Module 2

Kinematics and Dynamics of Fluids

KINEMATICS OF FLOW AND IDEAL FLOW

Kinematics is defined as that branch of science which deals with motion of particles without considering the force causing the motion.

Methods of describing fluid motion

The following are the different method of the fluid motion based on the particle motion

i. Langrangian method
ii. Eulerian method

Langrangian method

In this, a single fluid particle is followed during its motion and its velocity, acceleration, density etc are described.

In the cartesian system, the position of the fluid particle in space (x,y,z) at any time t from its position (a,b,c) at time t=0 shall be given as:

$x=f_1(a, b, c, t)$, $y= f_2(a, b, c, t)$ & $z= f_3(a, b, c, t)$

The velocity and acceleration components (obtained by taking derivates with respect time) are given by

$$u = \frac{\partial x}{\partial t}, v = \frac{\partial y}{\partial t} \,\&\, w = \frac{\partial z}{\partial t}$$ velocity component

$$a_x = \frac{\partial^2 x}{\partial t^2}, a_y = \frac{\partial^2 y}{\partial t^2} \,\&\, a_z = \frac{\partial^2 z}{\partial t^2}$$ acceleration component

Eulerian method

In case of eulerian method, velocity, acceleration, pressure and density etc are described at a point in the flow field.

Methods of describing based on fluid flow

The following are the different method of the fluid motion based on the fluid flow

i. Steady and unsteady flow
ii. Uniform and nonuniform flow
iii. Laminar and turbulent flow
iv. Compressible and incompressible flow

v. Rotational and irrotational flow

vi. One, two- and three-dimensional flows

Steady and unsteady flow

Steady flow is defined as that type of flow in which fluid is characteristics like velocity, pressure, density, etc at a point do not change with time. For steady flow, mathematically we have

$$\left(\frac{\partial \mathrm{v}}{\partial \mathrm{t}}\right)_{x_o,y_o,z_o} = 0,\ \left(\frac{\partial \mathrm{p}}{\partial \mathrm{t}}\right)_{x_o,y_o,z_o} = 0\ \&\left(\frac{\partial \rho}{\partial \mathrm{t}}\right)_{x_o,y_o,z_o} = 0$$

where x_o, y_o, z_o is a fixed point in fluid field.

Unsteady flow is defined as that type of flow in which fluid is characteristics like velocity, pressure, density, etc at a point changes with respect to time. For steady flow, mathematically we have

$$\left(\frac{\partial \mathrm{v}}{\partial \mathrm{t}}\right)_{x_o,y_o,z_o} \neq 0,\ \left(\frac{\partial \mathrm{p}}{\partial \mathrm{t}}\right)_{x_o,y_o,z_o} \neq 0\ \&\left(\frac{\partial \rho}{\partial \mathrm{t}}\right)_{x_o,y_o,z_o} \neq 0$$

Uniform and nonuniform flow

Uniform flow is defined as that type of flow in which velocity at any given time does not change with respect to space (i.e length of direction of flow). Mathematically

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}} = 0$$

Where ∂v = change of velocity ∂s = length of flow in direction S

Uniform flow is defined as that type of flow in which velocity at any given time does not change with respect to space (i.e length of direction of flow). Mathematically

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}} \neq 0$$

Where ∂v = change of velocity ∂s = length of flow in direction S

Laminar and Turbulent flow

Laminar flow is that type of flow in which fluid particles move along the well-defined paths or stream line and all stream lines are straight and parallel. Thus, the particles move in laminar or layers gliding smoothly over the adjacent layers. It is also called as stream line flow or viscous flow.

Turbulent flow is that type of flow in which fluid particles move in a zig zag way. Due to movement of fluid particles in a zigzag way, eddies formation takes place which are responsible for high energy loss. For a pipe flow , the type of flow is determined by a nondimensional number called Reynold number

$R_e = \frac{VD}{\gamma}$ where R_e – Reynold number, D- Diameter of the pipe, V-Mean velocity of flow in pipe and γ- Kinematic viscosity of fluid

If Reynold number < 2000, flow is laminar flow

If Reynold number >2000, flow is turbulent flow

If Reynold number is between 2000 and 4000, it may be laminar or turbulent flow.

Compressible and incompressible flow

Compressible is that type of flow in which density of fluid changes from point to point or in other words, density ρ is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{constant}$$

Incompressible is that type of flow in which density of fluid changes from point to point or in other words, density ρ is constant for the fluid. Thus, mathematically, for incompressible flow

$$\rho = \text{constant}$$

Rotational and irrotational flow

Rotational flow is that type of flow in which the fluid particles while flowing along the stream lines, also rotate about their own axis.

Irrotational flow is that type of flow in which the fluid particles while flowing along the stream lines, not rotate about their own axis.

One-two and three-dimensional flows

One dimensional flow, type of flow is one which the flow parameter such as velocity is a function of time and one space coordinate only, say x.

The mathematically for one dimensional flow is expressed as

$u = f(x), v = 0$ and $w = 0$

where u, v and w are the velocity components in x, y and z directions respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space coordinates say x and y.

$u = f_1(x, y), v = f_2(x, y)$ and $w = 0$

Three-dimensional flow is a type of flow in which velocity is a function of time and three mutually perpendicular directions. Fluid parameters are function of three space coordinates.

