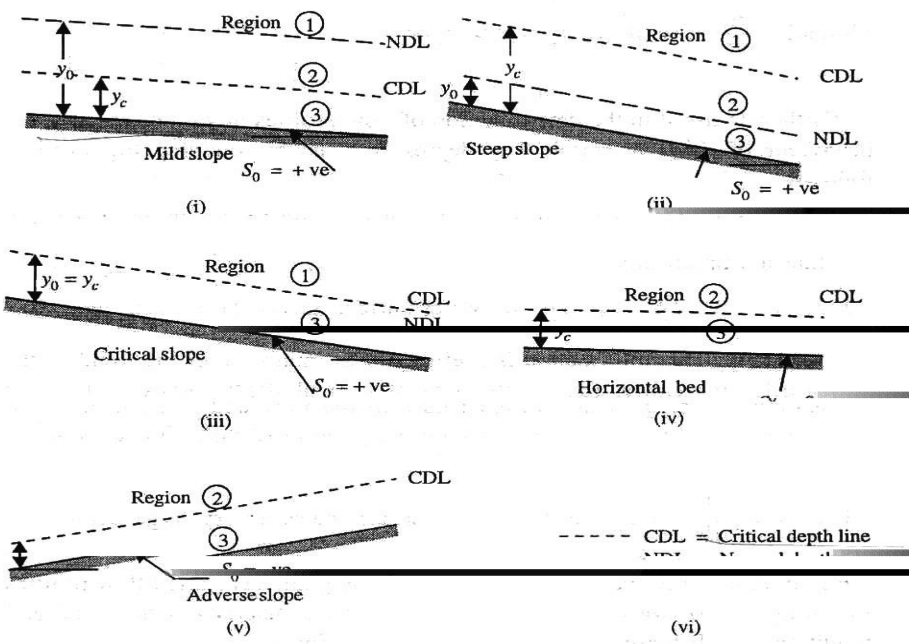


Figure 3.11 Classifications of Channel beds

Horizontal and adverse slopes are special because no normal depth exists for them. How flow-transitions through gradually varied flow depends on the relative position of the depth (*y*) to the normal depth (y_n) and the critical depth (y_c). The Table-3.1 gives details of water profile type, nomenclature and its main characteristics based on the slope of the bed

Table-3.1 Water Profile Classification based on Slopes

Sl. No.	Channel category	Symbol	Characteristics condition	Remarks
1.	MS	M	$y_0 > y_c$	Sub-critical flow at normal depth
2.	SS	S	$y_c > y_0$	Super-critical flow at normal depth
3.	CS	C	$y_0 = y_c$	Critical flow at normal depth
4.	HS	H	$S_0 = 0$	Can't sustain uniform flow
5.	AS	A	$S_0 < 0$	Can't sustain uniform flow

