MODULE 3:

Orifice and Flow through Pipes

ORIFICES AND MOUTHPIECE

ORIFICES

- An orifice is a small aperture through which the fluid passes. The thickness of an orifice in the direction of flow is very small in comparison to its other dimensions.
- If a tank containing a liquid has a hole made on the side or base through which liquid flows, then such a hole may be termed as an orifice.
- The rate of flow of the liquid through such an orifice at a given time will depend partly on the shape, size and form of the orifice
- An orifice usually has a sharp edge so that there is minimum contact with the fluid and consequently minimum frictional resistance at the sides of the orifice. If a sharp edge is not provided, the flow depends on the thickness of the orifice and the roughness of its boundary surface too.

Classification of Orifices

The orifices are classified as follow:

- According to shape of orifice
  1. Small orifice
  2. Large orifice

  An orifice is termed as small orifice when the head of the liquid from the center of orifice is more than five times the depth of the orifice, the orifice is called small orifice.

  And an orifice is termed as large orifice if the head of the liquid is less than five times the depth of orifice , it is known as large orifice.

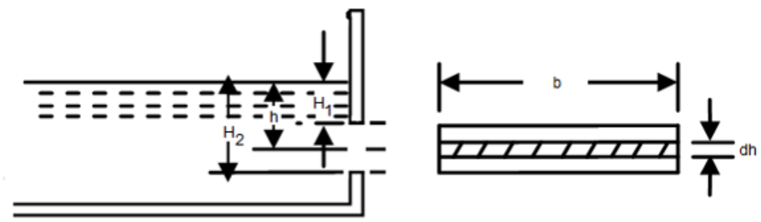
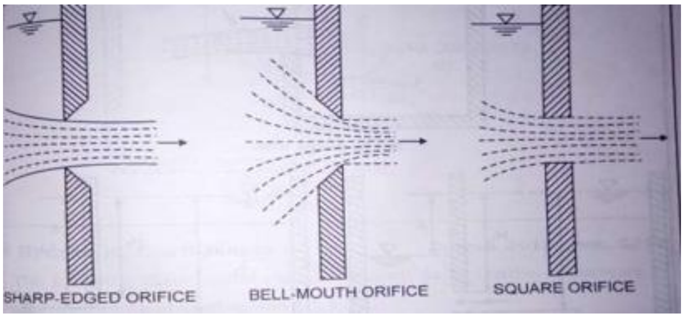


Fig: Large rectangular orifice

- According to size of orifice
  1. Circular orifice
  2. Triangular orifice

3. Rectangular orifice
4. Square orifice

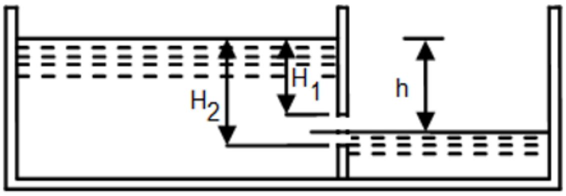
- According to Shape of edge
    1. Sharp edged orifice
    2. Bell mounted orifice



- According to Nature of discharge
    1. Free discharge orifice
    2. Full submerged orifice
    3. Partial submerged orifice

On orifice or mouth piece is said to be **discharging free**, when it discharges into atmosphere.

It is said to be **submerged** when it discharges into another liquid



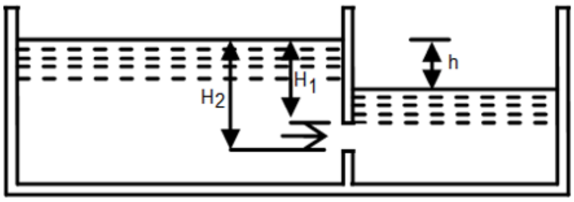


Fig: Partially drowned/ submerged orifice   Fig: Fully drowned/ submerged orifice

Flow through an orifice

Fig shows a small circular orifice with sharp edge in the side wall of a tank, discharging into the atmosphere.

- Let the orifice be at a depth h, below the free surface.
- As the fluid flow through the orifice, it contacts and attain a parallel form (i.e stream line become parallel) at a distance d/2 from the plane of orifice.

- The point at which the streamlines first become parallel is termed as “Vena Contracta” (the cross section of jet at jet at vena contracta is less than that of orifice)
- Beyond this section, the jet discharges and is attracted in the downward direction by gravity.

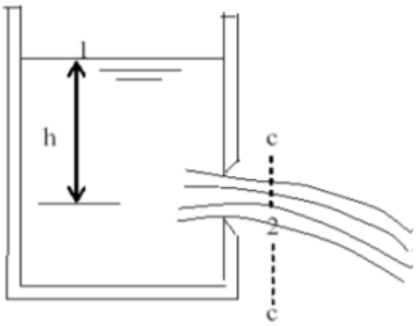


Fig: Tank with an Orifice

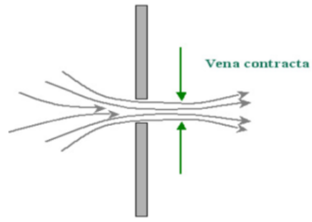


Fig: Formation of Vena Contracta

Consider points (1) and (2) as shown in figure,

Applying Bernoulli’s equation at point (a) and (2), we have

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

But $Z_1 = Z_2$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$\frac{p_1}{\rho g}$ **= H and** $\frac{p_2}{\rho g}$ **= 0 (atmospheric pressure)**

If the area of the tank is large enough as compared to that of the orifice, the velocity at point 1 becomes negligibly small

$$\boldsymbol{H + 0 = 0 +} \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2gh}$$

This is the theoretical velocity but the actual velocity will be less than this value

$$V_{(act)} = C_v.\sqrt{2gh}$$

Hydraulic Co- efficient of an Orifice

The hydraulic co-efficient are:

1. Co-efficient of Velocity, C_v
2. Co-efficient of Discharge, C_d
3. Co-efficient of Discharge, C_d

Co-efficient of Velocity, C_v

It is defined as the ratio of actual velocity of jet of liquid at vena contracta (V_{act}) to the theoretical velocity of jet (V_{th}).

$$C_v = \frac{V_{act}}{V_{th}} = \frac{V}{\sqrt{2gh}}$$

Value varies from 0.95 to 0.99 for orifices

Co-efficient of Discharge, C_d

It is defined as the ratio of actual discharge (Q_{act}) to the theoretical discharge (Q_{th})

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{Q_{act}}{a \times \sqrt{2gh}}$$

Value varies from 0.61 to 0.65 for orifices, generally taken as **0.62**

Co-efficient of Contraction, C_c

It is defined as the ratio of the area of cross section of the jet at Vena of cross section of the jet at Vena Contracta (a_c) to the area of the orifice (a).

$$C_c = \frac{a_c}{a}$$

Value varies from 0.61 to 0.69 for orifices, generally taken as **0.64**

NUMERICALS

1. An orifice 50mm dia is discharging water, under a head of 10m, if C_d = 0.6 and C_v= 0.97, find

a) Actual discharge

b) Actual velocity at vena contracta

Solution:

Given Dia of orifice, d = 50mm= 0.05m

Area of orifice A= $\frac{\pi}{4} \times 0.05^2$ = 0.001963m^2

Head, h = 10m C_d = 0.6 C_v= 0.97

a) Actual discharge

We have, $C_d = \frac{Q_{act}}{Q_{th}}$

Theoretical Discharge , Q_{th} = area of orifice × theoretical velocity

= a × $\sqrt{2gh}$

= 0.001963 × $\sqrt{2 \times 9.81 \times 10}$ = 0.02749 m^3/sec

Actual Discharge, $Q_{act} = C_d . Q_{th}$

= 0.6 × 0.0274 = 0.01649 m^3/sec

b) Actual velocity at vena contracta

$$C_v = \frac{V_{act}}{V_{th}}$$

Actual Velocity = C_v × theoretical Velocity

= $C_v \times \sqrt{2gh}$

$$= 0.97 \times \sqrt{2 \times 9.81 \times 10} = 13.58 \text{ m/sec}$$

2. The head of water over the centre of an orifice of dia 30mm is 1.5m. The actual discharge through the orifice is 2.55 lt/sec. Find the efficiency of discharge.

Solution:

Given Dia of orifice, d = 30mm= 0.03m

Area of orifice, a= $\frac{\pi}{4} \times 0.03^2$ = 0.0007068m^2

Head, h = 1.5m

actual discharge through the orifice, Q_{act}= 2.55 lt/sec = 0.00255 m^3/sec

Theoretical Discharge , Q_{th} = area of orifice × theoretical velocity

$$= a \times \sqrt{2gh}$$

$$= 0.007068 \times \sqrt{2 \times 9.81 \times 1.5}$$

$$= 0.004166 \text{ m}^3\text{/sec}$$

Actual Discharge, $Q_{act} = C_d . Q_{th}$

$$C_d = \frac{Q_{act}}{Q_{th}}$$

$$= \frac{0.00255}{0.004166} = 0.612$$

Experimental determination of Hydraulic Co-efficient

Fig shows a tank containing water at constant level. Let the water flows out through an orifice fitted at one side of tank.

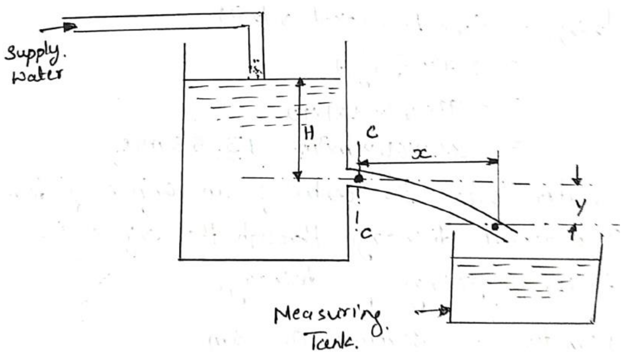
Let section c-c represents the point of vena contracta. Consider a particle of water in jet

Let, x= horizontal distance travelled by the particle

y = vertical distance b/w c-c and P

V = actual velocity of the jet at vena contracta

h = constant water head



(i) Determination of Co-efficient of Velocity

Then horizontal distance, $x = v \times t$, $t = \frac{x}{v}$

And vertical distance, $y = \frac{1}{2}gt^2$

$$y = \frac{1}{2}g\left(\frac{x}{v}\right)^2 = \frac{gx^2}{2v^2}$$

$$v^2 = \frac{gx^2}{2y}$$

$$v = \sqrt{\frac{gx^2}{2y}}$$

But theoretical velocity, $v_{th} = \sqrt{2gh}$

Co-efficient of velocity, $C_v = C_v = \frac{\sqrt{\frac{gx^2}{2y}}}{\sqrt{2gh}} = \sqrt{\frac{x^2}{4yh}}$

$$C_v = \frac{x}{\sqrt{4yh}}$$

(ii) Determination of Co-efficient of Discharge

- The water flowing through orifice at constant head, H is collected in measuring tank for a known time 't'.
- The rise of water in measuring tank is noted down

Discharge, $Q_{act} = \frac{area\ of\ measuring\ tank \times rise\ of\ water\ in\ measuring\ tank}{time\ (t)}$

Theoretical Discharge ,

Q_{th} = area of orifice $\times$ theoretical velocity

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{Q_{act}}{a \times \sqrt{2gh}}$$

(iii) Determination of Co-efficient of Discharge

The co-efficient of contraction, C_c determined as

$$C_d = C_c \times C_v$$

$$C_c = \frac{C_d}{C_v}$$

NUMARICALS

1. A vertical sharp-edged orifice 120mm in dia is discharging water at the rate of 98.2 lt/sec. Under a constant head of 10m. A point on the jet, measured from the vena contracta of the jet has co-ordinates 4.5m horizontal and 0.54m vertical. Find the following for the orifice, C_v, C_c, and C_d.

Solutions:

Dia of orifice, d = 120mm= 0.12m

Area of orifice A= $\frac{\pi}{4} \times 0.12^2$ = 0.01131m^2

Discharge, Q = 98.2 lt/sec = 0.0982 m^3/sec

Head, H = 10m

horizontal distance of point on the jet, measured from the vena contracta , x= 4.5m

Vertical distance, y= 0.54m

(i) Theoretical velocity, $v_{th} = \sqrt{2gh}$

$$= \sqrt{2 \times 9.81 \times 10} = 14 m/sec$$

Co-efficient of velocity, $C_v = \frac{x}{\sqrt{4yh}}$

$$= \frac{4.5}{\sqrt{4 \times 0.54 \times 10}} = 0.968$$

(ii)Theoretical Discharge ,Q_{th} = area of orifice × theoretical velocity

$$= 0.01131 \times 14 = 0.1583 \text{ m}^3/\text{sec}$$

Co-efficient of Discharge, $C_d = \frac{Q_{act}}{Q_{th}}$

$$= \frac{0.0982}{0.1583} = 0.62$$

(iii) Co-efficient of Contraction, $C_c = \frac{C_d}{C_v}$

$$= \frac{0.62}{0.968} = 0.64$$

2. The head of water over an orifice of diameter 100mm in dia is 12m. The water carrying out from the orifice is collected in a rectangular tank 23×0.9m. The rise of water level in this tank is 1.2m in 30sec. Find the co-efficient of discharge.

Solution:

Dia of orifice, d = 100mm= 0.1m

Area of orifice A= $\frac{\pi}{4} \times 0.1^2$ = 0.00785m^2

Area of measuring tank, a = 23×0.9 = 1.8m^2

Head, H = 12m

Rise of water level in collecting tank, h= 1.2m in t=30sec

(i) Co-efficient of discharge, C_d

Theoretical velocity, $v_{th} = \sqrt{2gh}$

$$= \sqrt{2 \times 9.81 \times 12} = 15.34\ m/sec$$

Theoretical Discharge ,Q_{th} = area of orifice × theoretical velocity

$$= a \times \sqrt{2gh}$$

$$= 0.00785 \times 15.34 = 0.1204 \text{ m}^3/\text{sec}$$

Actual discharge, $Q_{th} = \frac{area\ of\ measuring\ tank \times rise\ of\ water\ in\ measuring\ tank}{time\ (t)}$

$$= \frac{1.8 \times 1.2}{30} = 0.072 \text{ m}^3/\text{sec}$$

Co-efficient of discharge, $C_d = \frac{Q_{act}}{Q_{th}}$

$$= \frac{0.072}{0.1204} = 0.6$$

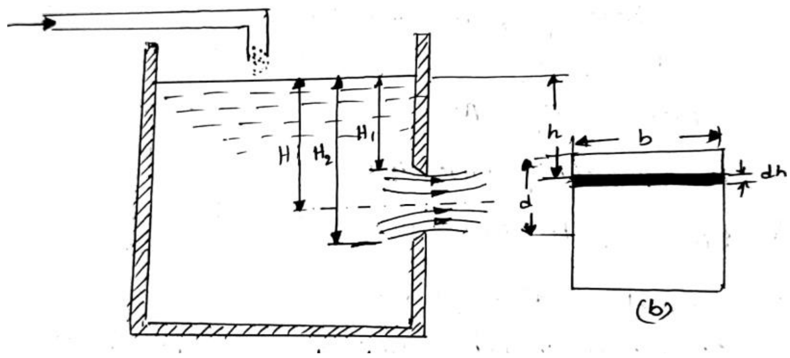
Discharge through Large Rectangular Orifice

When available head of the liquid is less than five times the depth of orifice , it is known as **large orifice**.