$u = f_1(x, y), v = f_2(x, y)$ and $w = f_3(x, y)$

Rate of flow or discharge (Q)

It is defined as quantity of a fluid flowing per second through a section of pipe or channel. Mathematically it is expressed as

$$Q = A * V$$

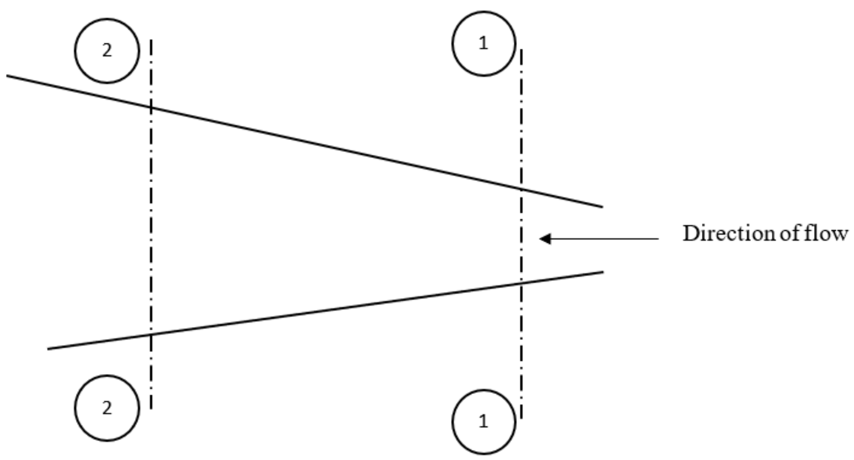
where Q – Discharge of fluid unit is lt/sec or m^3/sec

A – Cross section area of the pipe

V – average velocity of fluid across section

Continuity Equation

For a fluid flowing in a pipe at all the cross section, quantity of fluid flowing per second is constant. Equation based on principle of conservation of mass is called continuity equation.



Consider two cross sections of a pipe,

V_1 = average velocity at cross section 1-1

ρ_1 = Density at section 1-1

A_1 = area of the pipe at section 1-1

V_2, ρ_2 and A_2 are the corresponding values at 2-2

Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 1-1

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2} \quad \longrightarrow \quad (1)$$

Eq (1) is applicable to compressible as well as incompressible fluids and is called continuity equation

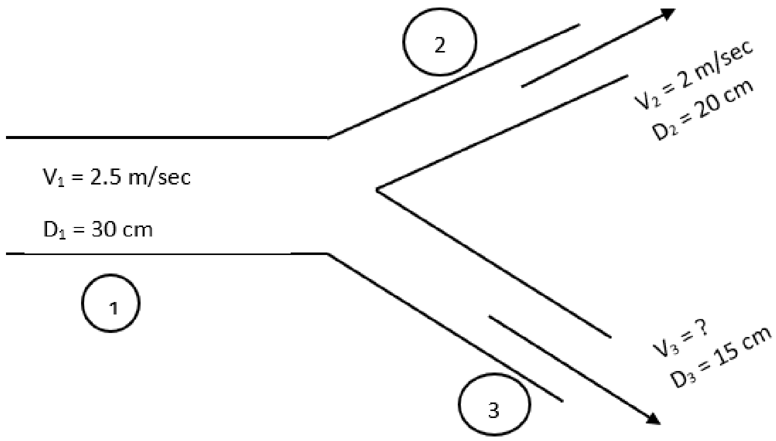
If fluid is incompressible fluids, then $\rho_1 = \rho_2$ and Equ (1) is reduced to

$$\boxed{A_1 V_1 = A_2 V_2} \quad \longrightarrow \quad (2)$$

NUMERICAL

1. A 30 cm diameter pipe, conveying water, branches into two pipes of diameter 20 cm and 15 cm respectively. If the average velocity in 30 cm diameter pipe is 2.5 m/sec, Find the discharge in the pipe, also determine the velocity in 15 cm pipe if average velocity in 20 cm diameter pipe is 2m/sec.

Solution:



D_1 =30cm =0.3m , $A_1 = \frac{\pi}{4} D^2$ =0.07068 m^2, V_1 = 2.5 m/sec

D_2 =20cm =0.2m , $A_1 = \frac{\pi}{4} D^2$ =0.0314 m^2, V_1 = 2 m/sec

D_2 =15cm =0.15m , $A_1 = \frac{\pi}{4} D^2$ =0.01767 m^2

Let Q_1, Q_2 & Q_3 are the discharge in the pipe 1, 2 &3

According to continuity equation

$Q_1 = Q_2 + Q_3 \dashrightarrow$ (1)

i. Discharge in Q_1 in pipe 1

$$Q_1 = A_1 V_1 = 0.07068 \times 2.5 = 0.1757 \ m^3/sec$$

ii. Value of V_3

$$Q_2 = A_2 V_2 = 0.0314 \times 2 = 0.0628 \ m^3/sec$$

Substituting values of Q_1 & Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$$Q_3 = 0.1139 \ m^3/sec$$