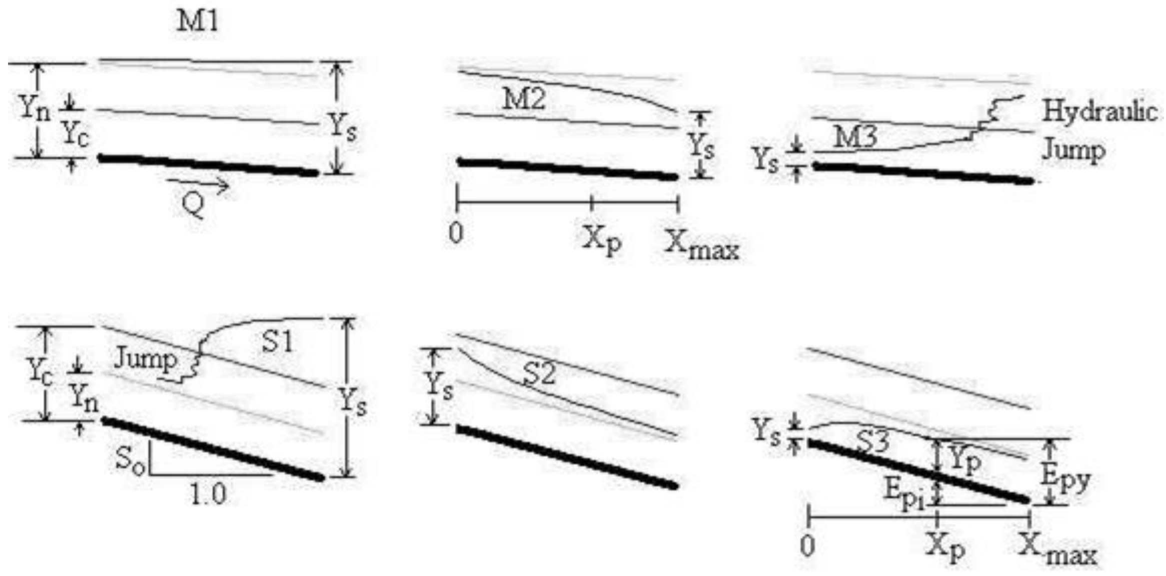


Figure 3.12 Gradually Varied Flow Profiles:

In mild slopes (M), the bottom slope is smaller than the critical slope S_C.

Type M flow:

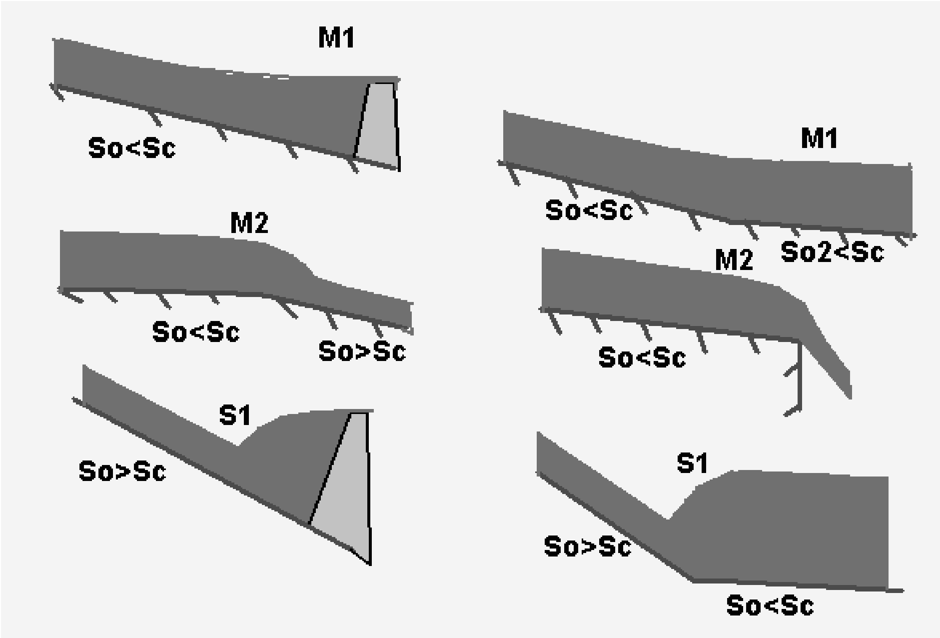


Figure 3.13 Different flow types.

In long prismatic (constant cross-sectional geometry) channels, flowing water will attempt to reach the "normal depth" (also known as the "uniform flow depth"). Normal depth is the water depth determined using Manning's equation (please see our other web page for design of trapezoidal channels using Manning's equation). A gradually varied flow (GVF) profile is a plot of water depth versus distance along the channel as the water depth gradually achieves normal depth. A GVF computation in a trapezoidal channel involves starting at a known depth Y_s and making successive water depth computations at small distance intervals. The method involves the continuity equation and energy slope equations. The calculations based on the solution of gradually varied flow equation initially computes normal depth, critical depth, and GVF profile type. Then, it computes the water depth profile.

The calculation also displays channel properties (depth, velocity, Froude number, etc.) at a desired specific location X_p.. A GVF profile is also known as a water depth profile, backwater calculation, and non-uniform flow computation. It is for steady state flows (discharge remains constant).