In case of large orifice, the velocity of a liquid flowing through the orifice varies with the available head of the liquid and hence cannot be calculated as

$$Q_{act} = C_d \sqrt{2gH}$$

Consider a large rectangular orifice in one side of the tank, discharging water freely into atmosphere as shown in fig:



Let H_1= height of liquid above top of the orifice

H_2= height of liquid above bottom of the orifice

b = breadth of the orifice (width)

d = depth of the orifice

C_d = co-efficient of discharge

Consider an elementary strip of depth 'dh' at the depth of 'h' below the water level as shown in fig:

Area of strip = b × dh

Theoretical velocity of water through strip, $v_{th} = \sqrt{2gh}$

Discharge of water through strip , dQ = C_d × area of strip × velocity

$$dQ = C_d \times b \times dh \times \sqrt{2gh}$$

Total discharge through the whole orifice may be found out by integrating the above equation b/w limits H_1 and H_2

$$Q_{act} = \int_{H_1}^{H_2} C_d \times b \times dh \times \sqrt{2gh}$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}.\, dh$$

$$= C_d \times b \times \sqrt{2g} \times \frac{3}{2} \times [h^{(\frac{3}{2})}]_{H_1}^{H_2}$$

$$\boldsymbol{Q_{act} = \frac{3}{2} \times C_d \times b \times \sqrt{2g}\, [H_2^{3/2} - H_1^{3/2}]}$$

NUMERICALS

1. Find the discharge through a rectangular orifice 3.0m wide and 2.0m deep fitted to a water tank. The water level in the tank is 4m above top edge of the orifice . Take $C_d = 0.62$

Solution:

Given: Width of orifice, b= 3.0m

Depth of orifice, d= 2.0m

Height of water above top of the orifice, H_1= 4.0m

Height of water above bottom of the orifice, $H_2 = H_1 + d = 4+2 = 6$m

Co-efficient of discharge, $C_d = 0.62$

Discharge through large rectangular orifice, $Q_{act} = \frac{3}{2} \times C_d \times b \times \sqrt{2g}\, [H_2{}^{3/2} - H_1{}^{3/2}]$

$= \frac{3}{2} \times 0.62 \times 3 \times \sqrt{2 \times 9.81}\, [(6)^{3/2} - (4)^{3/2}]$

= 36.78 m^3/sec

2. A rectangular orifice 0.6m wide and 0.8m deep is discharging water from a vessel. The top edge of the orifice is 0.4m below the water surface in the vessel. Find

(i) The discharge through the orifice, $C_d = 0.62$

(ii) The present error, if the orifice is treated as small orifice.

Solution:

Width of orifice, b= 0.6m

Depth of orifice, d= 0.8m

Height of water above top of the orifice, H_1= 0.4m

Height of water above bottom of the orifice, $H_2 = H_1 + d = 0.4+0.8 = 1.2$m

Co-efficient of discharge, $C_d = 0.62$

(i) The discharge through the orifice

Discharge through large rectangular orifice, $Q_{act} = \frac{3}{2} \times C_d \times b \times \sqrt{2g}\, [H_2{}^{3/2} - H_1{}^{3/2}]$

$= \frac{3}{2} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81}\, [(1.2)^{3/2} - (0.4)^{3/2}]$

= 1.165 m^3/sec

(ii) The present error, if the orifice is treated as small orifice.

area of orifice = (0.6 × 0.8)

Head, h = H_1 + d/2 = 0.4 + (0.8/2) = 0.8m

Discharge through small orifice, $Q' = C_d \times a \times \sqrt{2gh}$

$= 0.62 \times (0.6 \times 0.8) \times \sqrt{2 \times 9.81 \times 0.8}$

= 11.79 m^3/sec

Percent error = $\frac{Q' - Q}{Q} = \frac{11.79 - 1.165}{1.165}$ = 0.012 or 1.2 %

MOUTHPIECE

A mouthpiece is a short length of a pipe, which is two or three times its diameter in length, fitted in a tank or vessel containing the fluid.

Orifices as well as mouthpiece are used for measuring the rate of flow of fluid.

Classification of Mouthpiece

Mouthpiece are classified as

a) Depending upon their position with respect to the tank or vessel to which they are fitted

i. External Mouthpiece
ii. Internal Mouthpiece

b) Depending upon their shapes

i. Cylindrical Mouthpiece
ii. Convergent Mouthpiece
iii. Convergent-divergent Mouthpiece

c) Depending upon their nature of discharge at the outlet

i. Mouthpiece Running-full
ii. Mouthpiece Running-free

This classification is only for internal mouthpiece which are known as Borda's or Re-entrant mouthpieces

NOTCHES & WEIRS

Flow Over Notches & Weirs

Notches, often termed sharp crested or thin plate weirs, are little more than steel plates inserted vertically into a channel section and with a crest plate of some non-rusting metal (usually brass) with a precisely machined bevel.

The openings in such notches can be made triangular, for accurate measurement of small flow rates, or rectangular when larger flow rates have to be passed. The accuracy achievable with thin plate weirs is by far the highest of any type of weir (about 1 or 2 percent) but their use is rather restricted because they are unable, structurally, to withstand the forces met in real life situations and their crest units are easily damaged by floating debris and vandalism.

A notch is an opening made in the side wall of a tank such that the liquid surface in the tank is below the upper edge of the opening. Generally notches are made of metallic plates and their use is limited to laboratory channels.

A weir is a masonry/concrete structures built across an open channel so as to rise the water level on the upstream side and to allow the excess water to flow over the entire length onto the downstream side.

Classification of Notches and Weirs

a) Depending on shape:

- i) Rectangular
- ii) Triangular
- iii) Trapezoidal

b) Depending on the shape of the crest

- i) Sharp crested
- ii) Broad crested.

c) Depending on flow

- i) Free
- ii) Submerged

d) Depending on Ventilation

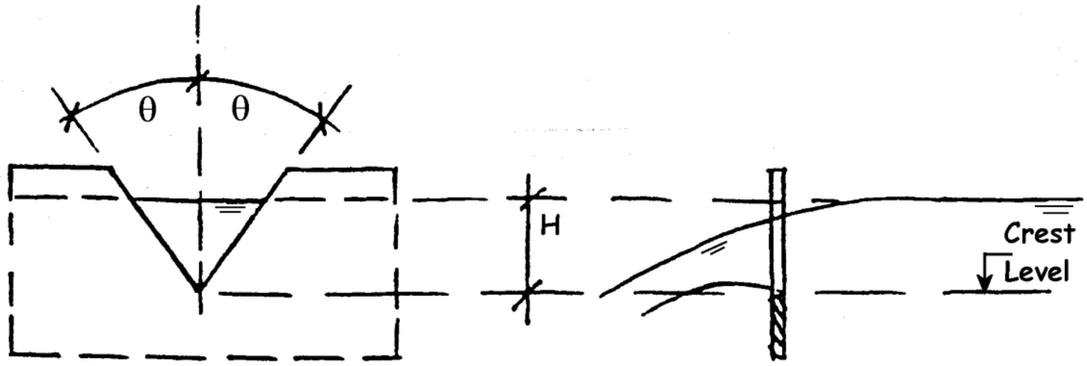
- i) Fully aerated
- ii) Depressed
- iii) Clinging or Drowned.

The equations for notches are:

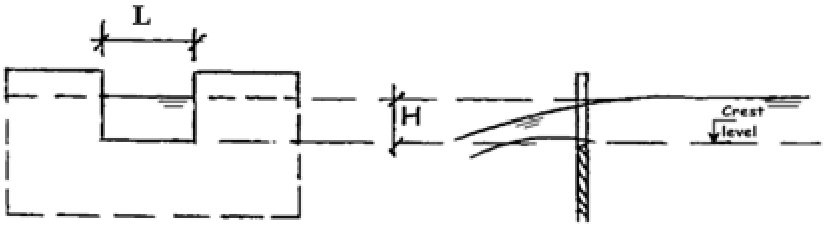
For the triangular notch $Q = C_d \frac{8}{15} \tan\frac{\theta}{2} \sqrt{2g}\, H^{\frac{5}{2}}$

For the rectangular notch $Q = C_d \frac{2}{3} b \sqrt{2g}\, H^{\frac{3}{2}}$

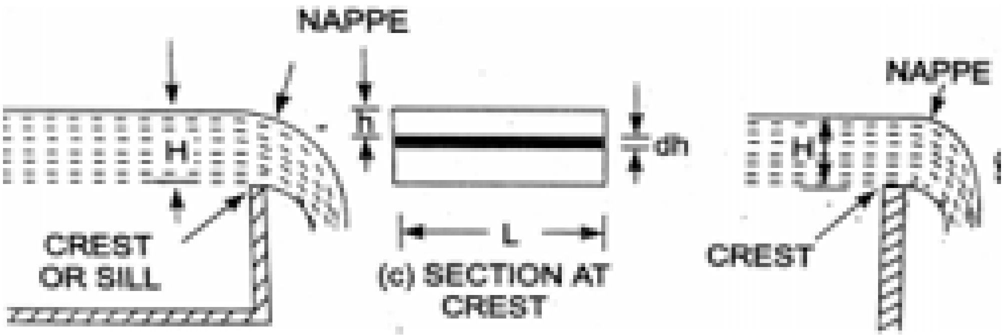
Flow over a Triangular Notch



Discharge over a Rectangular Notch



Consider a rectangular notch or weir provided in a channel carrying water as shown in fig



Let,

L=length of the notch or weir

H=head over the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth form the free surface of water as shown in fig

The area of strip = Lxdh

And theoretical velocity of water flowing through strip= $\sqrt{2gh}$

Discharge through the strip $dq = C_d X Area \times Velocity$

$$dq = C_d x L x dh x \sqrt{2gh}$$

Total discharge $\int_0^Q dq = C_d x L \sqrt{2g} \int_0^H h^{\frac{1}{2}} dh$

$$Q = C_d x L \sqrt{2g} \frac{2}{3} h^{\frac{3}{2}} \int_0^H$$

$$\therefore Q = C_d \frac{2}{3} L \sqrt{2g} H^{\frac{3}{2}}$$

$$Q_{act} = \frac{2}{3} C_d \sqrt{2g} L H^{\frac{3}{2}} ---(1)$$

C_d=Coefficient of discharge, its average value is about 0.62.

NUMERICALS

1. Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300mm. Take C_d= 0.60

Solution: Given

L=length of the notch= 2.0m

H=head over the notch = 300mm= 0.3m

$$Q_{act} = \frac{2}{3} C_d \sqrt{2g} L H^{\frac{3}{2}}$$

$$Q_{act} = \frac{2}{3} X 0.60 X \sqrt{2g} X 2.0 X 0.30^{\frac{3}{2}}$$

$$Q_{act} = 0.582\ m^3/s$$

2. The head of Water over a rectangular notch is 900mm. The discharge is 300litres/s.Find the length of the notch, when C_d= 0.62

Solution:

Given: Head over the notch, H= 90cm=0.9m

Discharge, Q= 300litres/s= 0.3 m^3/s

Coefficient of discharge C_d= 0.62

Length of notch = L

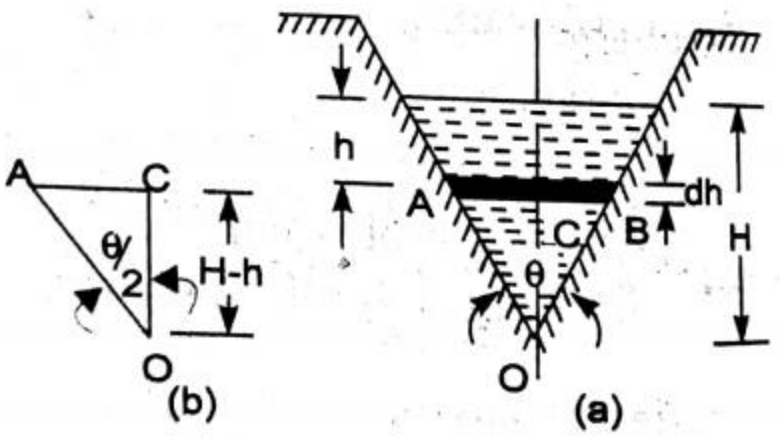
We have,

$$Q_{act} = \frac{2}{3} C_d \sqrt{2g} L H^{\frac{3}{2}}$$

$$0.3 = \frac{2}{3} x 0.62 x L x \sqrt{2g} x 0.9^{\frac{3}{2}}$$

L= 0.192m=192mm

Discharge over a Triangular Notch or Weir



The expression for the discharge over a triangular notch or weir is the same. It is derived as

H=head of water above the V- notch

θ= angle of notch

Consider a horizontal strip of water of thickness ‘dh’ at a depth of h from the free surface of water as shown in fig

From the figure, we have

$$tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

AC=(H-h) $tan\frac{\theta}{2}$

Width of the strip= AB=2AC=2(H-h) $tan\frac{\theta}{2}$

Area of the strip= 2(H-h) $tan\frac{\theta}{2}$xdh

theoretical velocity of water flowing through strip= $\sqrt{2gh}$

Discharge through the strip $dq = C_d\, X\, Area \times Velocity$

$$dq = C_d\, X\, 2(\mathrm{H} - \mathrm{h}) tan\frac{\theta}{2} \text{x dh} \times \sqrt{2gh}$$

$$dq = 2C_d\, (\mathrm{H} - \mathrm{h}) tan\frac{\theta}{2} \text{x dh} \times \sqrt{2gh}$$

Total discharge Q $is \int_0^Q dq = \int_0^H 2C_d\, (\mathrm{H} - \mathrm{h}) tan\frac{\theta}{2} \text{x dh} \times \sqrt{2gh}$

$= 2C_d\, tan\frac{\theta}{2}\sqrt{2g} \int_0^H (\mathrm{H} - \mathrm{h}) \text{x dh} \times h^{\frac{1}{2}}$

$=2C_d\, tan\frac{\theta}{2}\sqrt{2g} \int_0^H (\mathrm{H} - \mathrm{h}) x h^{\frac{1}{2}} \text{x dh}$

$=2C_d\, tan\frac{\theta}{2}\sqrt{2g} \int_0^H \left(\mathrm{H}h^{\frac{1}{2}} - h^{\frac{3}{2}}\right) \text{x dh}$

$$= 2C_d\, tan\frac{\theta}{2}\sqrt{2g}\left[\frac{2}{3}HH^{\frac{3}{2}} - \frac{2}{5}H^{\frac{5}{2}}\right]$$

$$= 2C_d\, tan\frac{\theta}{2}\sqrt{2g}\left[\frac{2}{3}H^{\frac{5}{2}} - \frac{2}{5}H^{\frac{5}{2}}\right]$$

$$= 2C_d\, tan\frac{\theta}{2}\sqrt{2g}\left[\frac{4}{15}H^{\frac{5}{2}}\right]$$

$$= \frac{8}{15}C_d\, tan\frac{\theta}{2}\sqrt{2g}\left[H^{\frac{5}{2}}\right]$$

For a right angled V- notch, if $C_d = 0.6$

$\theta = 90^0$, $tan\frac{\theta}{2} = 1$

Discharge, $Q = \frac{8}{15}C_d\, tan\frac{\theta}{2}\sqrt{2g}\left[H^{\frac{5}{2}}\right]$

$$Q = \frac{8}{15} X 0.6 X 1 X \sqrt{2g}\left[H^{\frac{5}{2}}\right]$$

$$Q = 1.417\left[H^{\frac{5}{2}}\right]$$

NUMERICAL

1. Find the discharge of water flowing over a triangular notch of angle 60^0 when the head over the notch is 0.3m. Take C_d= 0.60

Solution:

Given :

θ=Angle of the V-notch= 60^0

H=head over the notch = 0.3m

C_d= 0.60

Discharge $Q = \frac{8}{15} C_d \, tan\frac{\theta}{2} \sqrt{2g} \left[H^{\frac{5}{2}}\right]$

$$Q = \frac{8}{15} x0.6xtan\frac{60}{2} \sqrt{2x9.81} \left[0.3^{\frac{5}{2}}\right]$$

Q = 0.040 m³/s

2. Water flows over a rectangular weir 1m wide at a depth of 150mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively. Find the depth over the triangular weir.