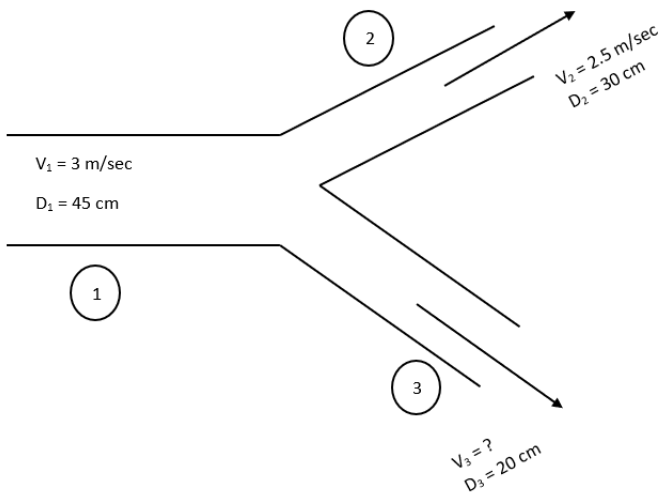
$$Q_3 = A_3 V_3$$

$$V_3 = {Q_3}/{A_3} = {0.1139}/{0.01767} = 6.44 m/\text{sec}$$

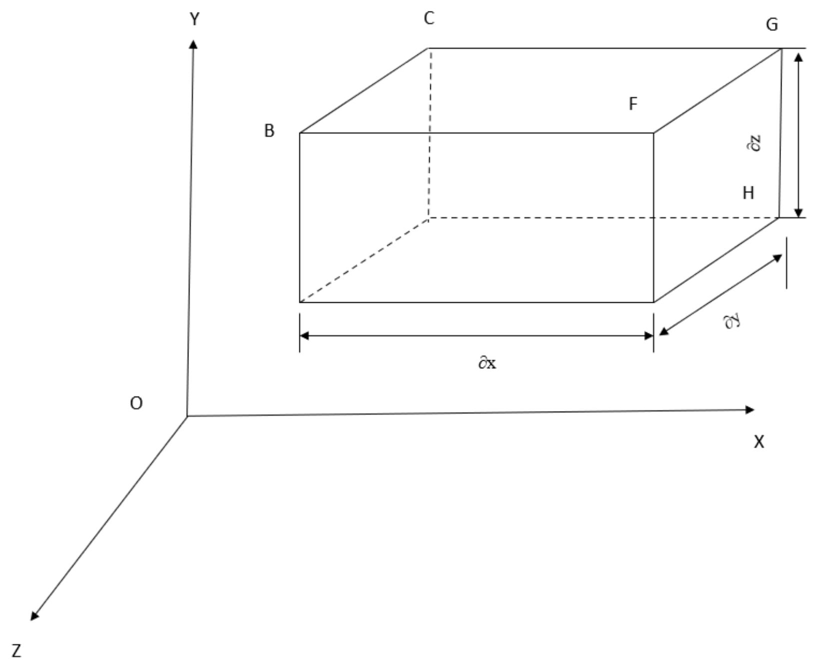
2. A pipe (1) 450mm in diameter branches into two pipes (2 and 3) of diameter 300mm and 200mm respectively as shown in below figure. If the average velocity in 450mm diameter pipe is 3 m/sec. Find:

a) Discharge through 450mm diameter of the pipe;

b) Velocity in 200mm diameter pipe if the average velocity in 300 mm pipe is 2.5 m/sec.



Continuity Equation in Three Dimensions



Consider a fluid element of length dx, dy &dz in the direction x, y and z. Let u,v and w are the inlet velocity components in x, y and z direction respectively. Mass of the fluid entering face ABCD per second.

$$= \rho \times \text{velocity in x direction} \times \text{area of ABCD}$$

$$= \rho \times u \times \partial y \times \partial z$$

Then mass of fluid leaving face EFGH per second $= \rho \times u \times \partial y \times \partial z + \frac{\partial(\rho\, u\, \partial y \partial z)dx}{\partial x}$

Therefore, mass gain in x-direction

$= \text{Mass through ABCD} - \text{Mass through EFGH per second}$

$= \rho \times u \times \partial y \times \partial z - (\quad \rho * u * \partial y * \partial z + \partial(\rho\, u\, \partial y \partial z) dx/\partial x)$

$= -\frac{\partial(\rho u)}{\partial x} dxdydz$

Similarly, net gain in y direction

$$= -\frac{\partial(\rho v)}{\partial y} dxdydz$$

and in Z direction

$$= -\frac{\partial(\rho w)}{\partial z} dxdydz$$

Therefore, Net gain of masses = $-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] dxdydz \dashrightarrow (1)$

Since mass is neither created nor destroyed in fluid element, the net increase of mass per unit time in fluid element must be equal to rate of increase of mass of fluid in the element. But mass of fluid element is [ρ×dx×dy×dz] and its rate of increase with time is $\frac{\partial[\rho * dx * dy * dz]}{\partial t}$ or

$$\frac{\partial \emptyset}{\partial t} dx * dy * dz \dashrightarrow (2)$$

Equating two expression

$$\frac{\partial \emptyset}{\partial t} dx * dy * dz = -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] dxdydz$$

$$\frac{\partial \emptyset}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \dashrightarrow (3)$$

From (3) this equation is continuity equation in cartesian coordinates in its most general form. This equation is applicable to

a) Steady and Unsteady flow
b) Uniform and non-uniform flow
c) Compressible and incompressible flow

For steady flow $\frac{\partial \emptyset}{\partial t} = 0$ and hence equation (3) becomes

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

If fluid is incompressible, then ρ is constant and above equation becomes

$$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} = 0$$

Velocity and Acceleration

Let V is the resultant velocity at any point in a fluid flow. Let u, v and w are its component in x, y and z directions.

$$u = f_1(x, y, z, t) \ , v = f_2(x, y, z, t) \ \& \ w = f_3(x, y, z, t)$$

and resultant velocity $V = ui + vj + wk = \sqrt{u^2 + v^2 + w^2}$

Let a_x, a_y & a_z are the total acceleration in x, y and z direction respectively.

Then by chain rule, we have

$$a_x = \frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt} + \frac{du}{dy}\frac{dy}{dt} + \frac{du}{dz}\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} + \frac{dv}{dy}\frac{dy}{dt} + \frac{dv}{dz}\frac{dz}{dt} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = \frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dy}{dt} + \frac{dw}{dz}\frac{dz}{dt} + \frac{\partial w}{\partial t}$$

for steady flow, $\frac{\partial v}{\partial t}=0,$ where V is the resultant velocity

$$\frac{\partial u}{\partial t}=0, \ \frac{\partial v}{\partial t}=0 \text{ and } \frac{\partial w}{\partial t}=0$$

Hence acceleration in x, y and z directions becomes

$$a_x = \frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt} + \frac{du}{dy}\frac{dy}{dt} + \frac{du}{dz}\frac{dz}{dt}$$

$$a_y = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} + \frac{dv}{dy}\frac{dy}{dt} + \frac{dv}{dz}\frac{dz}{dt}$$

$$a_z = \frac{dw}{dt} = \frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dy}{dt} + \frac{dw}{dz}\frac{dz}{dt}$$

Acceleration vector

$$A = a_x i + a_y j + a_z k$$

$$= \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$$

NUMERICAL

1. The velocity vector in a fluid flow is given $V=4x^3i-10x^2yj+2tk$. Find the velocity and acceleration of a fluid particle at (2,1,3) at time t=1.