The flow depth 'y' increase with distance if $\frac{dy}{dx}$ is positive, (the profile is known as backwater curve) and 'y' decreases with distance if $\frac{dy}{dx}$ is negative, (the profile is known as draw down curve). Thus by determine the signs of the numerator and denominator of the equation $\frac{dy}{dx} = \frac{S_0 - S_f}{1-(F_r)^2}$; we can say whether the flow depth for a particular profile increases (or) decrease with depth.

We already known that for uniform flow $S_f = S_w = S_0$ when $y = y_n$

It is clear from Manning's (or) Chezy's formula for a given "Q"

$S_f > S_0$ if $y < y_n$

$S_f < S_0$ if $y > y_n$

By using these inequalities, we can determine sign of numerator of above equation. Similarly $F_r > 1$ (super critical) or $F_r < 1$ (sub critical) we can determine the sign of denominator.

We can discuss how surface profile approaches the normal and critical depths and channel bottom.

$\frac{dy}{dx} > 0$; i) $y > y_n$ and $y > y_c$

ii) $y < y_n$ and $y < y_c$

$\frac{dy}{dx} < 0$; i) $y_c > y > y_n$

ii) $y_n > y > y_c$

Further

i) As $y \rightarrow y_n$; $S_f \rightarrow S_0$

$\Rightarrow \frac{dy}{dx} \rightarrow 0$ provided $Fr \neq 1$ (flow is not critical)

in other words surface profile approaches NDL asymptotically

ii) As $y \rightarrow y_c$; $Fr \rightarrow 1 \Rightarrow$ denominator tends to 0

$\Rightarrow \frac{dy}{dx}$ Tends to '∞', which means water surface profile approach the critical depth line vertically. (Physically impossible, water surface has a sharp curvature, hydrostatic pressure is not possible) hydraulic jump will occur.

iii) As $y \rightarrow \infty$; $V \rightarrow 0$; consequently both Fr and S_f tends to 0

$$(v = \frac{q}{y}; v = c\sqrt{RS_f}; \quad F_e = \frac{v}{\sqrt{gy}} = 0)$$

$\Rightarrow \frac{dy}{dx} \rightarrow S_0$ (small) which means, water surface meets a very large depth as a horizontal asymptote.

Based on this information, various possible GVF profiles are grouped into 12 types (Table 3.2). The characteristics shapes and end conditions of all these profiles are indicated in the figs.

Table-3.2 Classification of GVF profiles

Channel	Region	Condition	Type	Remarks
Mild slope	1	$y > y_0 > y_c$	M_1	Backwater
	2	$y_0 > y > y_c$	M_2	Draw down
	3	$y_0 > y_c > y$	M_3	Backwater
Steep slope	1	$y > y_c > y_0$	S_1	Backwater
	2	$y_c > y > y_0$	S_2	Draw down
	3	$y_c > y_0 > y$	S_3	Backwater
Critical slope	1	$y > y_0 = y_c$	C_1	Backwater
	3	$y < y_0 = y_c$	C_3	Backwater
Horizontal bed	2	$y > y_c$	H_2	Draw down
	3	$y < y_c$	H_3	Backwater
Adverse slope (imaginary)	2	y >yc	A_2	Draw down
	3	y <yc	A_3	Backwater

3.4. Feature of Open Channel flow profiles:

a) Type M - profiles:

The most common type of GVF profiles is M_1 type which is sub-critical flow condition. Obstructions to flow such as weirs, dams, and control structures produce M_1 back water curves.

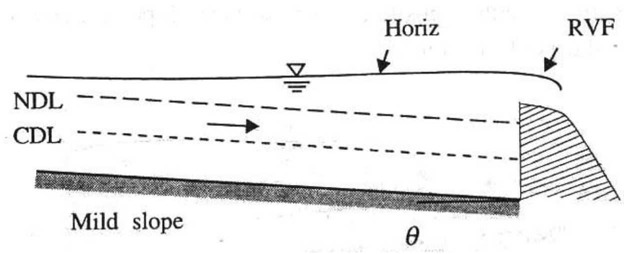
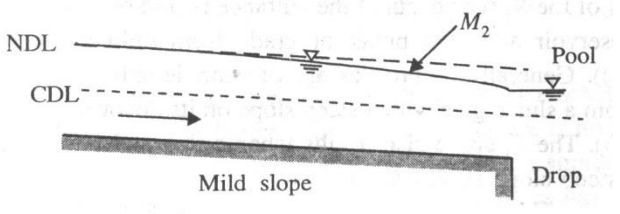