Solution:

Given : for rectangular weir

L=length of the notch= 1m

H=Depth of water , H =150mm= 0.15m

C_d= 0.62

For triangular weir, θ= 90^0 , C_d= 0.59

Let depth over triangular weir= H_1

The discharge over triangular weir is given by equation as

$$Q_{act} = \frac{2}{3} C_d \sqrt{2g} L H^{\frac{3}{2}}$$

$$Q_{act} = \frac{2}{3} X0.62X\sqrt{2g}X1.0X0.15^{\frac{3}{2}}$$

$$Q_{act} = 0.10635 \text{ m}^3/\text{s}$$

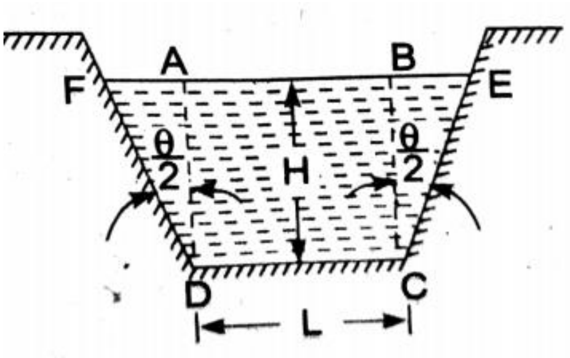
The same discharge passes through the triangular right angled weir, but the discharge Q is given by the equation for triangular weir.

Discharge $Q = \frac{8}{15} C_d \, tan\frac{\theta}{2} \sqrt{2g} \left[H_1^{\frac{5}{2}}\right]$

$$0.10635 = \frac{8}{15} x0.59xtan\frac{90}{2} \sqrt{2x9.81} \left[H_1^{\frac{5}{2}}\right]$$

H_1= 0.3572m

Discharge over a trapezoidal notch or weir



As shown in figure , a trapezoidal notch is a combination of rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular weir or notch.

Let L=length of the crest of the notch or weir

H=head of water over the notch or weir

C_{d1}= Coefficient of discharge for rectangular portion ABCD of figure

C_{d2}= Coefficient of discharge for triangular portion [FAD and BCE]

The discharge through rectangular portion ABCD is given by

$$Q_1 = \frac{2}{3} X C_{d1} XLX\sqrt{2g}XH^{\frac{3}{2}}$$

The discharge through two triangular notches FAD and BCE is equal to the discharge through the single triangular notch of angle θ and it is given by equation

Discharge $Q_2 = \frac{8}{15} X C_{d2} X \tan\frac{\theta}{2} X\sqrt{2g}X\left[H^{\frac{5}{2}}\right]$

Discharge through trapezoidal notch or weir FDCEF = Q_1+Q_2

$$Q = Q_1+Q_2 = \frac{2}{3} X\ C_{d1}\ XLX\sqrt{2g}XH^{\frac{3}{2}} + \frac{8}{15} X C_{d2} X \tan\frac{\theta}{2} X\sqrt{2g}X\left[H^{\frac{5}{2}}\right]$$

NUMERICAL

1. Find the discharge through a trapezoidal notch which is 1m wide at the top and 0.40m at the bottom and is 0.3cm in height. The head of water on the notch is 20cm. Taking C_d for the rectangular portion and triangular portion as 0.62 and 0.60 respectively.

Solution:

Given: Top width = AE=1m

Base width, CD= L= 0.4m

Head of water, H= 0.20m

For rectangular portion C_{d1} = 0.62

For triangular portion C_{d2} = 0.60

For Triangular ABC, we have $tan\frac{\theta}{2}$= AB/BC= $\frac{(AE - CD)/2}{H} = \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = 1$

Discharge through trapezoidal notch is given by equation

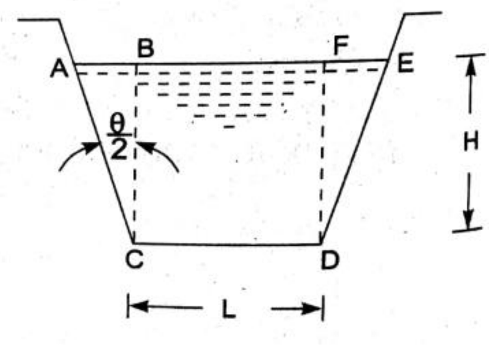
$$Q = Q_1+Q_2= \frac{2}{3}X\, C_{d1}\, XLX\sqrt{2g}XH^{\frac{3}{2}} + \frac{8}{15}XC_{d2}X\, tan\frac{\theta}{2}X\sqrt{2g}X\left[H^{\frac{5}{2}}\right]$$

$$Q = Q_1+Q_2= \frac{2}{3}X0.62X0.4X\sqrt{29.81}X0.2^{\frac{3}{2}} + \frac{8}{15}X0.60X\ 1X\sqrt{2x9.81}X\left[0.2^{\frac{5}{2}}\right]$$

Q= 0.06549+0.02535=0.09084m^3/s= 90.84litres/s

Discharge over Cipolletti notch or weir

Cipolletti weir is a trapezoidal weir, which has side slope of 1 horizontal to 4 vertical as shown in figure.



Thus in ΔABC $tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{H/4}{H} = \frac{1}{4}$

$$\frac{\theta}{2} = tan\frac{1}{4} = 14^0.2'$$

By giving this slope to the sides, an increase in discharge through the triangular portions ABC and DEF of the weir is obtained. If the slope is not provided the weir would be rectangular one, and due to the end contraction, the discharge would decrease, thus in case of Cipolletti weir, the factor of end contraction is not required which is shown below.

The discharge through a rectangular weir with two end contraction is

$$Q = \frac{2}{3} C_d \sqrt{2g}(L - 0.2H)H^{3/2}$$

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{\frac{3}{2}} - \frac{2}{15} C_d \sqrt{2g} H^{\frac{5}{2}}$$

Thus, due to end contraction, the discharge decreases by $\frac{2}{15} C_d \sqrt{2g} H^{\frac{5}{2}}$. This decrease in discharge can be compensated by giving such a slope to the sides that the discharge through two triangular portions is equal to $\frac{2}{15} C_d \sqrt{2g} H^{\frac{5}{2}}$. Let the slope is given by $\frac{\theta}{2}$.

The discharge through a V-notch of angle is given by

Discharge $Q = \frac{8}{15} C_d \, tan\frac{\theta}{2} \sqrt{2g} \left[H^{\frac{5}{2}}\right]$

$\frac{8}{15} C_d \, tan\frac{\theta}{2} \sqrt{2g} \left[H^{\frac{5}{2}}\right] = \frac{2}{15} C_d \sqrt{2g} H^{\frac{5}{2}}$

$tan\frac{\theta}{2} = \frac{2}{15} x \frac{15}{8} = \frac{1}{4}$ or $\frac{\theta}{2} = \tan^{-1}\frac{1}{4} = 14^0.2'$

Thus the discharge through cipolleti weir is

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{3/2}$$

If velocity of approach, Va is to be taken into consideration,

$$Q = \frac{2}{3} C_d \sqrt{2g} L \left\{(H + h_a)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right\}$$

NUMERICAL

1. Find the discharge over a Cipolletti weir of length 2.0m when the head over the weir is 1m. Take $C_d = 0.62$

Solution:

Given, L=length of the weir= 2m

H=Head over the weir, H =1.0m

C_d= 0.62

The discharge is given as

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{3/2}$$

$$Q = \frac{2}{3} X 0.62 X \sqrt{2x9.81}\, X 1^{3/2} = 3.661 m^3/s$$

2. A Cipolletti weir of crest length 60cm discharges water. The head of water over the weir is 360mm. Find the discharge over the weir if the channel is 80cm wide and 50cm deep. Take $C_d = 0.60$

Solution:

Given, L=length of the weir= 60cm = 0.60m

H=Head of water, H =360mm= 0.36m

Channel width= 80cm=0.80m

Channel depth= 50cm=0.50m

C_d= 0.60

A= Cross sectional area of channel= 0.8x0.5=0.4m^2

To find the velocity of approach, first determine the discharge over the weir as

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{3/2}$$

The velocity of approach Va= Q/A

$$Q = \frac{2}{3} X 0.60 X 0.60 \sqrt{2x9.81}\, X 0.36^{3/2} = 0.2296 m^3/s$$

Va= Q/A= 0.2296/0.40= 0.574m/s

Head due to velocity of approach,

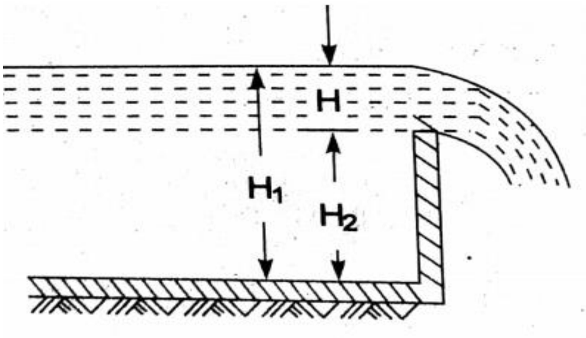
$$Ha = Va^2/2g = (0.574)^{2/}/(2x9.81) = 0.0168m$$

The discharge is given as

$$Q = \frac{2}{3} C_d \sqrt{2g} L \left\{ (H + h_a)^{\frac{3}{2}} - ha^{\frac{3}{2}} \right\}$$

$$Q = \frac{2}{3}x0.60x\sqrt{2x9.81x}0.60\left\{(0.36 + 0.0168)^{\frac{3}{2}} - 0.0168^{\frac{3}{2}}\right\} = 0.2435m^3/s$$

Discharge over a broad crested weir



A weir having a wide crest is known as broad crested weir.

H= Height of water, above the crest

L= length of the crest.

If 2L> H the weir is called broad crested weir

If 2L< H the weir is called as narrow crested weir

Shows a broad crested weir.

Let h= head of water at the middle of the weir which is constant

V= velocity of flow over the weir

Applying Bernoulli's equation to the still water surface on the upstream side and running water at the end of weir,

$$0+0+H= 0+v^2/2g+h$$

$$v^2/2g= H-h$$

$$v= \sqrt{2g\{H-h)}$$

The discharge over weir Q= $C_d X\ Area\ of\ flow X\ velocity$

$$Q= C_d XLXhX\sqrt{2g\{H-h)}$$

$$Q = C_d X L X \sqrt{2g}\{Hh^2 - h^3)$$

The discharge will be maximum if $(Hh^2 - h^3)$is maximum

$\frac{d(Hh^2-h^3)}{dh}$=0 or 2hxH-3h= 0 or 2H=3h

h= $\frac{2}{3}$H

Q_{max} will be obtained by substituting this value of h in equation as

$$Q_{max} = C_d X L X \sqrt{2g\{H(\frac{2}{3}H)^2 - \left(\frac{2}{3}H\right)^3\}}$$

$$Q_{max} = C_d X L X \sqrt{2g\{H(\frac{4}{9}H)^2 - \left(\frac{8}{27}H\right)^3\}}$$

$$Q_{max} = C_d X L X \sqrt{2g}\sqrt{\frac{4}{9}H^3 - \frac{8}{27}H^3}$$

$$Q_{max} = C_d X L X \sqrt{2g}\sqrt{\frac{4}{27}H^3}$$

$$Q_{max} = C_d X L X \sqrt{2g} X 0.3849 X H^{\frac{3}{2}}$$

$$Q_{max} = C_d X L X \sqrt{2x9.81} X 0.3849 X H^{\frac{3}{2}}$$

$$Q_{max} = 1.705 X C_d X L X H^{\frac{3}{2}}$$

NUMERICAL

1. A broad crested weir of 50m length, has 50cm height of water above its crest Find the maximum discharge take Cd= 0.60

Solution:

Given

L=length of the weir= 50m

H=Head of water, H =50cm= 0.5m

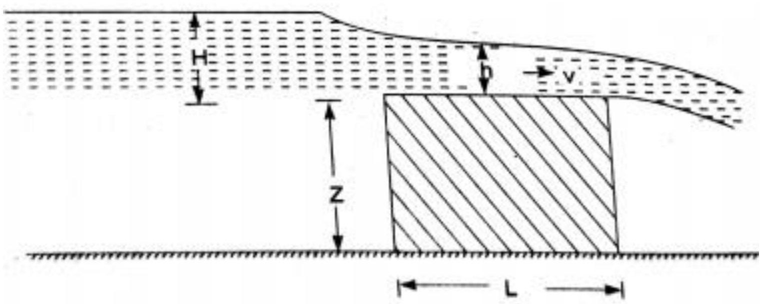
Cd= 0.60

The maximum discharge is given by

$Q_{max} = 1.705 X C_d X L X H^{\frac{3}{2}} = 1.705 X 0.60 X 50 X 0.5^{\frac{3}{2}} = 18.084 m^3/s$

Discharge over a Submerged or Drowned weir

When the water level on the downstream side of the weir is above the crest of the weir, then the weir is called as submerged or drowned weir. The total discharge over the weir is obtained by dividing the weir into two parts. The portion between the upstream and downstream water surface may be treated as free weir and portion between downstream water surface and crest of weir as drowned weir.



Let H= height of water on upstream of the weir

h= height of water on upstream of the weir

Then Q_1= Discharge over upper portion

$Q_1 = \frac{2}{3} X C_{d1} X L X \sqrt{2g}\{H - h\}^{\frac{3}{2}}$

Q_2= Discharge over drowned portion

$Q_2 = C_d X \; Area \; of \; flow X \; velocity$

$Q_2 = C_d X L X h X \sqrt{2g}\{H - h)$

Total discharge Q= $Q_1 + Q_2$

$$Q = \frac{2}{3} X C_{d1} X L X \sqrt{2g\{H - h\}^{\frac{3}{2}}} + C_{d2} X L X h X \sqrt{2g}\{H - h)$$

NUMERICAL

1. The heights of water on the upstream and downstream side of submerged weir of 3m length are 20cm and 10cm respectively. If the Cd for free and drowned portions ar3e 0.6 and 0.8 respectively, find the discharge over the weir.

Solution:

Let height of water on upstream side= H=20cm=0.20m

height of water on upstream side= h=10cm=0.10m

Length of weir, L=3m

C_{d1} = 0.60, C_{d2} = 0.80

Total discharge Q is sum of discharge through free portion and discharge through the drowned portion. This is given by equation as

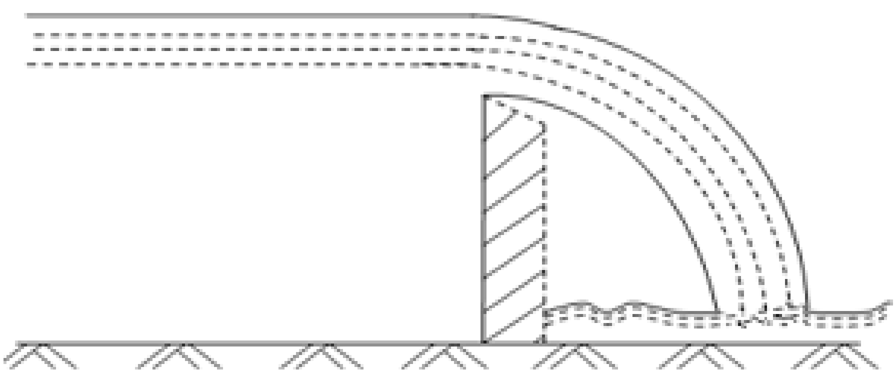
$$Q = \frac{2}{3} X C_{d1} X L X \sqrt{2g\{H - h\}^{\frac{3}{2}}} + C_{d2} X L X h X \sqrt{2}g\{H - h)$$

$$Q = 2/3X0.6X30X\sqrt{2x9.81\{0.20 - 0.10\}^{\frac{3}{2}}} + 0.8X3X0.10X\sqrt{2}x9.81\{.2 - 0.1)$$

Q= 0.168+0.336= 0.504m^3/s.