Solution:

Velocity components u,v and w are

$u = 4x^3$, $v = -10x^2$ and $w = 2t$

Hence velocity component at [2,1,3] are

$u= 4\times(2)^3=32$ Units, $v=-10\times(2)^2 =-40$ Units & $w=2\times1=2$ Units

There fore velocity vector V at (2,1,3) = 32i-40j+2k

Resultant velocity, V= $\sqrt{32^2+(-40)^2+2^2}$ =51.26 Units

Accelerations;

$$a_x = \frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt} + \frac{du}{dy}\frac{dy}{dt} + \frac{du}{dz}\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} + \frac{dv}{dy}\frac{dy}{dt} + \frac{dv}{dz}\frac{dz}{dt} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = \frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dy}{dt} + \frac{dw}{dz}\frac{dz}{dt} + \frac{\partial w}{\partial t}$$

From velocity components, we have

$\frac{du}{dx} = 12x^2$, $\frac{du}{dy} = 0 = \frac{du}{dz} = \frac{du}{dt}$

$\frac{dv}{dx} = -20xy$, $\frac{dv}{dy} = -10x^2$, $\frac{dv}{dz} = 0$ & $\frac{dv}{dt} = 0$

$\frac{dw}{dx} = \frac{dw}{dy} = \frac{dw}{dz} = 0$ and $\frac{dw}{dt} = 2$

Substitute [2,1,3] at time t=1;

$a_x = 4x^3 \times (12x^2) + (-10x^2y)(0) + 2t \times 0 + 0$

$= 48x^5 = 48 \times (2)^5 = 48 \times 32 = 1536$ units

$a_y = 4x^3 \times (-20xy) + (-10x^2y)(-10x^2) + 2 + 0 + 0$

$= -1280 + 1600 = 320$ units

$a_z = 4x^3(0) + (-10x^2y) \times (0) + (2t) \times 0 + 2.0 = 2$ units

Accelerations

A= 1536i+320j+2k

A= $\sqrt{1536^2+320^2+2^2}$ =1568.9 units

NUMERICAL

1. Velocity vector in a fluid flow is given by $V=2x^3i-5x^2yj+4tk$. Find the velocity and acceleration of a fluid particle at (1,2,3) at time t=1.

Solution:

Same procedure of the previous problem

V=10.95 units and A =16.12 units

In a fluid, the velocity field is given by $V=(3x+2y)i+(2z+3x^2)j+(2t-3z)k$. Determine The velocity components u,v and w at any point in the flow field

a. The speed at a point (1,1,1)
b. The speed at time t=2 sec at a point (0,0,2).

Same procedure of the previous problem

2. Find the velocity and acceleration at a point (1,2,3) after 1 sec , for a three dimensional flow given by u=yz+t, v=xz-t & w=xy m/sec.

Ans: v = 7.55 m/sec and A = 29.34 m/sec^2

3. Given the velocity field V=(6+2xy+t^2)i-(xy^2+10t)j+25k. What is the acceleration of the particle at (3,0,2) at time t=1?

Same procedure of the previous problem, A=62.8 m/sec^2

4. The following cases represents the two velocity components, Determine the third component of velocity, such that it satisfy the continuity equation

a. $u=x^2+y^2+z^2$; $v=xy^2-yz^2+xy$
b. $v=2y^2$, $w=2xyz$

The continuity equation for incompressible fluid is given by equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \dashrightarrow (1)$$

Case a) $u=x^2+y^2+z^2$; v=xy2-yz2+xy

$$\frac{\partial u}{\partial x}=2x,\ \frac{\partial v}{\partial y}=2xy-z^2+x$$

from (1)

$$2x+(2xy-z^2+x)+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial w}{\partial z}=-3x-2xy+Z^2$$

$$\partial w=(-3x-2xy+Z^2)\partial z$$

$$\int \partial w=\int(-3x-2xy+Z^2)\partial z$$

$$\boxed{w = (-3xz-2xyz+z^3/3)+C}$$

Case b) $v=2y^2$, $w=2xyz$

$$\frac{\partial v}{\partial x}=0,\ \frac{\partial v}{\partial y}=4y,\ \frac{\partial w}{\partial z}=2xy$$

from (1)

$$\frac{\partial u}{\partial x}+4y+2xy=0$$

$$\frac{\partial \mathrm{u}}{\partial \mathrm{x}} = -(4\mathrm{y} + 2\mathrm{xy})$$

$$\partial \mathrm{u} = -(4\mathrm{y} + 2\mathrm{xy})\, \partial \mathrm{x}$$

Integrating above equation

$$\boxed{\mathbf{u = -(4xy + x^2y) + C}}$$

5. A fluid flow field is given by V=x^2yi+y^2zj-(2xyz+yz^2)k. Calculate the velocity and acceleration at the point (2,1,3).