Fig 3.14 Type M - Profile (Back Water curve)

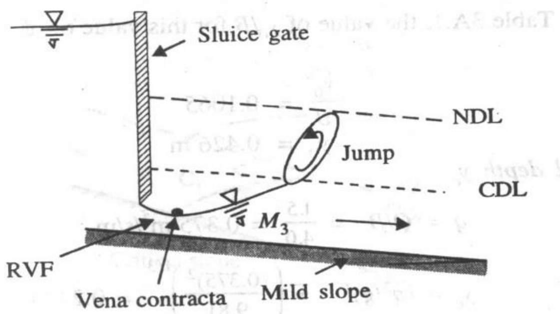
M_2 profiles occur at a sudden drop in the bed of channel and at the canal outlet into pools.



M_2 Profile

Fig 3.15 Type M - Profile (Draw Down Curve)

When a supercritical stream enters a mild channel the M_3 type of profile occurs. The flow leading from a spillway or a sluice to a mild slope forms an example. (Hydraulic jump)



M_3 Profile

Fig. 3.16 Hydraulic Jump

b) Type S - profile:

The S, profile is produced when the flow from a steep channel is terminated by a deep pool created by an obstruction (wear (or dam). At the beginning the flow changes from (super critical) flow to sub-critical flow thro a hydraulic jump.)

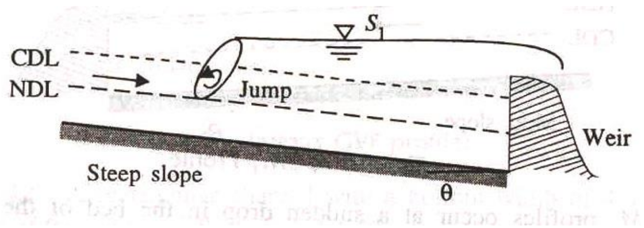


Fig 3.17 Type S - Profile

S_2 type profile occurs at the entrance region of a steep channel leading from a reservoir and a brake of grade from mild slope to steep slope.

c) Critical, Horizontal and Adverse slope:

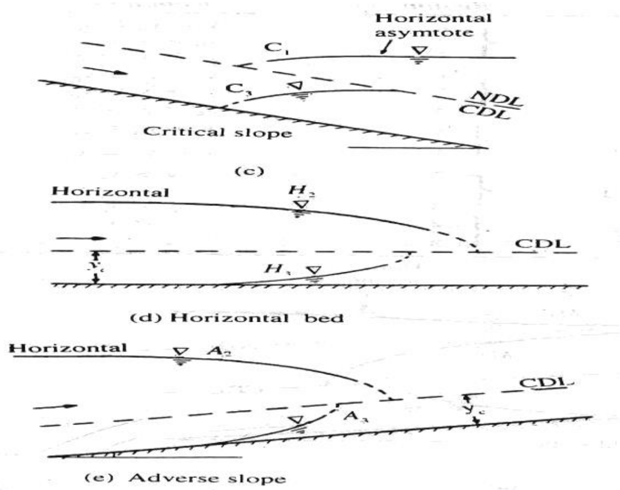


Fig 3.18 Critical, Horizontal and Adverse slope

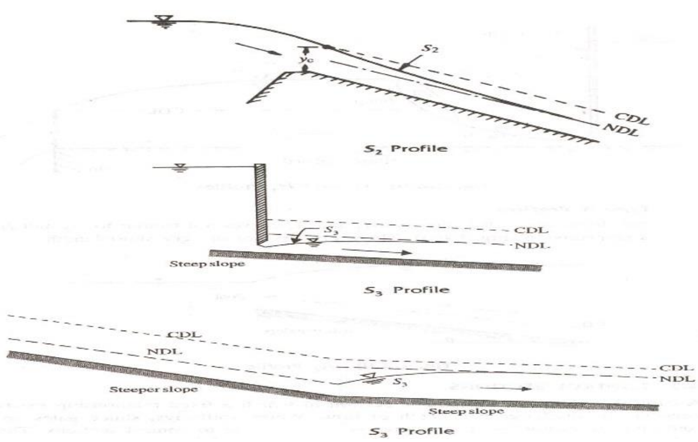


Fig 3.19 Steep Slope Profiles

Free flow from sluice gate with a steeper slope on its d/s is of S_3 type. S_3 curve also occurs – flow exits from steeper slope to a less steep slope.

Type C profiles:

C_1 and C_3 profiles are very rare and highly unstable.

Type H profile:

There is no region 1 for horizontal channel as $y_0 \rightarrow \propto$. The H2 and H3 profiles are similar to M2 and M3 profiles. However H2 curve has a horizontal asymptote.

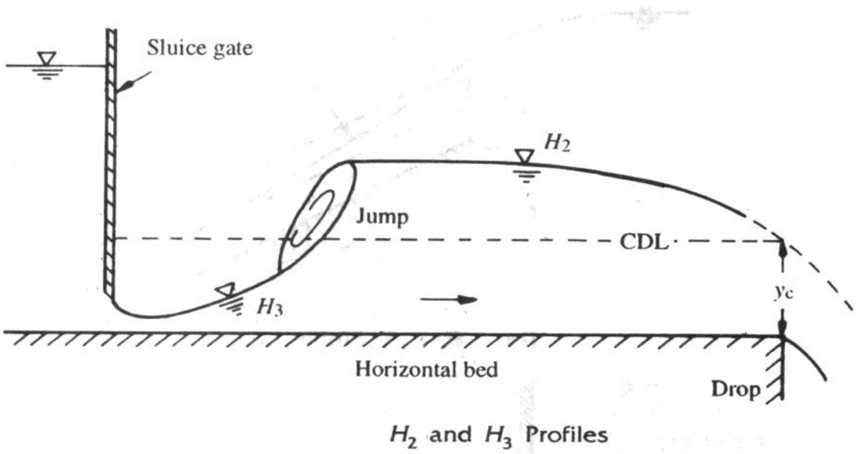


Fig 3.20 Horizontal Profiles

Type A - profile:

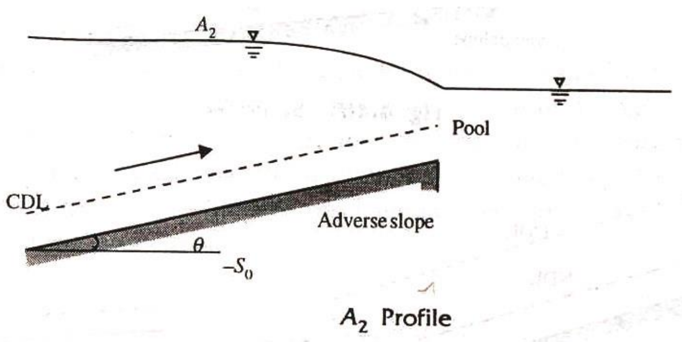


Fig. 3.21 Adverse Profiles

Adverse slopes are rather rare and A_2 and A_3 curved are similar to H_2 and H_3 curves.

3.5. Draw Down and Back Water Curves:

The general differential equation for gradually varied flow is given by,

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \alpha F_r^2} \qquad \text{Eq...3.30}$$

Where, $\frac{dy}{dx}$ Represent the slope of the water surface with respect to bottom of the channel S_0 and $S_f$represent bed slope and friction slope of the channel respectively 'α' is kinetic energy correction co efficient 'F_r' is Froude number of flow which is given by, $F_r = \frac{V}{\sqrt{gD}}$

3.5.1 Backwater Curve:<u>**For a backwater curve,** $\frac{dy}{dx}$ is positive.</u> For this condition, the above equation indicates two possible cases:

(i) $(S_0 - S_e) > 0$ **and** $(1 - F_r^{\,2}) > 0$ and

(ii) $(S_0 - S_e) < 0$ **and** $(1 - F_r^{\,2}) < 0$

These two conditions are combined together represents three surface profiles M_1, S_1 and S_3 types.