Ventilation of Rectangular Weirs

It has been observed that whenever water is flowing over a rectangular weir, having no end contractions, the nappe (i.e., the sheet of water flowing over the weir) touches the side walls of the channel. After flowing over the weir, the nappe falls away from the weir, thus creating a space beneath the water as shown in fig. In such a case, some air is trapped beneath the weir.



This air is carried away by the flowing water, which results in creating a negative pressure beneath the nappe. The negative pressure drags the lower side of the nappe towards the surface of the weir wall. This results in more discharge than the normal discharge. In order the keep the atmospheric pressure in the space below the nappe holes are made through the channel walls which are connected through the pipes to the atmosphere as shown in figure. Such holes are called 'Ventilation' of a weir. Though there are many types of the nappes, yet the following are important from the subject point of view :

- Free nappe
- Depressed nappe
- Clinging nappe

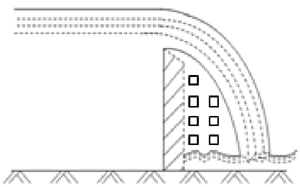


Fig-2(a) : Free Nappe

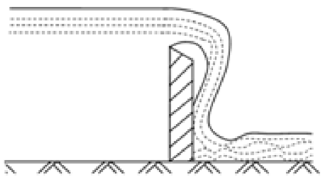


Fig-2(b) : Depressed Nappe

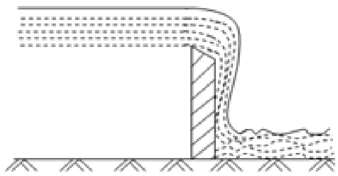


Fig-2(c) : Clinging Nappe

Free Nappe

If the atmospheric pressure exists beneath the nappe, it is known as a free nappe as shown in fig-2(a). A free nappe is obtained by ventilating a weir.

Depressed Nappe

Sometimes a weir is not fully ventilated, but is partially ventilated as shown in fig-2(b). If the pressure below the nappe is negative, it is called a depressed nappe.

The discharge of the nappe, in this case, depends upon the amount of ventilation and the negative pressure. Generally, the discharge of a depressed nappe is 6% to 7% more than that of a free nappe.

Clinging Nappe

Sometimes, no air is left below the water, and the nappe adheres or clings to the downstream side of the weir as shown in fig-2(c). Such a nappe is called clinging nappe or an adhering nappe. The discharge of a clinging nappe is 25% to 30% more than that of a free nappe.

FLOW THROUGH PIPES

If a fluid flowing in a closed boundary due to pressure then the flow is known as pipe flow

Losses of Pipe Flow

There are two types of losses in flow through the pipes

- Major Loss
- Minor Loss

Major Loss

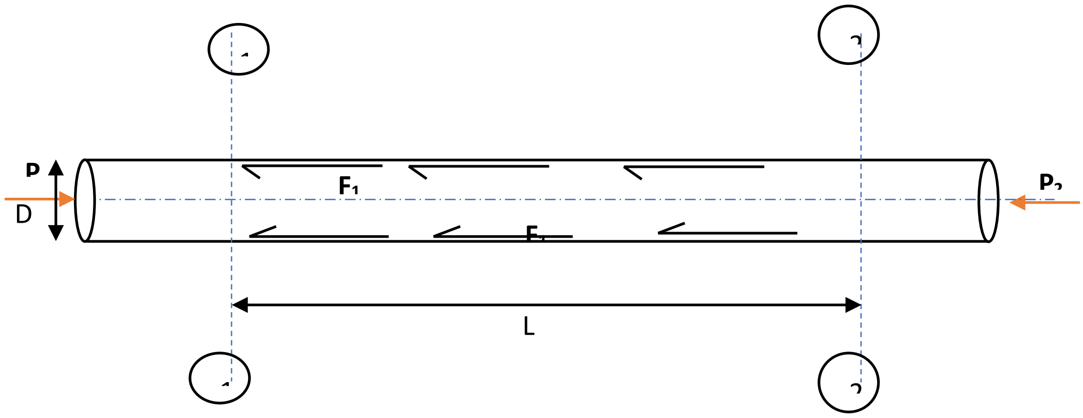
The loss occurs in the flow through pipes is due to friction, then the losses is known as Major Loss

Minor Loss

The loss occurs in the flow through pipe due to following

Bend, Obstruction, fitting, contraction and exit

DARCY'S WEISBACH EQUATION



Consider a pipe having a diameter D. Consider the two section of the pipe as shown in above figure.

Let V_1 – be the velocity of the pipe in the section 1 -1

P_1 – be the pressure of the pipe in the section 1-1

L – be the length of the pipe between two section

V_2, P_2 are the corresponding value of the pipe in section 2-2

F_1 – frictional force per unit wetted area.

Applying Bernoulli's equation between section 1-1 and 2-2

$$\frac{P_1}{\rho g} + \frac{v_1{}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2{}^2}{2g} + Z_2 + h_f$$

where h_f be the loss of head due to friction

$V_1 = V_2 = V$ and $Z_1 = Z_2 = Z$ Because Datum lines remains same in both the sections and Velocity of flow is constant for entire length of the pipe

$$\boxed{\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f} \dashrightarrow (1)$$

The various forces are acting on the pipe flow are

a. Pressure force at the section 1-1
b. Pressure force at the section 2-2
c. Frictional force F_1

Frictional force $= F_1 = f^1 v^2 (\pi dL)$ [Because $V_1 = V_2 = V$]

V = Velocity

πdL = wetted area

πd = wetted Perimeter = p

$F_1 = f^1 v^2\, pL$

$$\boxed{F_1 = f^1 v^2\, pL} \dashrightarrow (2)$$

Resolving all the forces in x direction

$P_1A - P_2A - F_1 = 0$

$(P_1 - P_2)A = F_1$

$(P_1 - P_2)A = f^1 V^2 pL$

$$\boxed{P_1 - P_2 = \frac{f^1 v2\, pL}{A}} \dashrightarrow (3)$$

From equation (1)

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h_f$$

$$\boxed{\mathbf{P_1 - P_2 = \rho g h_f}} \dashrightarrow (4)$$

Comparing equation (3) and (4) , we get

$$\rho g h_f = \frac{f^1 v2\ pL}{A}$$

$$h_f = \frac{f^1 V^2 pL}{A\rho g}$$

$$h_f = \left(\frac{f^1}{\rho}\right) \frac{LV^2 \pi d}{g\frac{\pi D^2}{4}}$$

$$h_f = \left(\frac{f^1}{\rho}\right) \frac{4LV^2 \pi d}{g\pi d^2}$$

$$\frac{f^1}{\rho} = \frac{f}{2}$$

where f = coefficient of friction

$$h_f = \frac{f}{2} \frac{4LV^2}{gd}$$

$$\boxed{\mathbf{h_f = \frac{4fLV^2}{2gd}}}$$

where h_f – Loss of head due to friction

f is the coefficient of friction depends on Reynolds number

R_e is Reynolds number

$R_e \longrightarrow$ 0-2000 then $f = \frac{16}{R_e}$

$R_e \longrightarrow$ 2000-10^6 then $f = \frac{0.079}{R_e^{1/4}}$

d is the diameter of the pipe

V is the velocity of fluid flowing in pipe

L length of the pipe

NUMERICAL

1.Find the head loss due to friction in a pipe of diameter 300mm and length 50m through which water is flowing at a velocity of 3m/sec. Using Darcy's formula. Take Kinematic viscosity = 0.01stoke

Solution:

D = Diameter of the pipe = 300mm =0.3m

L= Length of the pipe = 50m

V= Velocity of the water = 3m/sec

ν = Kinematic viscosity = 0.01 stoke =$0.01*10^{-4}$ m^2/sec

According to Darcy's Weisbach equation

$$\mathbf{h_f = \frac{4fLV^2}{2gd}}$$

f is the coefficient of friction

$$R_e = \frac{\mathbf{Vd}}{\boldsymbol{\gamma}} = \frac{\mathbf{3*0.3}}{\mathbf{0.01*10^{-4}}} = 9*10^5$$

For

$$R_e \longrightarrow 2000\text{-}10^6 \text{ then } f = \frac{0.079}{R_e^{1/4}}$$

$$f = \frac{0.079}{9*10^{5^{1/4}}} = 2.56*10^{-3}$$

therefore

$$h_f = \frac{4*2.56*10^{-3}*50*3^2}{2*9.81*0.3} = 0.78m$$

2. Water is flowing through a pipe of diameter of 300mm with a velocity of 5m/sec. If the coefficient of friction is given by f= $0.015+\frac{0.08}{{R_e}^{0.3}}$ where R_e is the Reynolds number. find the head loss due to friction for a length of 10m. Take Kinematic viscosity = 0.01stoke.

Solution:

D- Diameter of the pipe – 0.3m

V- Velocity of water – 5m/sec

f= $0.015+\frac{0.08}{{R_e}^{0.3}}$

L- Length of the pipe = 10m

ν = Kinematic viscosity = 0.01 stoke =$0.01*10^{-4}$ m^2/sec

According to Darcy's Weisbach equation

$$\mathbf{h_f = \frac{4fLV^2}{2gd}}$$

f is the coefficient of friction

$R_e = \frac{\mathbf{Vd}}{\boldsymbol{\gamma}} = \frac{\mathbf{5*0.3}}{\mathbf{0.01*10^{-4}}} = 15*10^5$

f= $0.015+\frac{0.08}{(15*10^5)^{0.3}}$ = 0.016

$h_f = \frac{4*0.016*10*5^2}{2*9.81*0.3}$ = 2.718m

3.A crude oil of Kinematic viscosity is 0.4 stoke is flowing through the pipe of diameter 300mm at the rate of 300 lt/sec. Find the head loss due to friction for a length of 50m of the pipe.

Solution:

ν = Kinematic viscosity of crude oil = 0.4 stoke =$4*10^{-5}$ m^2/sec

D = Diameter of the pipe = 300mm =0.3m

Q = Discharge of oil = 300lt/sec = 0.3 m^3/sec

L = Length of the pipe = 50m

A= $\frac{\pi D^2}{4}$ = 0.0707 m^2

w.k.t

$Q = A*V$

$V = \frac{Q}{A} = \frac{0.3}{0.0707} = 4.243$ m/sec

$R_e = \frac{\mathbf{Vd}}{\boldsymbol{\gamma}} = \frac{\mathbf{4.2.3*0.3}}{\mathbf{4*10^{-5}}} = 31800$

For

$R_e \longrightarrow 2000\text{-}10^6$ then $f = \frac{0.079}{{R_e}^{1/4}}$

$f = \frac{0.079}{31830^{1/4}} = 5.19*10^{-3}$

According to Darcy's Weisbach equation

$$\mathbf{h_f = \frac{4fLV^2}{2gd}}$$

therefore

$h_f = \frac{4*5.91*10^{-3}*50*4.243^2}{2*9.81*0.3} = 3.62\text{m}$

4. An oil of specific gravity 0.7 is flowing through a pipe of diameter 300mm at the rate of 500 lt/sec. Find the head loss due to friction and power required to maintain the flow for a length of 1000m. Take Kinematic viscosity is 0.29 stoke.

Solution:

S_{oil} = Specific gravity of oil = 0.7

D = Diameter of the pipe = 300mm =0.3m

Q= Discharge of oil = 500 lt/sec = $500*10^{-3}$ m^3/sec

L = Length of flow of the pipe = 1000m

ν = Kinematic viscosity = 0.29 stoke =$0.29*10^{-4}$ m^2/sec

h_f = ?

P =?

Q= A*V

$V = \frac{Q}{A} = \frac{0.5}{0.0707} = 7.07$ m/sec

$R_e = \frac{\mathbf{Vd}}{\boldsymbol{\gamma}} = \frac{\mathbf{7.07*0.3}}{\mathbf{2.9*10^{-5}}} = 73.13*10^3$

$f = \frac{0.079}{9*10^{5\,1/4}} = 2.56*10^{-3}$

According to Darcy's Weisbach equation

$$\mathbf{h_f = \frac{4fLV^2}{2gd}}$$

therefore

$h_f = \frac{4*4.8*10^{-3}*1000*7.07^2}{2*9.81*0.3} = 163.05m$

Now,

$S = \frac{\omega_{oil}}{\omega_{water}}$

$\omega_{oil} = S * \omega_{water} = 0.7*9810 = 6867\ N/m^3$

W.K.T

$P = \omega_{oil}*Q*h_f = 6867*0.5*163.2 = 560\ kW$

5. Water is flowing through a pipe of diameter 200mm with a velocity of 3m/sec. Find the head loss due to friction for a length of 5m if the coefficient of friction is given by $f = 0.02 + \frac{0.09}{{R_e}^{0.3}}$ where R_e is the Reynolds number. find the head loss due to friction. Take Kinematic viscosity = 0.01stoke.

Solution:

D- Diameter of the pipe – 0.2m

V- Velocity of water – 3m/sec

$f = 0.02 + \frac{0.09}{{R_e}^{0.3}}$

L- Length of the pipe = 5m

ν = Kinematic viscosity = 0.01 stoke $= 0.01*10^{-4}\ m^2/sec$

According to Darcy's Weisbach equation

$$\mathbf{h_f = \frac{4fLV^2}{2gd}}$$

f is the coefficient of friction

$R_e = \frac{Vd}{\gamma} = \frac{3*0.2}{0.01*10^{-4}} = 6*10^5$

$f = 0.02 + \frac{0.09}{(6*10^5)^{0.3}} = 0.022$

$h_f = \frac{4*0.022*5*3^2}{2*9.81*0.2} = 1m \text{ or } 0.97m$

6.Water is to be supplied to the inhabitants of a college campus through a main supply. The following data gives distance of the reservoir from the campus = 3000m, number of inhabitants =4000, consumption of water per day of each inhabitant is 180lt/sec, loss of head due to friction for the pipe, f=0.007. If half of the daily supply is from the 8 hours. Determine the size of the main supply.