Same procedure of the previous problem

V=21.587 units and A =126.18 units

Velocity Potential Function and Stream Function

Velocity Potential Function

It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is defined by Ø (Phi). Mathematically, the velocity, potential is defined as Ø =f(x, y,z)

For steady flow

$$\mathrm{u} = \frac{-\partial \emptyset}{\partial \mathrm{x}},\ \mathrm{v} = \frac{-\partial \emptyset}{\partial \mathrm{y}}\ \&\ \mathrm{w} = \frac{-\partial \emptyset}{\partial \mathrm{z}} \dashrightarrow \quad (1)$$

where u, v and w are the component of the velocity in x, y and z direction respectively. Velocity components in cylindrical polar co-ordinates in terms of velocity potential function are

$$\mathrm{u}_r = \frac{\partial \emptyset}{\partial \mathrm{r}},\ \mathrm{u}_\theta = \frac{1}{r}\frac{\partial \emptyset}{\partial \theta} \dashrightarrow \quad (2)$$

where u_r – velocity component in radial direction (that is in r direction)

u_θ– velocity component in tangential direction (that is in θ direction)

W.K.T

$$\frac{\partial \mathrm{u}}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Substituting values of u, v and w,

We get

$$\frac{\partial\left(\frac{-\partial \emptyset}{\partial \mathrm{x}}\right)}{\partial x} + \frac{\partial\left(\frac{-\partial \emptyset}{\partial y}\right)}{\partial y} + \frac{\partial\left(\frac{-\partial \emptyset}{\partial z}\right)}{\partial z} = 0$$

$$\frac{\partial \emptyset^2}{\partial \mathrm{x}^2} + \frac{\partial \emptyset^2}{\partial \mathrm{y}^2} + \frac{\partial \emptyset^2}{\partial \mathrm{z}^2} = 0 \dashrightarrow (3)$$

Eq(3) is Laplace equation

For two dimensions case (2) reduces to

$$\frac{\partial \emptyset^2}{\partial x^2} + \frac{\partial \emptyset^2}{\partial y^2} = 0$$

If any value of ∅ that satisfies Laplace equation, we will correspond to some case of fluid flow

Properties of Potential Function

Rotational components are given by

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right],\ \omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right] \text{ and } \omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] \dashrightarrow \qquad (4)$$

Substituting values of u, v and w in above equations we get

$$\omega_z = \frac{1}{2}\left[\frac{\partial\left(-{\partial \emptyset}/{\partial y}\right)}{\partial x} - \frac{\partial\left(-{\partial \emptyset}/{\partial x}\right)}{\partial y}\right] = \frac{1}{2}\left[\frac{-\partial \emptyset^2}{\partial x\, \partial y} + \frac{\partial \emptyset^2}{\partial y\, \partial x}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial\left(-{\partial \emptyset}/{\partial x}\right)}{\partial z} - \frac{\partial\left(-{\partial \emptyset}/{\partial z}\right)}{\partial x}\right] = \frac{1}{2}\left[\frac{-\partial \emptyset^2}{\partial z\, \partial x} + \frac{\partial \emptyset^2}{\partial x\, \partial z}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial\left(-{\partial \emptyset}/{\partial z}\right)}{\partial y} - \frac{\partial\left(-{\partial \emptyset}/{\partial y}\right)}{\partial z}\right] = \frac{1}{2}\left[\frac{-\partial \emptyset^2}{\partial y\, \partial z} + \frac{\partial \emptyset^2}{\partial z\, \partial y}\right]$$

If ∅ is a continuous function,

then

$$\frac{\partial \emptyset^2}{\partial x\, \partial y} = \frac{\partial \emptyset^2}{\partial y\, \partial x};\ \frac{\partial \emptyset^2}{\partial z\, \partial x} = \frac{\partial \emptyset^2}{\partial x\, \partial z} \text{ etc}$$

$$\omega_x = \omega_y = \omega_z = 0$$

Then rotational components are zero, flow is called irrotational, hence properties of potential function are

- If velocity potential (Ø) exists, flow should be rotational
- If velocity potential (Ø) satisfies Laplace equation, it represents possible steady incompressible irrotational flow.

Stream Function

It is defined as scalar function of space and time, such that its partial derivate with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (psi) and defined only two-dimensional flow

$$\frac{\partial \psi}{\partial x} = v \text{ and } \frac{\partial \psi}{\partial y} = -u \dashrightarrow \quad (1)$$

The velocity component in cylindrical polar co-ordinates in terms of stream function are given as

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \text{ and } u_\theta = \frac{-\partial\psi}{\partial r}$$

where u_r- radial velocity and u_θ-tangential velocity continuity equation for two-dimension flow is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0$, substituting u & v in (1)

$$\frac{\partial\left(-{\partial\psi}/{\partial y}\right)}{\partial x} - \frac{\partial\left(-{\partial\psi}/{\partial x}\right)}{\partial y} \text{ or } \frac{-\partial\psi^2}{\partial x\,\partial y} + \frac{\partial\psi^2}{\partial x\,\partial y} = 0$$

Hence existence of ψ means a possible case of fluid flow. The flow may be rotational or irrotational.

Rotational component ω_z is given by $\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$ substitute u & v from (1)

$$\omega_z = \frac{1}{2}\left[\frac{\partial\left(-{\partial\psi}/{\partial x}\right)}{\partial x} - \frac{\partial\left(-{\partial\psi}/{\partial y}\right)}{\partial y}\right] = \frac{1}{2}\left[\frac{\partial\psi^2}{\partial x^2} + \frac{\partial\psi^2}{\partial y^2}\right]$$

For irrotational flow, $\omega_z = 0$. Hence above equation becomes

$$\frac{\partial\psi^2}{\partial x^2} + \frac{\partial\psi^2}{\partial y^2} = 0,$$ which is Laplace equation is ψ

Properties of stream function ψ are:

a. If the stream function exists, it is possible case of fluid flow which may be rotational or irrotational.
b. If stream function satisfies the Laplace equation, it is possible case of a irrotational flow.

Equipotential Line

A line along which velocity potential Ø is constant, is called equipotential line.