Analysis of Case (i)

Profile M_1: $y > y_n > y_c$

Profile S_1: $y > y_c > y_n$

Analysis of Case (ii)

Profile S_3: $y < y_n < y_c$

3.5.2 Drawdown Curve:<u>**For a drawdown curve,** $\frac{dy}{dx}$ is negative.</u> For this condition, the above equation indicates two possible cases:

(i) $(S_0 - S_e) > 0$ **and** $(1 - F_r^2) < 0$ and

(ii) $(S_0 - S_e) < 0$ **and** $(1 - F_r^2) > 0$

These two conditions are combined together represents three surface profiles S_2 and M_2 types.

Analysis of Case (i)

Profile S_2: $y_c > y > y_n$

Analysis of Case (ii)

Profile M_2: $y_n > y > y_c$

3.6 Analysis of Open Channel Flow profile:

The process of identification of possible flow profiles as a prelude to quantitative computations is known as analysis of flow profile. It is essentially a synthesis of the information about the GVF profiles and control sections discussed in the previous section.

A channel carrying a gradually-varied flow can in general contain different prismoidal-channel sections of varying hydraulic properties. There can be a number of control sections of various at various locations. To determine the resulting water-surface profile in a given case, one should be in a position to analyze the effects of various channel sections and controls connected in series.

3.6.1 Break in Grade:

<u>Simple situations of a series combination of two channel sections with differing bed slopes are considered.</u> In Fig 3.15, a break in grade from a mild channel to a milder channel is shown. It is necessary to first draw the critical-depth line (CDL) and the normal-depth line (NDL) for both slopes. Since, y_c does not depend upon the slope (as Q=constant), the CDL is at a constant height above the channel bed in both slopes. The normal depth y_{01} for the mild slope is lower than that of the milder slope. (y_{02}). In this case, y_{02} acts as a control similar to the weir or spillway case and an M1 backwater curve is produced in the mild-slope channel. Various combinations of slopes and the resulting GVF profiles are presented in Fig. 5.

In the examples indicated in Fig 5, the section where the grade changes acts as a control section and this can be classified as a natural control. It should be noted that even though the bed slope is considered as the only variable in the above examples, the same type of analysis would hold good for channel sections in which there is a marked change in the roughness characteristics with or without a change in the bed slope. A long reach of unlined canal followed by a lined reach serves as a typical example for the same.

<u>Serial Combination of channel sections:</u>

To analyze a general problem of any channel sections and controls, the following steps are to be adopted:

- Draw the longitudinal section of the systems.
- Calculate the critical depth and normal depth to draw CDL & NDL
- Make all the controls- both the imposed as well as natural controls.
- Identify the possible profiles.