Solution:

Length of the pipe =3000m

Number of inhabitants =4000

Loss of head due to friction = h_f = 18m

Discharge per one inhabitant = 180 lt/sec

Half of the discharge requires 8hrs/day

Therefore

The total discharge to the college campus = $\frac{1}{2}\left[\frac{720}{8*60*60}\right]$ = 0.0125 m^3/sec

W.K.T

Loss of the head due to friction,

$$\mathbf{h_f = \frac{4fLV^2}{2gd}} \dashrightarrow \mathbf{(1)}$$

$$V = \frac{Q}{A} = \frac{0.0125}{\frac{\pi}{4}D^2} = \frac{0.0125}{0.785D^2} = \frac{0.015}{D^2} \dashrightarrow \mathbf{(2)}$$

Put (2) in (1)

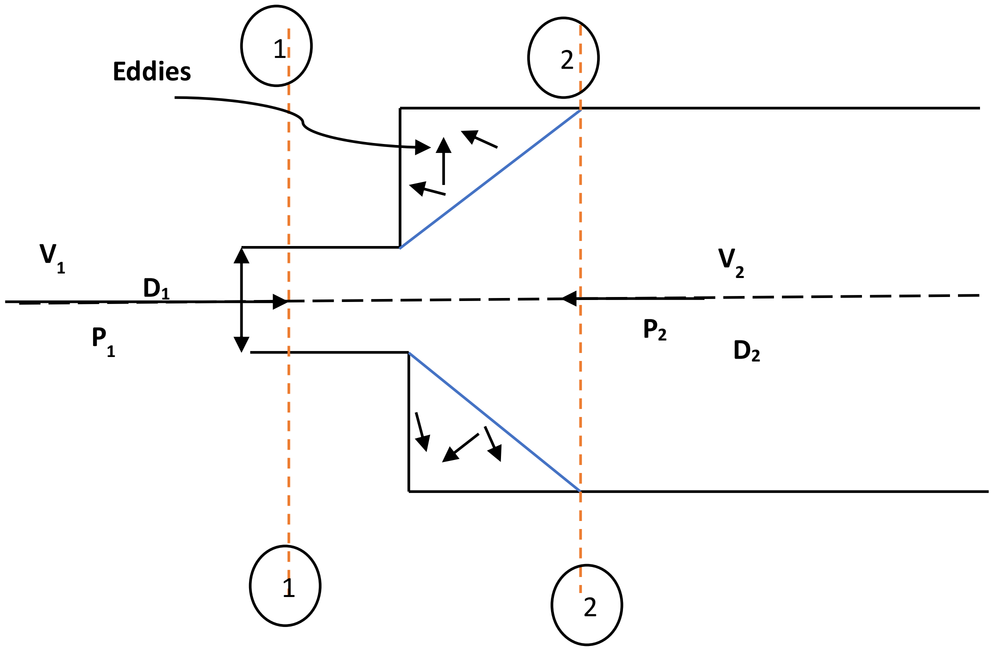
$$18 = \frac{4 * 0.007 * 3000 * 0.015^2}{2 * 9.81 * D * D^4}$$

$$18 = \frac{0.018}{19.62 D^5}$$

D =140mm =0.14m

MINOR LOSSES OF FLOW THROUGH PIPE

(a) Due to sudden Expansion



Consider a pipe having a diameter D_1, water is flowing with a velocity V_1 due to pressure P_1 at a section (1)-(1) as shown in above figure. The diameter of the pipe increases D_1 to D_2 due to sudden expansion of the pipe at the section (2)-(2) as shown in above figure. Let P_2, V_2 and D_2 are the corresponding values of the pipe at the section (2)-(2).

Due to sudden change in diameter loss of head occurs at the section (2)-(2) and pressure changes due to sudden changes in area P^1

Applying the Bernoulli's equations between the section (1)-(1) and (2)-(2)

$$\frac{P_1}{\rho g} + \frac{V_1{}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2{}^2}{2g} + Z_2 + h_e$$

where, h_e loss of head due to sudden expansion

$Z_1 = Z_2$ = Horizontal Pipe

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = h_e \text{ ------}\blacktriangleright \text{ (1)}$$

The following are the forces acting in sudden expansion pipe,

1.Pressure force, P_1A_1 at section (1)-(1)

2. Pressure force, P_2A_2 at section (2)-(2)

3.Loss of pressure force due to change in area, $P^1(A_2-A_1)$

Net forces acting in pipe along x direction

$$F_x = P_1A_1+ P^1(A_2-A_1)-P_2A_2 = P_1A_1-P_2A_2+ P^1A_2- P^1A_1$$

But $P^1= P_1$ experimental found that

$F_x = P_1A_1-P_2A_2+ P_1A_2- P_1A_1 = P_1A_2- P_2A_2$

$F_x = (P_1-P_2)A_2$ -----► (2)

Initial momentum of a water per sec at (1)-(1) = $\rho A_1 V_1^2$

Final momentum of a water per sec at (2)-(2) = $\rho A_2 V_2^2$

Change in momentum = Final momentum - Initial momentum = $\rho A_2 V_2^2$ - $\rho A_1 V_1^2$---►(3)

W.K.T

From the continuity equation

$Q_1 = Q_2$

$A_1V_1 = A_2V_2$

$A_1 = A_2\frac{V_2}{V_1}$ -------► (4)

Put (4) in (3)

Change in momentum = $\rho A_2 V_2^2$ - $\rho A_2 \frac{V_2}{V_1} V_1^2$

$$= [\rho A_2 V_2{}^2 - \rho A_2 V_1 V_2]$$

$$= \rho A_2 [V_2{}^2 - V_1 V_2] \text{-----}\blacktriangleright (5)$$

Comparing equation (5) and (2)

$$(P_1 - P_2) A_2 = \rho A_2 [V_2{}^2 - V_1 V_2]$$

$$\frac{P_1 - P_2}{\rho} = [V_2{}^2 - V_1 V_2]$$

Divide g on both side

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2{}^2}{2g} - \frac{V_1{}^2}{2g} \text{-------}\blacktriangleright (6)$$

Put (6) in (1)

$$\frac{V_2{}^2}{2g} - \frac{V_1{}^2}{2g} + \left(\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g}\right) = h_e$$

$$\frac{2V_2{}^2 - 2V_1V_2 + V_1{}^2 - V_2{}^2}{2g} = h_e$$

$$\frac{V_2{}^2 - 2V_1V_2 + V_1{}^2}{2g} = h_e$$

$$\boxed{\mathbf{\frac{(V_1 - V_2)^2}{2g} = h_e}}$$

TYPES OF MINOR LOSSES

1.Sudden Expansions, $h_e = \frac{(V_1 - V_2)^2}{2g}$

2.Sudden Contraction, $h_c = \frac{KV^2}{2g}$ where K $= \left[\frac{1}{C_c} - 1\right]^2$ & C_c – co efficient of contraction

3.Entry Loss, $h_{entry} = \frac{0.5V^2}{2g}$

4.Exit (Outlet) Loss, $h_o = \frac{V^2}{2g}$

5.Bend Loss, $h_b = \frac{KV^2}{2g}$ where K - co efficient of bend

6. Fitting Loss, $h_f = \frac{KV^2}{2g}$ where K - co efficient of fitting

7.Losses due to Obstruction

$h_{ob} = \left[\frac{A}{C_b(A-a)} - 1\right]^2$ where A- area of cross section of the pipe, C_b – Coefficient of Obstruction, a – area of Obstruction

NUMERICAL

1.Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water is 250 lt/sec.

Solution:

D_1 – Diameter of the smaller section = 0.2m

A_1 = Area of the smaller section = $\frac{\pi}{4} D^2$ = 0.03m^2

D_2 – Diameter of the larger section = 0.4m

A_2 = Area of the larger section = $\frac{\pi}{4} D^2$ = 0.125m^2

Q =250 lt/sec = 0.25 m^3/sec

$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03}$ = 8.06 m/sec

$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.126}$ = 1.98 m/sec

Loss of head due to sudden Expansions, $h_e = \frac{(V_1 - V_2)^2}{2g}$

$h_e = \frac{(8.06-1.98)^2}{2*9.81}$ =1.88m

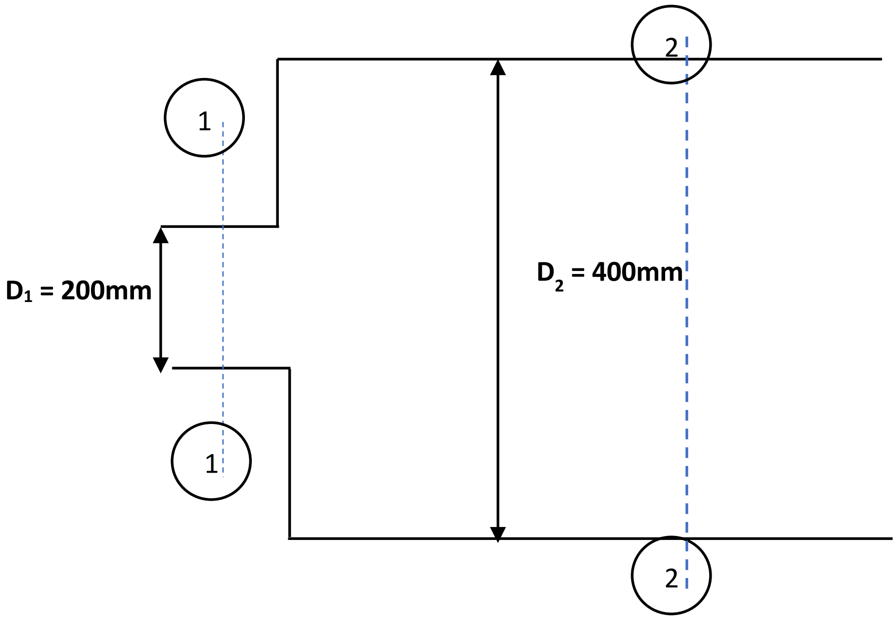
2.The rate of flow of water through horizontal pipe is 0.25m^3/sec. The diameter of the pipe is 200mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine

a. Loss of head due to sudden enlargement

b. Pressure intensity in the larger pipe

c. Power loss due to sudden enlargement

Solution:



Q =0.25 m^3/sec

$D_1 = 0.2m$

$A_1 = 0.03m^2$

$D_2 = 0.4m$

$A_1 = 0.126m^2$

Pressure at the section 1-1 = P_1 = 11.772 N/cm^2 = 11.772 *10^4 N/m^2

$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03} = 8.06$ m/sec

$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.126} = 1.98$ m/sec

a) Loss of head due to sudden expansions

Loss of head due to sudden Expansions, $h_e = \frac{(V_1 - V_2)^2}{2g}$

$$\mathbf{h_e = \frac{(8.06-1.98)^2}{2*9.81} = 1.88m}$$

b) Pressure at the section 2-2

Applying Bernoulli's equations between the section 1-1 and section 2-2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + Z_2 + h_e$$

Since $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + h_e$$

$$P_2 = \rho g\left[\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g} - h_e\right]$$

$$P_2 = 1000 * 9.81\left[\frac{11.772 * 10^4}{1000 * 9.81} + \frac{8.06^2}{2 * 9.81} - \frac{1.98^2}{2 * 9.81} - 1.88\right]$$

P_2 = 12.98 N/cm²

a) Power Loss

P= ω_{water}*Q*h_f = 9810*0.25*1.88 = **4.61 kW**

HYDRAULIC GRADIENT LINE

It is defined as the line gives the sum of pressure head and datum head measured from the reference line

$$H_{GL} = \frac{P}{\rho g} + Z$$

Total Energy Line or Energy Gradient Line

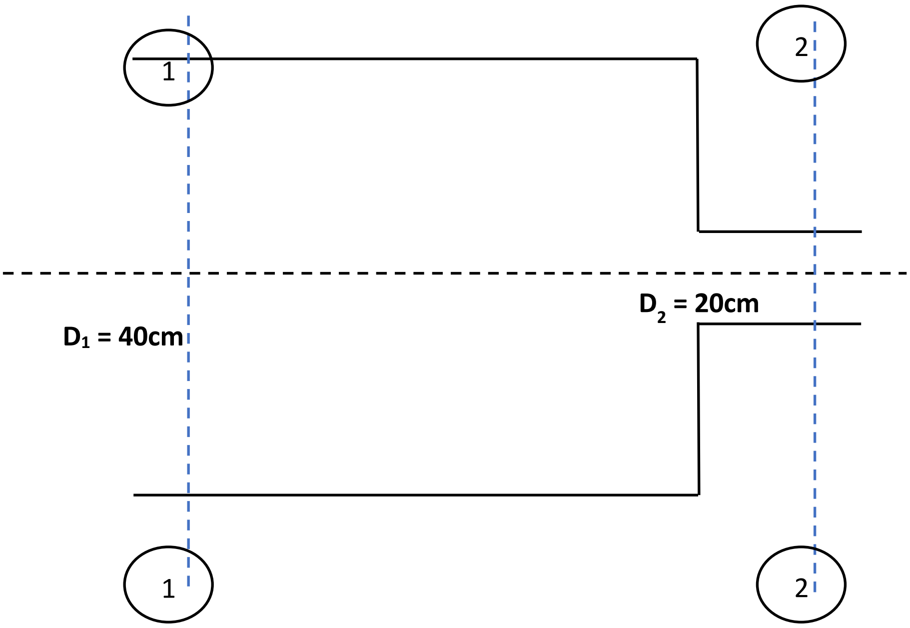
It is defined as the line sum of pressure head, kinetic head and datum head, measured from the reference line

$$H_{Tol} = \frac{P}{\rho g} + \frac{v^2}{2g} + Z$$

NUMERICAL

1.In the below figure, the sudden change in the pipe diameter from 40-20cm , causes the pressure water drop from 200kPa to 150 kPa. Compute the discharge , if the coefficient of contraction is 0.6

Solution:



Q =0.25m^3/sec

D_1 = 0.2m, A_1 = 0.03 m^2

D_2 = 0.4m, A_2 = 0.126 m^2

P_1= 11.772*10^4 N/m^2

C_c =0.6

Applying Bernoulli's equations between the section 1-1 and section 2-2

$$\frac{P_1}{\rho g} + \frac{V_1{}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2{}^2}{2g} + Z_2 + h_e \dashrightarrow \quad (1)$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{0.126}$$

$$V_2 = \frac{Q}{A_2} = \frac{Q}{0..031}$$

From equation (1)

$$\frac{200*10^3}{1000*9.81} + \frac{Q^2}{2*981*0.126^2} + Z_1 = \frac{150*10^3}{1000*9.81} + \frac{Q^2}{2*981*0.031^2} + Z_2 + h_c \dashrightarrow \quad (3)$$

Loss of head due to contraction

$h_c = \frac{KV^2}{2g}$ where $K = \left[\frac{1}{C_c} - 1\right]^2$ & C_c – co efficient of contraction

$$h_c = \frac{\left[\frac{1}{C_c} - 1\right]^2 V^2}{2g}$$

$$h_c = \frac{\left[\frac{1}{0.6}-1\right]^2 V^2}{2*9.81} = 0.023V^2$$

$$h_c = 0.023\left(\frac{Q^2}{9.61*10^{-4}}\right) = 23.57\ Q^2$$

Therefore equation (2) becomes

$20.39+3.21*Q^2 = 15.29+53.04*Q^2+23.57*Q^2$

Q= 0.267 m^3/sec

2. The water flows at the rate of 170 lt/sec through a bend as shown below figure. The diameter of the bend is 150mm and the pressure drop across is 300 mm of mercury. Determine the resistance coefficient of the bend.

Solution:

Diameter = 150mm = 0.15m

A =area of the pipe = 0.0176 m^2

Pressure drop = 0.3m of Hg

Therefore in turns of water = $(0.3*S_{Hg}*\rho_{Hg}*g)$m of water = (0.3*13.6*1000*9.81)m of water

= 40.024kN/m^2

$$h_L = \frac{P_1-P_2}{\rho g} = \frac{40024}{1000*9.81} = 0.408 \text{ m of water}$$

$$V = \frac{Q}{A} = \frac{0.17}{0.0176} = 9.65\text{m/sec}$$

Minor loss of bend

$$h_L = \frac{KV^2}{2g}$$

$$K = \frac{h_L*2g}{V^2} = \frac{2*9.81*0.408}{9.65^2} = \mathbf{0.85}$$

3. At a sudden enlargement of water weir from 240mm to 480mm diameter. The hydraulic gradient rises by 10mm. Estimate the rate of flow.