For equipotential line

Ø = constant

dØ =0

Ø = f(x,y) for steady flow

$$d\text{Ø} = \frac{\partial\text{Ø}}{\partial x}dx + \frac{\partial\text{Ø}}{\partial y}dy$$

= -udx-vdy

= -(udx+vdy)

For equipotential line dØ = constant

-(udx+vdy) =0 or udx+vdy =0

$$\frac{\text{dy}}{\text{dx}} = \frac{-u}{v}$$

$\frac{\text{dy}}{\text{dx}}$= slope of equipotential line

Line of constant stream function

ψ = constant

$d\psi = 0$

$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$

$= v\,dx - u\,dy$ [because $\frac{\partial \psi}{\partial x} = v$; $\frac{\partial \psi}{\partial y} = -u$]

For a line of constant stream function

$= d\psi = 0$ or $vdx - udy = 0$

$$\frac{\text{dy}}{\text{dx}} = \frac{v}{u}$$

But $\frac{\text{dy}}{\text{dx}}$ is a slope of a stream line

Flow Net – A grid obtained by drawing a series of equipotential lines and stream lines is called a flow net.

NUMERICAL

1. The velocity potential function (Ø) is given by Ø = $\frac{xy^3}{3} - x^2 + \frac{yx^3}{3} + y^2$

a. Find the velocity components in x and y direction
b. Show that Ø represents a possible case of flow

Solution:

$$Ø = \frac{xy^3}{3} - x^2 + \frac{yx^3}{3} + y^2$$

Partial derivatives of Ø with respect to x and y are

$$\frac{\partial Ø}{\partial x} = -\frac{y^3}{3} - 2x + \frac{3x^2y}{3}$$

$$\frac{\partial Ø}{\partial y} = \frac{-3xy^2}{3} + \frac{x^3}{3} + 2y$$

Velocity components u and v are given

$$u = \frac{-\partial Ø}{\partial x} = -\left[-\frac{y^3}{3} - 2x + \frac{3x^2y}{3}\right] = \frac{y^3}{3} + 2x - \frac{3x^2y}{3} = \frac{y^3}{3} + 2x - x^2y$$

$$v = \frac{-\partial \emptyset}{\partial y} = -\left[\frac{-3y^2x}{3} + \frac{x^3}{3} + 2y\right] = y^2x - \frac{x^3}{3} - 2y$$

Ø represents a possible case of flow

$$\frac{\partial \emptyset^2}{\partial x^2} + \frac{\partial \emptyset^2}{\partial y^2} = 0$$

$$\frac{\partial \emptyset}{\partial x} = -\frac{y^3}{3} - 2x + \frac{3x^2y}{3}$$

$$\frac{\partial \emptyset^2}{\partial x^2} = -2 + 2xy$$

$$\frac{\partial \emptyset}{\partial y} = \frac{-3y^2x}{3} + \frac{x^3}{3} + 2y$$

$$\frac{\partial \emptyset^2}{\partial y^2} = 2 - 2xy$$

$$\frac{\partial \emptyset^2}{\partial x^2} + \frac{\partial \emptyset^2}{\partial y^2} = (-2 + 2xy + 2 - 2xy) = 0$$

Therefore, Laplace equation is satisfied and hence $\emptyset$ represents the possible case of flow.

Types of Motion

A fluid particle while moving may undergo any one or combination of following four types of displacement

a. Linear Translation or pure translation
b. Linear Deformation
c. Angular Deformation
d. Rotation

Linear Translation or pure translation

It is defined as the movement of a fluid element in such a way that it moves bodily from one position to another position and the two axes ab and cd represented in new positions by a'b' and c'd' are parallel as shown in below figure

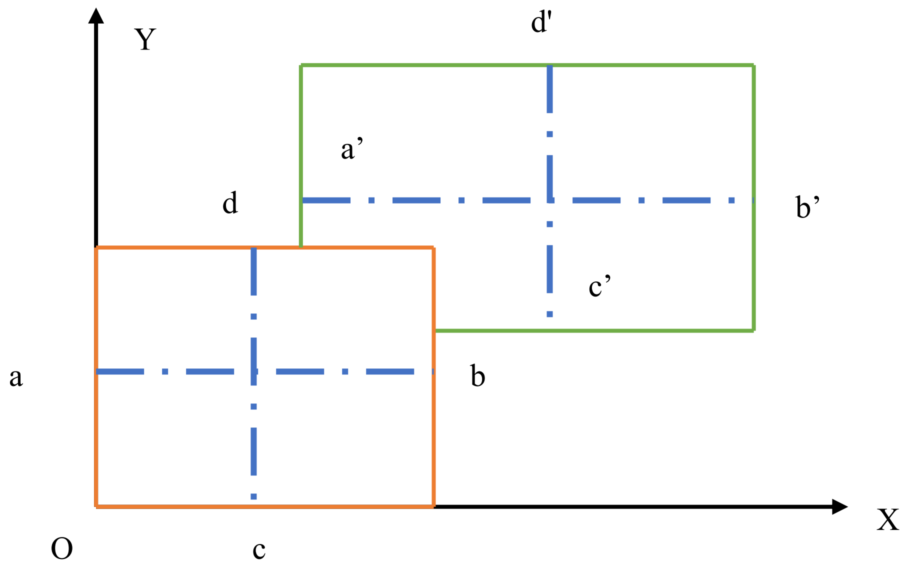


Figure: Linear Translation

Linear Deformation

It is defined as the deformation of a fluid element in linear direction when the element moves. The axes of the element in the deformed position.

Angular Deformation

It is defined as the change in the angle contained by two adjacent sides. Let $\Delta\theta_1$ and $\Delta\theta_2$ is the change in the angle between two adjacent sides of a fluid element, then the angular deformation $= \frac{1}{2}[\Delta\theta_1 + \Delta\theta_2]$

Rotation

It is defined as the movement of a fluid element in such a way that both of its axes rotate in the same direction. It is equal to $= \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$ for a two-dimensional element x-y plane. Rotational components are

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right], \omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] \text{ and } \omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right]$$

Vorticity

It is defined as the value twice of the rotation and hence it is given as 2w.