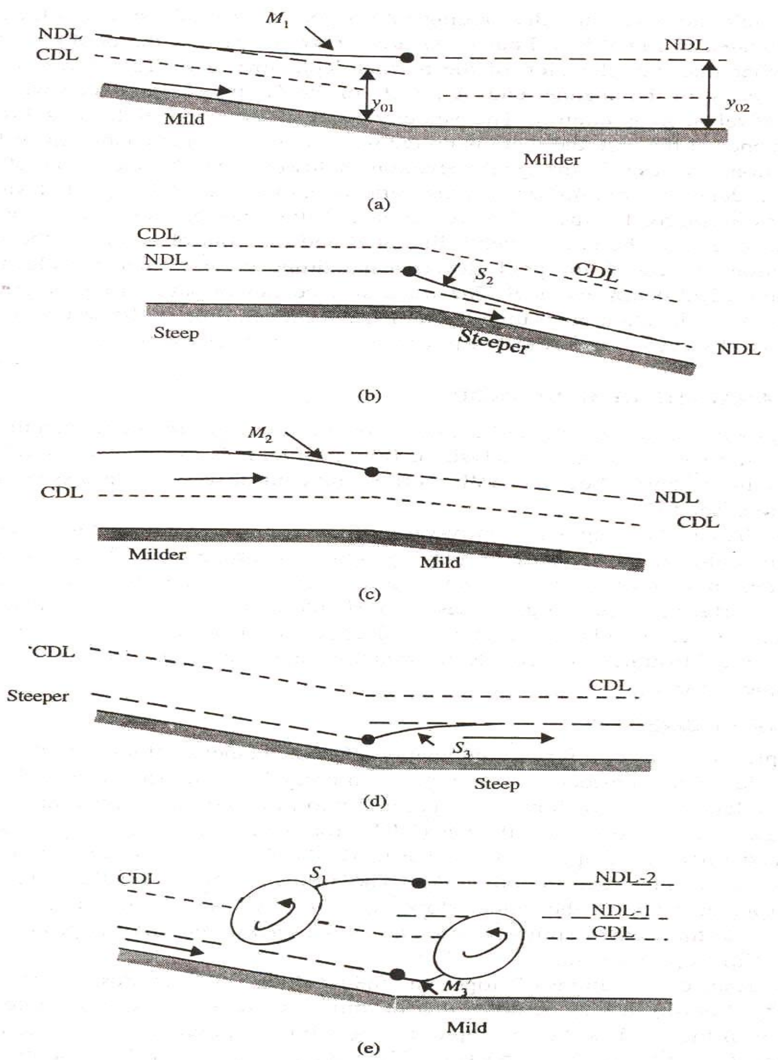
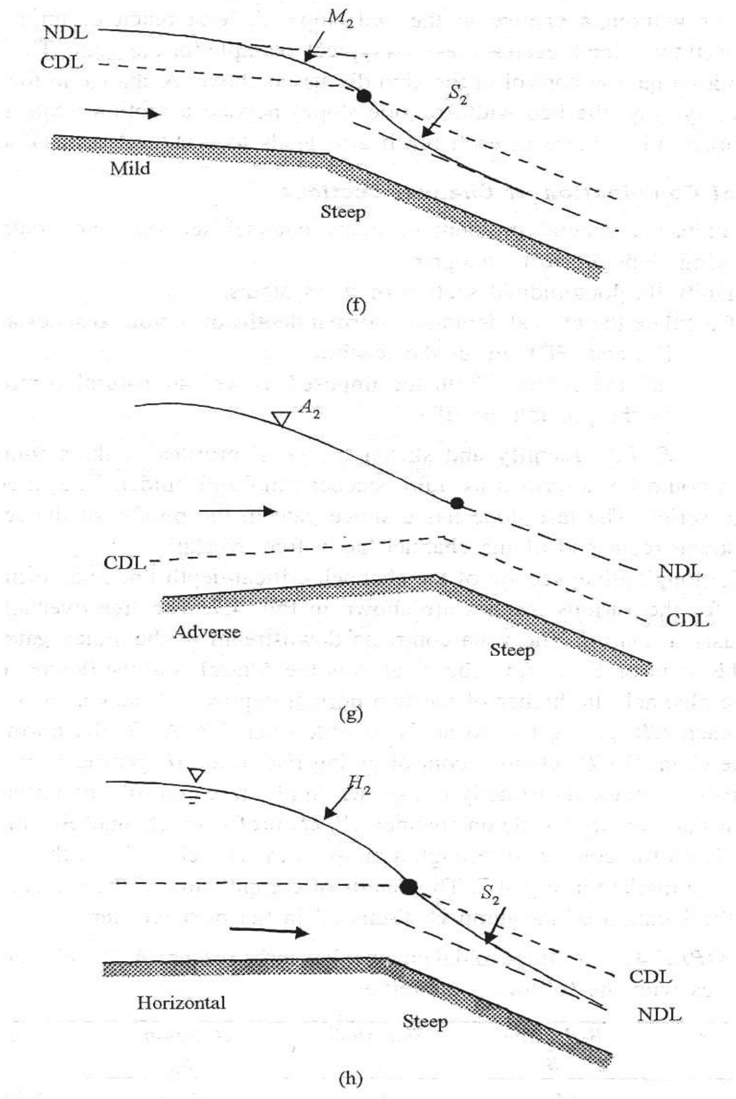