Solution:

$D_1 = 240mm = 0.24m$

$A_1 = 0.045\ m^2$

$D_2 = 480mm = 0.48m$

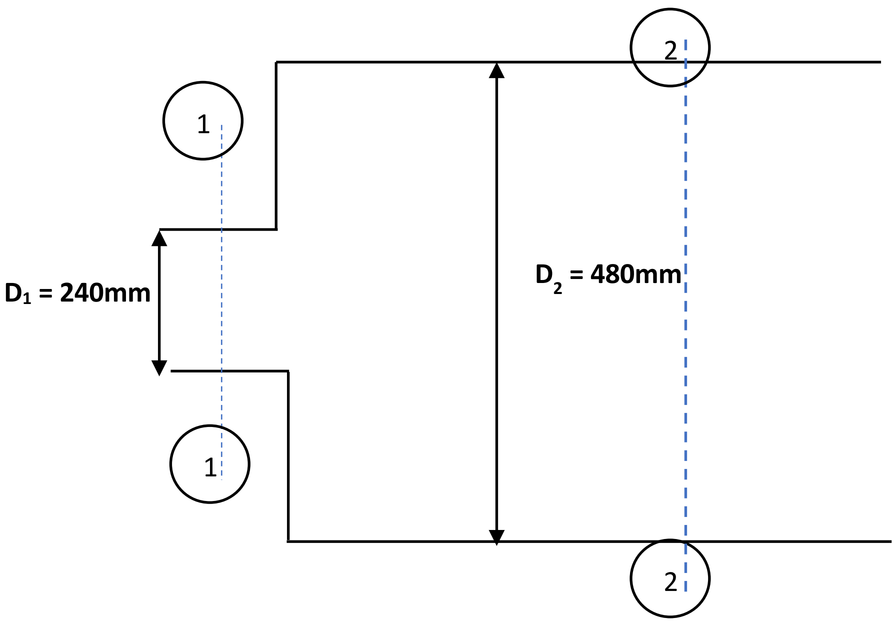
$A_2 = 0.181\ m^2$

Hydraulic gradient line = 10mm rise

Q =?

Applying Bernoulli's equations between section (1)-(1) and (2)-(2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e \dashrightarrow (1)$$



$Z_1 = Z_2$

$Q_1 = Q_2$

$A_1V_1 = A_2V_2$

$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2V_2}{\frac{\pi}{4}D_1^2} = \frac{0.48^2}{0.24^2}V_2 = 4V_2$$

Loss of head due to sudden Expansions, $h_e = \frac{(V_1 - V_2)^2}{2g}$

$$h_e = \frac{(4V_2 - V_2)^2}{2*9.81} = 0.459\ V_2^2$$

From equation (1)

$\frac{16V_2{}^2}{2g}$ -$\frac{V_2{}^2}{2g} - \frac{9V_2{}^2}{2g}$ =$\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right)$

$\frac{16V_2{}^2}{2g}$ -$\frac{V_2{}^2}{2g} - \frac{9V_2{}^2}{2g} = \frac{10}{1000}$ Because $\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right)$ = Hydraulic rise

V_2 = 0.181 m/sec

Q =A_2V_2 = 0.181*0.181 = 0.0327 m^3/sec = **32.7 lt/sec**

Compound Pipes

For a larger length of a pipe we cannot provide the same diameter, along the entire length. In order to overcome this pipe having different diameter, different length are provided by connecting in series or parallel.

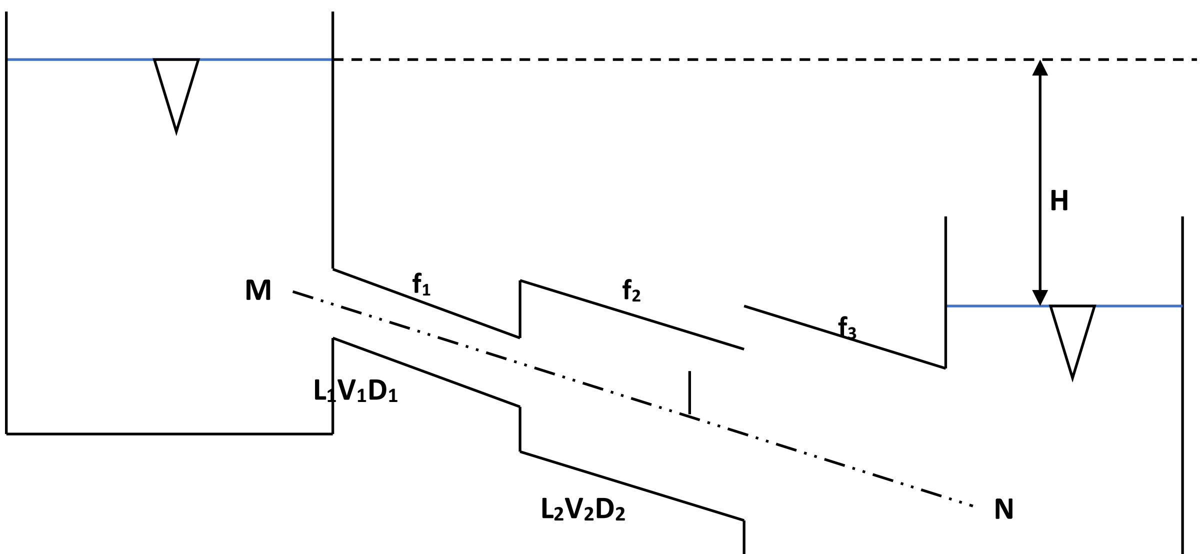
There are two types of connections

1.Pipe in series connection

2.Pipe in parallel connection

Pipe in Series Connections

It is defined as, if a pipe having different length, different diameter connected in end to end connection is known as pipe in series connections. When the pipe are in series the discharge in each pipe are remains same.



Consider a pipe MN is used to convey the fluid from one tank to another tank ia shown in above figure. The pipe MN consist of three different length. Let L_1 be the length of the first pipe, V_1 be the diameter of the first pipe, D_1 be the diameter of the first pipe and f_1 be the coefficient of friction of the first pipe. L_2, V_2, D_2, &f_3 and L_3, V_3, D_3 and f_3 are the corresponding values of the pipe 2 and 3 respectively.

Let H be the difference between of water level in the two tank

$Q_1 = Q_2 = Q_3$

$A_1V_1 = A_2V_2 = A_3V_3$

If minor loss is considered

$H = h_{ent} + h_{f1} + h_{e1,2} + h_{f2} + h_{e2,3} + h_{f3} + h_{exit}$

$$\mathbf{H} = \frac{0.5V^2{}_1}{2g} + \frac{4f_1L_1V_1{}^2}{2gD_1} + \frac{(V_1 - V_2)^2}{2g} + \frac{4f_2L_2V_2{}^2}{2gD_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3{}^2}{2gD_3} + \frac{V_3{}^2}{2g}$$

If minor loss is neglected

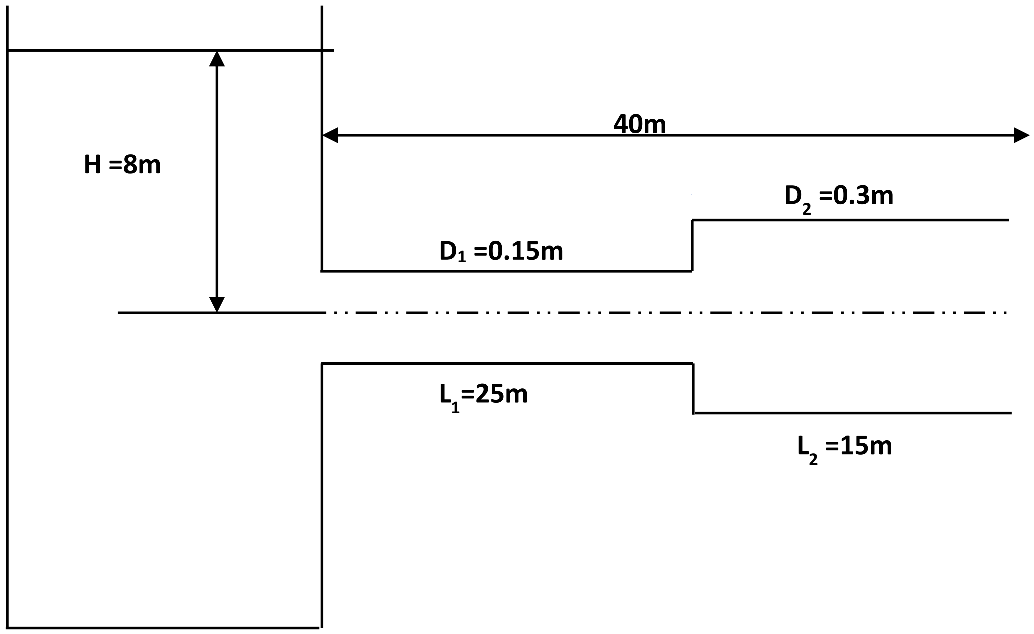
$H = h_{f1} + h_{f2} + h_{f3}$

$$H = \frac{4f_1L_1V_1^2}{2gD_1} + \frac{4f_2L_2V_2^2}{2gD_2} + \frac{4f_3L_3V_3^2}{2gD_3}$$

NUMERICAL

1. A horizontal pipe line 40m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end. For the first 25m of its length from the bottom of the tank the pipe of diameter is 150mm and its suddenly enlarged to 300mm. The height of the water level from the centre line of the pipe is 8m. Consider all the head loss is to be considered, determine the rate of flow. Take f =0.1 for both the pipe.

Solution:



Q =?

$f_1 = f_2 =?$

W.K.T

$H = h_{entrance} + h_{f1} + h_e + h_{f2} + h_{exit}$

$$H = \frac{0.5V^2{}_1}{2g} + \frac{4f_1L_1V_1{}^2}{2gD_1} + \frac{(V_1-V_2)^2}{2g} + \frac{4f_2L_2V_2{}^2}{2gD_2} + \frac{V_2{}^2}{2g} \dashrightarrow (1)$$

From

Continuity equation

$Q_1 = Q_2$

$A_1V_1 = A_2V_2$

$$V_1 = \frac{A_2}{A_1}V_2 = \frac{\left(\frac{\pi}{4}D_2{}^2\right)}{\frac{\pi}{4}D_1{}^2}V_2 = 4V_2$$

$V_1 = 4V_2$ $\dashrightarrow$ (2)

Put (2) in (1)

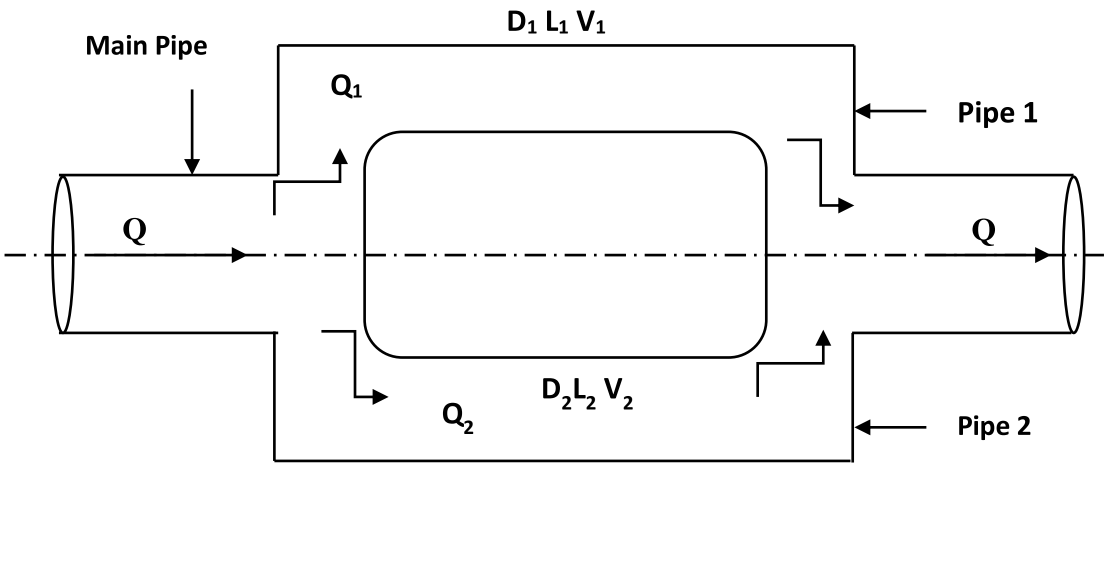
$$8 = \frac{0.5*16*V^2{}_2}{2*9.81} + \frac{4*0.1*25*16*V_2{}^2}{2*9.81*0.15} + \frac{(4V_2-V_2)^2}{2*9.81} + \frac{4*0.1*15*V_2{}^2}{2*9.81*0.3} + \frac{V_2{}^2}{2*9.81}$$

V_2 = 0.15 m/sec

$Q = A_2V_2 = \left(\frac{\pi}{4}0.3^2\right)*0.15 = 0.98\ m^3/sec$ = **10 lt/sec**

PIPE IN PARALLEL CONNECTIONS

The pipes are said to be parallel, when the main line divides into two or more parallel pipes which again join together downstream and continues as a main line. When the pipes are parallel the loss of head in each branch pipe remains same.



The conditions for the parallel connections are

1.Loss of head in each branch pipe is remains same ($H_1 = H_2$)

2.Total discharge is equal to sum of the discharge in branched pipe ($Q = Q_1 + Q_2$)

Consider a main pipe having a diameter D, which divides the main pipe into two branch pipe having a diameter D_1, Length L_1, velocity of water flowing is V_1 are the components of the pipe 1. L_2, D_2 and V_3 are the corresponding components of the Pipe 2 respectively.