NUMERICAL

1. A fluid flow is given by V= $8x^3$i-$10x^2$yj. Find the shear strain rate and state weather flow is rotational or irrotational

Solution:

$$V = 8x^3 i - 10x^2 y j$$

$$u = 8x^3, \frac{\partial u}{\partial x} = 24x^2, \frac{\partial u}{\partial y} = 0$$

$$v = -10x^2 y, \frac{\partial v}{\partial x} = -20xy, \frac{\partial v}{\partial y} = -10x^2$$

Shear strain rate

$$= \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] = \frac{1}{2}[-20xy + 0] = -10xy$$

Rotation in x-y plane , is given by

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}[-20xy + 0] = -10xy$$

As rotation $\omega_z \neq 0$, hence the flow is rotational

FLOW NET

A grid obtained by drawing a series of streamlines and equipotential lines is known as flow net.

Methods of Drawing flow nets

1.Analytical method

2.Graphical method

3.Electric analogy method

4.Hydraulic model

Use of flow nets

1. To determine the streamlines and equipotential lines
2. To determine quantity of seepage and upward lift pressure below hydraulic structure.
3. To determine the design of the outlets for the streamlining

4. To determine the velocity and pressure distribution for a given boundaries of flow.

Limitations of flow nets:

1. The flow net analysis cannot be applied in the region close to the boundary where the effects of viscosity are predominant
2. In case of a flow of a fluid past a solid body, while the flow net gives a fairly accurate picture of the flow pattern for the upstream part of the solid body, it can give little information concerning the flow conditions at the rear because of separation and eddies.

1. The stream function for 2D flow is given by ψ=2xy. Find the velocity function Ø

$$\psi = 2xy$$

$$u = \frac{-\partial\psi}{\partial y} = -2x, \quad v = \frac{\partial\psi}{\partial x} = 2y$$

Velocity potential function

$$\frac{\partial\emptyset}{\partial x} = -u = 2x \dashrightarrow (1)$$

$$\frac{\partial\emptyset}{\partial y} = -v = -2y \dashrightarrow (2)$$

Integrate (1)

$$\emptyset = \frac{2x^2}{2} + c = x^2 + C \dashrightarrow (3)$$

C is constant

$$\frac{\partial\emptyset}{\partial y} = \frac{\partial c}{\partial y} \dashrightarrow (4)$$

Compare (2) and (4)

$$\frac{\partial c}{\partial y} = -2y$$

$$-2y\,\partial y = \partial c$$

$$C = -y^2$$

Put in (3) we get

$$\emptyset = x^2 - y^2$$

DYNAMICS OF FLUID FLOW

Introduction

In the last unit, we studied the velocity and acceleration at a point in a fluid flow, without taking into consideration the forces causing the flow. This chapter includes the study of forces causing fluid flow. Thus, dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamics behavior of the fluid flow is analyzed by the Newton's second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non – viscous.

Fluid dynamics is that branch of fluid mechanics wherein we study the analysis of the fluid motion along with the forces generating them.

The fluid motion is analyzed by the **Newton's second law of motion,** which states that t**he force applied on a body along any direction is given by the rate of change of momentum along the same direction**.

$$F_x = \frac{(mV_2 - mV_1)}{t} = m\frac{(V_2 - V_1)}{t} = ma$$

The forces acting on the fluid can be classified as under:

$F_g \Rightarrow$ Gravity forces

$F_P \Rightarrow$ Pressure forces

$F_v \Rightarrow$ Viscous forces

$F_t \Rightarrow$ Turbulent forces

$F_e \Rightarrow$ Elastic forces

$F_c \Rightarrow$ Compressibility forces

The net force acting along x direction is given by

$F_x = (F_g)_x + (F_P)_x + (F_v)_x + (F_t)_x + (F_e)_x + (F_c)_x$ ……………………(01)

Equations of motion

Reynolds equation of motion: In the equation of motion (Eq. 01), the force due to compressibility is neglected and only forces due to gravity, pressure, viscosity and turbulence are considered, and the resulting equation is termed as **Reynolds equation of motion.**

Navier-Stokes equation of motion: In the equation of motion (Eq. 01), the forces due to compressibility and turbulence are neglected and only forces due to gravity, pressure and viscosity are considered, the resulting equation is termed as **Navier-Stokes equation of motion.**

Euler's equation of motion: In the equation of motion (Eq. 01), if the fluid is ideal, then the forces due to compressibility, turbulence and viscosity are neglected and only forces due to gravity and pressure are considered, the resulting equation is termed as **Euler's equation of motion.**

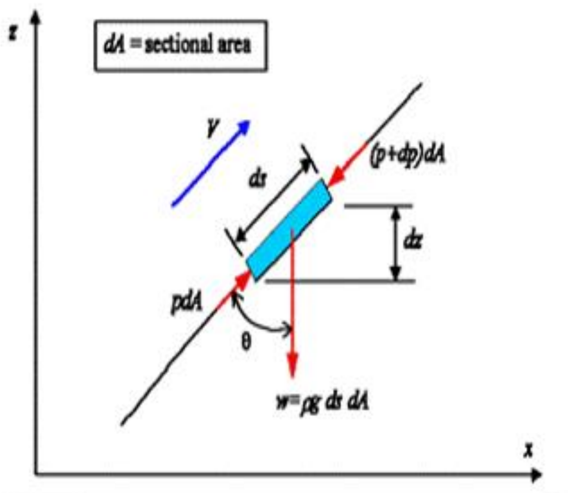
$$F_x = ma_x = F_g + F_P$$

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Assumptions:

1. Only Gravitational and Pressure forces are considered
2. Fluid motion along a stream line is considered
3. Flow is steady & Incompressible
4. Flow is irrotational
5. Flow is in viscid (Zero Viscosity)

Consider a stream line along direction ***S*** as shown in fig (a) below:



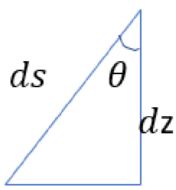


fig (a) fig (b)

Consider a cylindrical fluid element of cross-sectional area ***dA*** and length ***ds*** along the stream line direction.