Figure 3.22 Water surface Profiles based on Break in Grade



GVF profiles at break in grades

Figure 3.23 Water surface Profiles based on Break in Grade

3.6.2 Control Section:

A Control section is defined as a section in which a fixed relationship exists between the discharge and depth of flow. Weirs, spillways, sluice gates are some typical examples of structures which give rise to control sections. **The critical depth is also control point.** However, it is effective in a flow profile which changes from sub-critical to supercritical flow. In the reverse case of transition from supercritical flow to sub-critical flow, a hydraulic jump is usually formed bypassing the critical depth as a control point. Any GVF profile will have at least one control section.

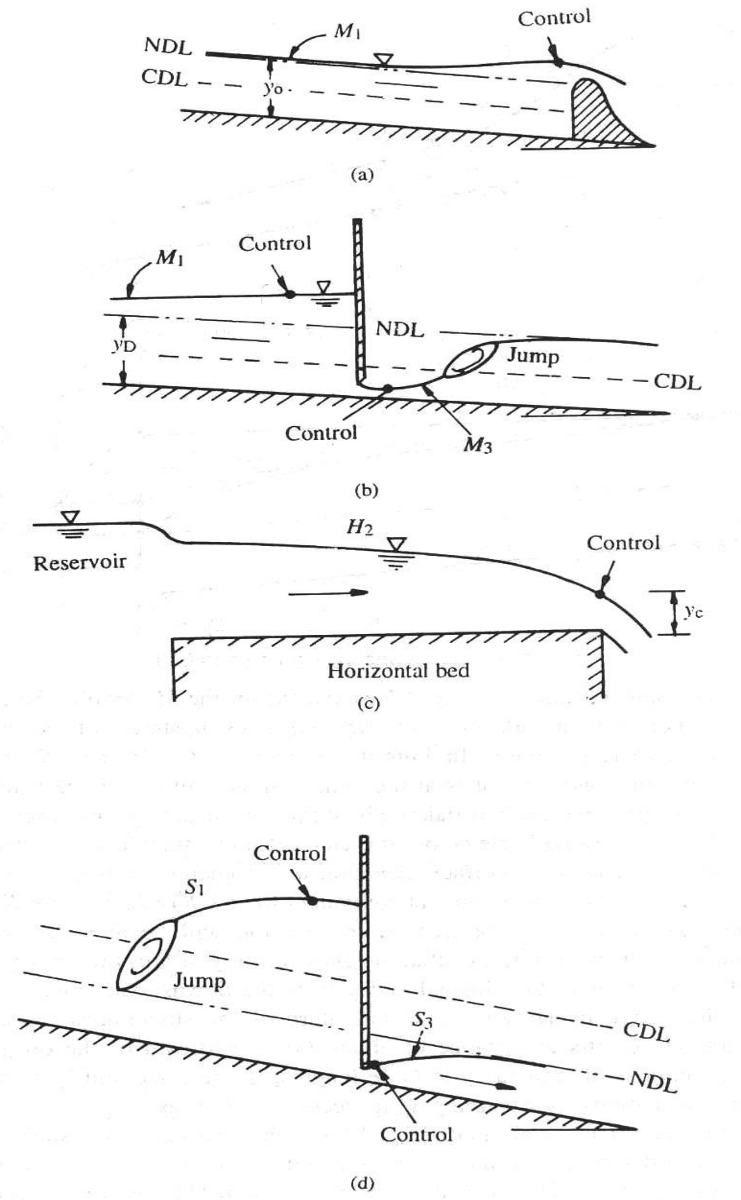


Figure 3.24 Water surface Profiles at Control Section

In the synthesis of GVF profile occurring in serially-connected channel elements, the control sections provide a key to the identification of proper profile shapes. A few typical control sections are indicated if Fig 4.5 (6). It may be noted that sub-critical flows have controls in the downstream end while supercritical flows are governed by control sections existing at the upstream end of the channel section.