Loss of head in branch pipe 1 = Loss of head in branch pipe 2

$$h_{f1} = h_{f2}$$

$$\frac{4f_1 L_1 V_1^2}{2gD_1} = \frac{4f_2 L_2 V_2^2}{2gD_2}$$

When $f_1 = f_2$

$$\boxed{\frac{L_1 V_1^2}{D_1} = \frac{L_2 V_2^2}{D_2}}$$

NUMERICAL

1.The main pipe divides into two parallel pipe which again forms one pipe. The data is as follows:

First parallel pipe: Length =1000m, diameter = 0.8m

Second parallel pipe: Length =1000m, diameter = 0.6m

Coefficient of friction of each parallel pipe =0.005

If the total discharge in the main pipe is $2m^3/sec$, find the discharge in each parallel pipe

Solution:

Length of first parallel pipe = L_1 = 1000m

Diameter of pipe 1 = D_1 = 0.8m

Length of second parallel pipe = L_2 = 1000m

Diameter of pipe 2 = D_2 = 0.6m

Total discharge of flow= $Q = 2m^3/sec$

Co-efficient of friction = $f_1 = f_2 = 0.005$

Rate of flow in each pipe

Q_1 – Rate of flow in pipe 1, Q_2 – Rate of flow in pipe 2, Q – Rate of flow in main pipe

$\mathbf{Q = Q_1 + Q_2}$ ----------→ (1)

$\mathbf{h_{f1} = h_{f2}}$ ----------→ (2)

$f_1 = f_2$ & $L_1 = L_2$

$$\frac{V_1^2}{0.8} = \frac{V_2^2}{0.6}$$

$V_1 = 1.15V_2$ ----------→ (3)

$Q_1 = A_1V_1 = \left(\frac{\pi}{4}0.8^2\right)*1.15V_2$ Because From (3)

$\mathbf{Q_1 = 0.578\ V_2}$ ----------→(4)

$Q_2 = A_2V_2 = \left(\frac{\pi}{4}0.6^2\right)*V_2$

$\mathbf{Q_1 = 0.283\ V_2}$ ----------→(5)

Put (4) & (5) in (3)

$Q = 0.578V_2 + 0.283V_2 = 2$

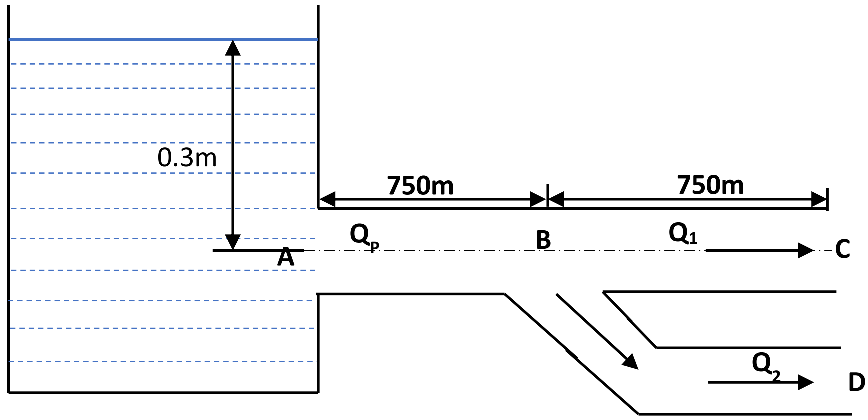
$\mathbf{V_2 = 2.32 m/sec}$

From equation (4) and (5) put the values of V_2

$\mathbf{Q_1 = 1.324\ m^3/sec}$ & Q_2= $\mathbf{0.658\ m^3/sec}$

2.A pipeline of 600mm diameter is 1.5km long. To increase the discharge another line of same diameter is introduced parallel to the first in the second half of the length. If the f =0.01 and head at inlet is 300mm calculate the increase in discharge. Neglect minor loss

Solution:



Diameter of pipe line =D =0.6m

Length of the pipe line =1.5km =1.5*1000 = 1500m

Co efficient of friction =f =0.01

Head at inlet = h =0.3m

Head at outlet = 0

Loss of head = h_f = 0.3

Length of another parallel pipe, $L_2 = L_1$ = 1500/2 = 750m

Diameter of another pipe line =$D_2 = D_1$ = 0.6m

The arrangement of the pipe as shown in above figure

Case I: Discharge (Q) for a single pipe length 1500m and diameter 0.6m

The loss of head due to friction for a single pipe line

$$h_f = \frac{4fLV^2}{2gd}$$

$$0.3 = \frac{4 * 0.01 * 1500 * V^2}{2 * 9.81 * 0.6}$$

V =0.243 m/sec

Therefore discharge, Q

Q =AV = $\left(\frac{\pi}{4} 0.6^2\right)$*0.243 = **0.0687 m^3/sec**

Case II: When additional pipe of length 750m and dimeter 0.6m is connected in parallel with the last half length of the pipe:

Let, Q_1 = Discharge in first parallel pipe, Q_2 = Discharge in second parallel pipe & Q_p = Discharge in the main pipe

Then,

$Q_P = Q_1 + Q_2$ ----------→ (1)

Then,

As the pipe in parallel have the same diameter and length,

$Q_1 = Q_2 = \frac{Q_P}{Q_P}$

Consider the flow through ABC or ABD;

Head lost due to friction ABC = Head lost in AB +Head lost in BC ----------→ (2)

Head lost in ABC =0.3m (given)

Head lost in AB due to friction = $h_{fAB} = \frac{4fLV_{AB}^2}{2gd} = \frac{4*0.01*750*V_{AB}^2}{2*9.81*2} = \mathbf{31.9Q_P^2}$ ------→ (3)

$V_{AB} = \frac{Q_P}{Area} = \frac{Q_P}{\left(\frac{\pi}{4}0.6^2\right)} = \mathbf{3.54Q_P}$

Head lost in BC due to friction = $h_{fBC} = \frac{4fLV_{BC}^2}{2gd} = \frac{4*0.01*750*(1.77Q_P)^2}{2*9.81*0.61} = \mathbf{7.98Q_P^2}$ ----→ (4)

$V_{BC} = \frac{Q_P/2}{Area} = \frac{Q_P/2}{\left(\frac{\pi}{4}0.6^2\right)} = \mathbf{1.77Q_P}$

Put (3) & (4) in (2)

$0.3 = 31.9Q_P^2 + 7.98Q_P^2$

$Q_P = 0.087 \ m^3/sec$

The increase in discharge = $Q_P - Q$ = 0.087-0.687 = **0.0183 m^3/sec**

EQUIVALENT PIPE

An equivalent pipe is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is known as the equivalent diameter of the series or compound pipe.

Let L_1, L_2, L_3, etc, = Lengths of pipe 1,2,3, etc.

D_1, D_2, D_3, etc, = Diameter of pipe 1,2,3, etc.

H = Total head loss

D = Diameter of the equivalent pipe

L = Length of the equivalent pipe

$$\mathbf{\frac{L}{D^5} = \frac{L_1}{{D_1}^5} + \frac{L_2}{{D_2}^5} + \frac{L_3}{{D_3}^5}}$$

NUMERICAL

1.A piping system consists of three pipes arranged in series; the length of pipes are 1200m,750m and 600m and diameter of the pipes are 750mm,600mm and 450mm respectively.

(i) Transform the system to an equivalent 450mm diameter pipe

(ii) Determine an equivalent diameter for the pipe, 2550m long

Solution:

Pipe 1: L_1 =1200m; D_1 =750 mm = 0.75m

Pipe 2: L_2 =750m; D_1 =600 mm = 0.6m

Pipe 3: L_3 =600m; D_1 =450 mm = 0.45m

i) Equivalent length, L:

Diameter of the equivalent pipe, D= 450mm =0.45m

$$\frac{L}{D^5} = \frac{L_1}{{D_1}^5} + \frac{L_2}{{D_2}^5} + \frac{L_3}{{D_3}^5}$$

$$\frac{L}{0.45^5} = \frac{1200}{0.75^5} + \frac{750}{0.6^5} + \frac{600}{0.45^5}$$

L =871.3m

ii) Equivalent Diameter:

Length of the equivalent pipe, L= 2550m

$$\frac{L}{D^5} = \frac{L_1}{{D_1}^5} + \frac{L_2}{{D_2}^5} + \frac{L_3}{{D_3}^5}$$

$$\frac{2550}{D^5} = \frac{1200}{0.75^5} + \frac{750}{0.6^5} + \frac{600}{0.45^5}$$

D = 0.5578m =557.8mm

PIPE NETWORK ANALYSIS

Analysis of water distribution system includes determining quantities of flow and head losses in the various pipe lines, and resulting residual pressures. In any pipe network, the following two conditions must be satisfied:

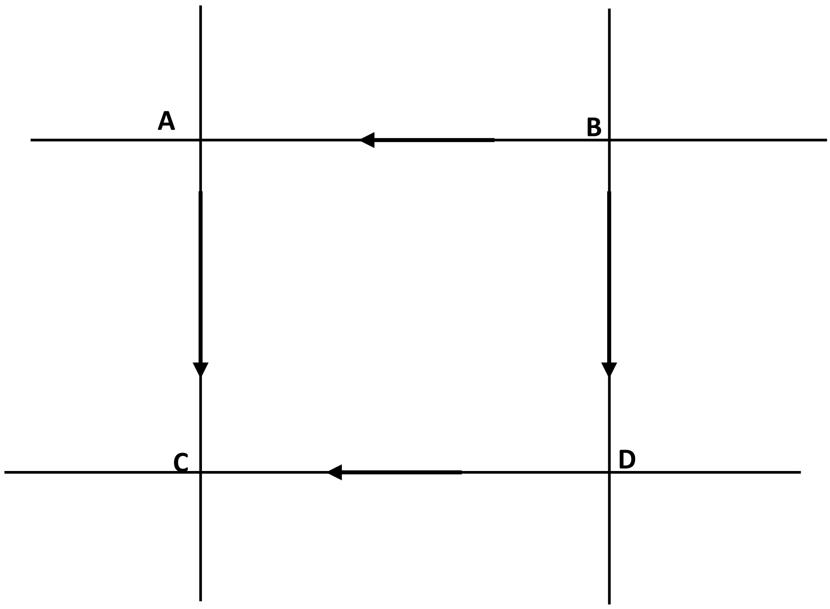
The algebraic sum of pressure drops around a closed loop must be zero, i.e. there can be no discontinuity in pressure. The flow entering a junction must be equal to the flow leaving that junction; i.e. the law of continuity must be satisfied. Based on these two basic principles, the pipe networks are generally solved by the methods of successive approximation. The widely used method of pipe network analysis is the Hardy-Cross method.

Hardy-Cross method

The Hardy Cross method is an iterative method for determining the flow in pipe network systems where the inputs and outputs are known, but the flow inside the network is unknown. The method was first published in November 1936 by its namesake, Hardy Cross, a structural engineering professor at the University of Illinois at Urbana–Champaign. The Hardy Cross method is an adaptation of the Moment distribution method, which was also developed by Hardy Cross as a way to determine the moments in indeterminate structures. The introduction of the Hardy Cross method for analysing pipe flow networks revolutionized municipal water supply design. Before the method was introduced, solving complex pipe systems for distribution was extremely difficult due to the nonlinear relationship between head loss and flow. The method was later made obsolete by computer solving algorithms employing the Newton-Raphson method or other solving methods that prevent the need to solve nonlinear systems of equations by hand. The Hardy Cross method provides a system for calculating the value of the correction to be made, each loop or junction being considered in turn and corrected assuming that conditions in the reminder of the network remain unaltered.

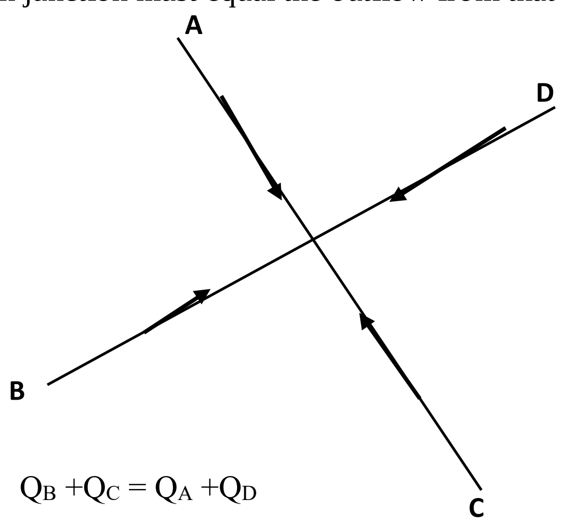
Two main assumptions used in Hardy Cross method:

1. The losses of head between any two junctions must be the same for all routes between these junctions.



Head loss in pipe BA + Head loss in pipe DC = Head loss in pipe AC + Head loss in pipe BD

2. The inflow to each junction must equal the outflow from that junction.



$Q_B + Q_C = Q_A + Q_D$

The method of balancing heads uses an initial guess that satisfies continuity of flow at each junction and then balances the flows until continuity of potential is also achieved over each loop in the system.

From previous lesson, major loss can be determined as:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D^5}\frac{16}{2g\pi^2}Q^2 = KQ^2$$

Head loss in pipe in which flow is clockwise;

$$\sum h_c = \sum K_C\, Q_C{}^2$$

Head loss in pipe in which flow is counter---clockwise;

$$\sum h_{cc} = \sum K_{CC}\, Q_{CC}{}^2$$

In first assumption, we assumed that the losses value is not balanced.

$\sum K_C\,.\,(Q_C - \Delta Q)^2 = \sum K_{CC}\,.\,(Q_{CC} + \Delta Q)^2$

ΔQ^2 is neglected

$$\Delta Q = \frac{\sum K_C . Q_C{}^2 - \sum K_{CC} . Q_{CC}{}^2}{2(\sum K_C . Q_C - K_{CC} . Q_{CC})}$$

$$h_f = KQ^2$$

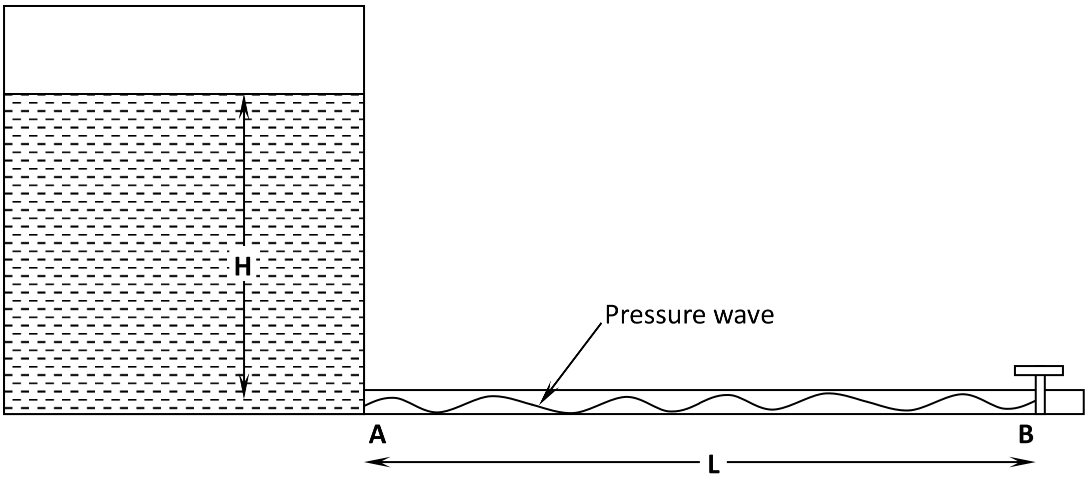
$$K.Q = \frac{h_f}{Q}$$

Finally,

$$\boldsymbol{\Delta Q} = \frac{\sum \mathbf{h_{fC}} - \sum \mathbf{h_{fCC}}}{\mathbf{2}\left(\sum \frac{\mathbf{h_{fC}}}{\mathbf{Q_C}} + \frac{\mathbf{h_{fCC}}}{\mathbf{Q_C C}}\right)^{\mathbf{2}}}$$

Calculation will be done many times until the value of ΔQ is approaching zero.

WATER HAMMER IN PIPES



If water is flowing along a long pipe and is suddenly brought to rest by the closing of a valve, or by any similar cause, there will be a sudden rise in pressure due to the momentum of the water being destroyed. This will cause a wave of high pressure to be transmitted along the pipe with a velocity equal to the sound wave, which may setup noises known as **Knocking.** The magnitude of this pressure will depend on (i) The mean pipe flow velocity (ii) The length of the pipe (iii) The time taken to close the valve and (iv) The elastic properties of the pipe material and that of water.

This sudden rise in pressure in the pipe due to the stoppage of the flow generating a high pressure wave, which will have a hammering effect on the walls of the pipe, is known as **Water Hammer.**

The cases that can be studied under this are:

- Gradual closure of valve

- Sudden closure of valve and
- Pipe is rigid
- Pipe is elastic

Critical Time:

It is defined as the time required for the pressure wave generated due to closure of valve to travel once from the point of origin to reservoir over the length of pipe and back to the point of origination.

If **T** is the time required by the pressure wave to travel once up and down the pipe and **C** is the velocity of the pressure wave equal to the velocity of sound wave in water also called as **Celerity**, then from Newton's law, we have

Distance traveled = Average velocity x time

i.e. $2L = C \times T$

Hence $T = \frac{2L}{C}$

If **t** is the actual time of closure, **T** is the critical time, then

if **t** > $T = \frac{2L}{C}$, then it is referred to as Gradual closure and

if **t** < $T = \frac{2L}{C}$, then it is referred to as Sudden closure

Instantaneous rise in pressure in a pipe running full due to Gradual closure of valve

Consider a pipe AB of length **L** connected to a tank at A and a valve at B with water flowing in it as shown in Fig. Let **V** be the mean flow velocity and **a** is the flow cross-sectional area, **p** the instantaneous rise in pressure due to gradual closure of valve and **t** be the actual time of closure of valve.

From Newton's second law of motion, the retarding force generated against the flow direction is given by the rate of change momentum of the liquid along the direction of the force.

Retardation of water = Change in velocity / Time $= \frac{(V - 0)}{t} = \frac{V}{t}$

Retarding force = Mass of water x Retardation = $\rho a L \frac{V}{t}$ (01)

The force generated due to pressure wave = Pressure intensity x area

$$= p_i \times a \qquad (02)$$

From Eqs. 1 and 2, we get

$$p_i a = \rho a L \frac{V}{t}$$

Hence $p_i = \frac{\rho L V}{t}$

Instantaneous rise in Pressure head = $H = \frac{p_i}{\rho g} = \frac{\rho L V}{\rho g t} = \frac{L V}{g t}$ (03)

The above equation is valid only for rigid pipes with incompressible fluids flowing through it.

Instantaneous rise in pressure in a pipe running full due to Sudden closure of valve when the pipe is rigid

When the valve provided at the downstream end is closed suddenly and the pipe is rigid, then the converted pressure energy from the kinetic energy due to closure is to be absorbed by the fluid due to its compressibility only.

{ Pressure Energy converted from Kinetic energy } = { Pressure energy absorbed by water due to its compressibility }

i.e. $\mathbf{E_k = E_w}$ (01)

Consider the pipe AB of length **L** and cross-sectional area **a** in which water of mass density **ρ,** weight density **γ** and bulk density **K** is flowing with a mean velocity **V** be suddenly stopped due to closure of valve provided at B.

The kinetic energy of flowing water before closure of valve will be converted to strain energy, when the effect of friction and elasticity of pipe material are ignored.

Loss of kinetic energy $\mathbf{E_k}$ = ½ **x** mass of water x $\mathbf{V^2}$

As mass = ρ **x volume = ρ x aL**

Loss of kinetic energy $\mathbf{E_k}$ = ½ x **ρ a L** x $\mathbf{V^2}$ (02)

Gain in strain energy $= \frac{1}{2}\left(\frac{p_i^2}{K}\right)\times \text{Volume} = \frac{1}{2}\left(\frac{p_i^2}{K}\right)\times aL$ (03)

From Eqs. 1, 2 and 3, we get

$$\rho a L V^2 = \frac{1}{2}\left(\frac{p_i^2}{K}\right)\times aL$$

or $p_i^2 = \rho K V^2$

or $p_i = V\sqrt{\rho K}$

But Celerity $C = \sqrt{\frac{K}{\rho}}$.

Substituting for the value of **C** in the above equation for pressure rise, we get

$\mathbf{p_i = \rho\ V\ C}$

Instantaneous rise in pressure in a pipe running full due to Sudden closure of valve when the pipe is elastic

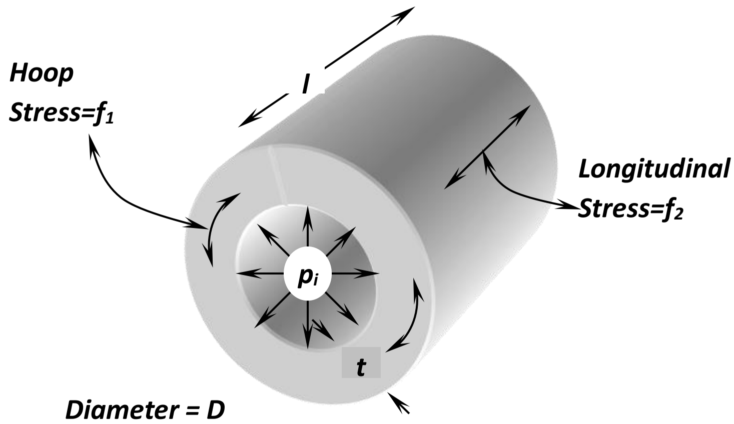
When the valve provided at the downstream end is closed suddenly and the pipe is elastic, then the converted pressure energy from the kinetic energy due to the valve closure is to be absorbed by both the fluid due to its compressibility and the elasticity of the pipe.

$$\left\{\begin{matrix}\text{Pressure Energy}\\ \text{converted from}\\ \text{Kinetic energy}\end{matrix}\right\} = \left\{\begin{matrix}\text{Pressure energy}\\ \text{absorbed by water due}\\ \text{to its compressibility}\end{matrix}\right\} + \left\{\begin{matrix}\text{Pressure energy absorbed}\\ \text{by the Elastic pipe due to}\\ \text{its expansion}\end{matrix}\right\}$$

i.e. $\mathbf{E_k = E_w + E_p}$ (01)

$\mathbf{E_k}$ and $\mathbf{E_w}$ can be computed as in the previous derivation.

Computation of $\mathbf{E_p}$ can be done by simulating the situation to the thick cylinder subjected to internal fluid pressure. Let **t** be the thickness of the elastic pipe wall and assume that it is small compared to its diameter **D.** Let $\mathbf{f_1}$ be the hoop or circumferential



and stress $\mathbf{f_2}$ be the longitudinal stress as shown in figure.

Let the Young's modulous of the pipe material be **E** and poisons ration $\mathbf{1/m}$

Let the instantaneous fluid pressure be $\mathbf{p_i}$**.**

From the knowledge oaf Strength of materials, we can write that

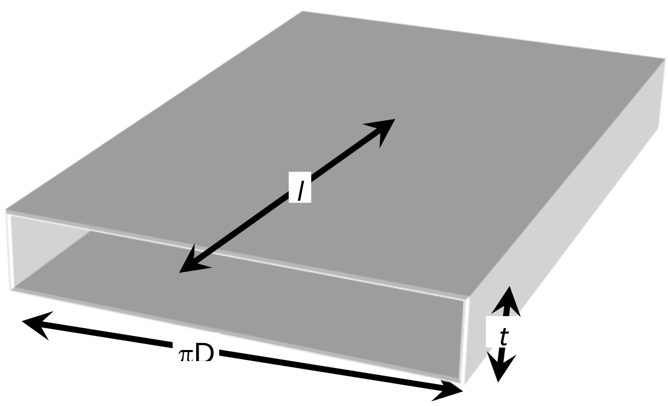
$$f_1 = \frac{p_i D}{2t} \text{ and } f_2 = \frac{p_i D}{4t}$$

Hence $\mathbf{f_1 = 2\ f_2}$

Further, the strain energy stored in pipe per unit volume is given by

$$E_p \big/ V_1 = \frac{1}{2E}\left[f_1^2 + f_2^2 - \frac{2 f_1 f_2}{m}\right]$$

Substituting $\mathbf{f_1 = 2\ f_2}$**,** we get



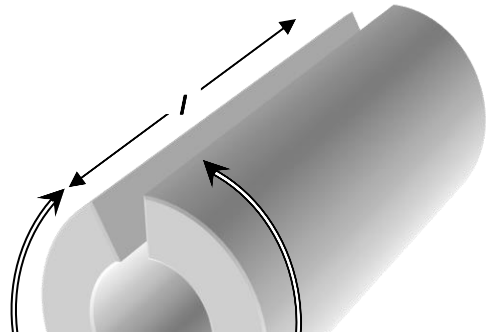
$$E_p \big/ V_1 = \frac{1}{2E}\left[4 f_2^2 + f_2^2 - \frac{4 f_2^2}{m}\right]$$

$$E_p \big/ V_1 = \frac{f_2^2}{2E}\left[5 - \frac{4}{m}\right]$$

Substituting for $\mathbf{f_2}$**,** and $\mathbf{V_1 = \pi Dtl}$ we get

$$E_p = \frac{p_i^2 D^2}{16 t^2} \frac{1}{2E}\left[5 - \frac{4}{m}\right] \pi D t l \qquad (04)$$

From Eqs. 02 and 03, we have



E_k = ½ x ρ a L x **V^2** (02)

$$E_k = \frac{1}{2} \times \rho\, a\, L \times V^2$$

$$E_w = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times \text{Volume} = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times aL \qquad (03)$$

Substituting in Eq. 01, we have

$$\frac{1}{2} \times \rho\, a\, L \times V^2 = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times aL + \frac{p_i^2 D^2}{16t^2} \frac{1}{2E}\left[5 - \frac{4}{m}\right] \pi D t L$$

Simplifying, we get

$$\frac{1}{2} \times \rho\, a\, L \times V^2 = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times aL + \frac{p_i^2 D}{4t} \frac{1}{2E}\left[5 - \frac{4}{m}\right] \frac{\pi D^2}{4} L$$

But $a = \frac{\pi D^2}{4}$ and hence **aL/2** gets canceled on both sides

$$\rho V^2 = \left(\frac{p_i^2}{K}\right) + \frac{p_i^2 D}{4t} \frac{1}{E}\left[5 - \frac{4}{m}\right] = p_i^2\left[\left(\frac{1}{K}\right) + \frac{D}{4t\,E}\left(5 - \frac{4}{m}\right)\right]$$

$$p_i = V\sqrt{\frac{\rho}{\left[\left(\frac{1}{K}\right) + \frac{D}{4t\,E}\left(5 - \frac{4}{m}\right)\right]}} = V\sqrt{\frac{\rho}{\left[\left(\frac{1}{K}\right) + \frac{D}{t\,E}\left(\frac{5}{4} - \frac{1}{m}\right)\right]}} \qquad (05)$$

The above expression gives the instantaneous rise in pressure in an elastic pipe due to sudden closure of Valve.

If the Poisons ration is not given, it can be assumed as ¼. Then Eq. 04 reduces to

$$p_i = V\sqrt{\frac{\rho}{\left[\frac{1}{K} + \frac{D}{t\,E}\right]}} \qquad (06)$$

NUMERICAL

1. A hydraulic pipeline 3 km long, 500 mm diameter is used is used to convey water with a velocity of 1.5 m/s. Determine the pressure growth of the valve provided at the outflow end is closed in (i) 20 s (ii) 3.5 s. Consider pipe to be rigid and take bulk modulous of elasticity of water as $K_{water} = 20 \times 10^8$ N/m^2

Solution:

(10)

L = 3000 m; d = 0.5 m; V = 1.5 m/s; t_1 = 20 s; t_2 = 3.5 s;

K = 20 x 10^8 N/m^2; ρ = 1000 kg/m^3 (Assumed)

Critical time $T = \frac{2L}{C}$, where Celerity $C = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2\times10^9}{1000}} = 1414.2 \text{m/s}$

Hence $T = \frac{2\times3000}{1414.2} = 4.24\,\text{s}$

Case (i)

t_1 > T, Hence the valve closure is gradual.

Instantaneous rise in pressure is given by

$$p_i = \frac{\rho LV}{t} = \frac{1000\times3000\times1.5}{20} = 225\,\text{kPa}\ \ (\text{Ans})$$

Case (ii)

t_2 < T, Hence the valve closure is Sudden with pipe rigid.

Instantaneous rise in pressure is given by

$$p_i = \rho VC = 1000\times1.5\times1414.2 = 2.1213\,\text{MPa}\ (\text{Ans})$$

2. Water flowing with a velocity of 1.5 m/s in a rigid pipe of diameter 500 mm is suddenly brought to rest. Find the instantaneous rise in pressure if bulk modulous of water is 1.962 GPa

Solution:

V = 1.5 m/s**; K = 1.962** GPa; **ρ = 1000** kg/m^3 (Assumed)

Celerity $C = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{1.962 \times 10^9}{1000}} = 1400.7 \text{m/s}$

Instantaneous rise in pressure is given by

$p_i = \rho V C = 1000 \times 1.5 \times 1400.7 = 2.1 \, \text{MPa}$ (Ans)

3. A steel penstock of 1000 mm diameter has a thickness of 20 mm. Water is flowing initially with a velocity of 2.0 m/s. Flow velocity is brought to rest by closing a valve at the end of the pipeline. Bulk modulous of water is 2 x 10^9 N/m^2 and elastic modulous of pipe material is 2 x 10^{11} N/m^2. If the length of the pipe is 2000 m, find the pressure rise in terms of head of water when:

Water is compressible and pipe is elastic

Water is compressible and pipe is rigid

Solution:

V = 2.0 m/s; d = 1 m; t =20 x 10^{-3} m ; K = 2.0 GPa;

E = 200 GPa; $\frac{1}{m} = \frac{1}{4}$ and ρ = 1000 kg/m^3 (Assumed)

Case (i)

The valve closure is Sudden with pipe Elastic.

$$p_i = V \sqrt{\frac{\rho}{\left[\left(\frac{1}{K}\right) + \frac{D}{tE}\right]}} = 2.0 \sqrt{\frac{1000}{\left[\left(\frac{1}{2}\right) + \frac{1}{20 \times 10^{-3} \times 200}\right] \frac{1}{10^9}}} = 2.309 \, \text{MPa} \ \text{(Ans)}$$

Instantaneous rise in Pressure head =

$$\frac{p_i}{\rho g} = \frac{2.309 \times 10^6}{1000 \times 10} = 230.9 \, \text{m of water} \ \text{(Ans)}$$

Case (ii)

The valve closure is Sudden with pipe rigid.

Celerity $C = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2 \times 10^9}{1000}} = 1414.2 \text{m/s}$

Instantaneous rise in pressure is given by

$$p_i = \rho V C = 1000 \times 2.0 \times 1414.2 = 2.828 \text{ MPa}$$

Instantaneous rise in Pressure head =

$$\frac{p_i}{\rho g} = \frac{2.828 \times 10^6}{1000 \times 10} = 282.8 \text{ m of water (Ans)}$$

4. What is the maximum pressure rise due to sudden closure of a valve in a pipe of 300 mm diameter conveying water with a velocity of 1.8 m/s? The pipe wall is 18 mm thick. The E_{pipe} = 210 GPa and K_{water} = 2.1 GPa. Also find the hoop stress developed.

Solution:

V = 1.8 m/s; d = 0.3 m; t =18 x 10^{-3} m ; K = 2.1 GPa;

E = 210 GPa;

$\frac{1}{m} = \frac{1}{4}$ and **ρ = 1000** kg/m^3 (Assumed)

The valve closure is Sudden with pipe Elastic.

$$p_i = V \sqrt{\frac{\rho}{\left[\left(\frac{1}{K}\right) + \frac{D}{tE}\right]}} = 1.8 \sqrt{\frac{1000}{\left[\left(\frac{1}{2.1}\right) + \frac{1}{18 \times 10^{-3} \times 210}\right] \frac{1}{10^9}}} = 2.091 \text{ MPa (Ans)}$$

Hoop stress developed is given by

$$f_1 = \frac{p_i d}{2t} = \frac{2.091 \times 10^6 \times 0.3}{2 \times 18 \times 10^{-3}} = 17.425 \text{ MPa (Ans)}$$