The forces acting on the fluid element are:

- The pressure force ***pdA*** along the flow direction ***S***
- The pressure force (***p***+$\boldsymbol{dp}$)***dA*** against the flow direction ***s***
- Weight of the fluid element = ***ρ.g.dA.ds*** acting vertically downwards at an angle ***θ*** with the vertical.

Mass of the fluid element = ***ρ.dA.ds***

Let '**θ**' is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of **S** must be equal to the mass of fluid element x acceleration in the direction ***S*** *(Newton's law, F=m.a)*

$$\mathbf{p.dA} - (\boldsymbol{p} + \boldsymbol{dp})\boldsymbol{dA} - \boldsymbol{\rho g.dA.dS.\ cos\theta} = \boldsymbol{\rho.dA.dS.\ a_s}$$

$$\mathbf{p.dA} - \mathbf{p}\boldsymbol{dA}\mathbf{-dp.da} - \boldsymbol{\rho g.dA.dS.\ cos\theta} = \boldsymbol{\rho.dA.dS.\ a_s}$$

where a_s is acceleration in the direction of S

Now, $a_s = \frac{dv}{dt}$, where v is the function of S and t

$$a_s = \frac{dv}{ds}\,\frac{ds}{dt} = v\,\frac{dv}{ds}$$

Substituting the value of a_s n

$$-dp.dA - \rho g.dA.dS.\ cos\theta = \rho.dA.dS.v\ \frac{dv}{ds}$$

Dividing by $-\rho.\ dA$.ds

$$\frac{1}{\rho}\frac{dp}{ds} + g.\ cos\theta + v\ \frac{dv}{ds} = 0$$

From fig(b) $cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\frac{dp}{ds} + g.\frac{dz}{ds} + v\ \frac{dv}{ds} = 0$$

$$\boxed{\boldsymbol{\frac{dp}{\rho} + g.dz + v.dv = 0}}$$ ------------ Eulers's equation of Motion

Bernoulli's Equation

Bernoulli's Equation is obtained by integration Euler's equation

$$\int \frac{dp}{\rho} + \int g.dz + \int v.dv = constant$$

If flow is incompressible, ρ is constant

$$\frac{p}{\rho} + g.\ z + \frac{v^2}{2} = constant$$

Dividing by 'g'

$$\boxed{\boldsymbol{\frac{p}{\rho g} + z + \frac{v^2}{2g} = constant}}$$

$\boldsymbol{\frac{p}{\rho g}}$ = *pressure energy per unit weight of fluid or* ***pressure head***

$\boldsymbol{\frac{v^2}{2g}}$= *kinetic energy per unit weight of fluid or* ***kinetic head***

$\boldsymbol{z}$ = *potential energy per unit weight of fluid or* ***potential head/ datum head***

Assumptions:

- Fluid motion along a stream line is considered
- Flow is steady & Incompressible
- Flow is irrotational
- Flow is ideal (Zero Viscosity*)*
- Forces acting are gravitational and pressure forces

Limitations of Bernoulli's Equation:

- External forces are neglected (Only Gravitational and Pressure forces are considered)
- Flow is continuous and velocity is uniform over the section

NUMERICALS

1. Water is flowing through a pipe of 5cm dia, under a pressure of 29.43 N/cm^2 and with mean velocity of 2m/sec. Find the total head or total energy per unit weight of water at a cross section which is 5m above the datum line.

Solution:

Given, dia of pipe, d = 5cm =0.5m

pressure, p = of 29.43 N/cm^2 = of 29.43 × 10^4 N/m^2

velocity, v = 2m/s

Total head = pressure head + kinetic head + datum head

Pressure head = $\frac{p}{\rho g}$

$= \frac{29.43 \times 104}{1000 \times 9.81} = 30\text{m}$

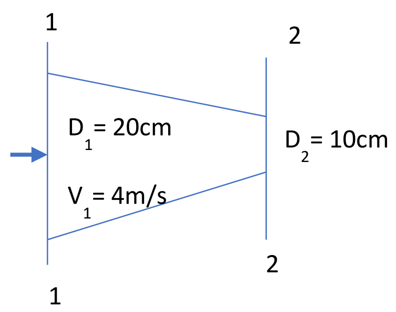
Kinetic head = $\frac{v^2}{2g}$

$= \frac{2^2}{2 \times 9.81} = 0.204\text{m}$

Datum head, Z =5m (given)

Total head = 30+0.204+5 = 35.204m

2. A pipe through which water is flowing is having diameters, 20cm and 10cm at the cross section 1 & 2 respectively. The velocity of water at section 1 is given 4.0m/s. Find the velocity head at section 1 & 2 and also rate of discharge.



Solution:

Given,

$D_1 = 20\text{cm} = 0.2\text{m}$ $\quad$ $D_2 = 10\text{cm} = 0.1\text{m}$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314\text{m}^2$ $\quad$ $A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785\text{m}^2$

$V_1 = 4.0\text{m/s}$ $\quad$ $V_2 = ?$

(i) Velocity head at section (1) = $\frac{v_1^2}{2g}$

$= \frac{4^2}{2 \times 9.81} = 0.815\text{m}$

Velocity head at section (2) = $\frac{v_2^2}{2g}$

$= \frac{v_2^2}{2 \times 9.81}$

Applying continuity equation at section (1) and (2)

$$A_1 V_1 = A_2 V_2$$

$$0.0314 \times 4 = 0.00785 \times \frac{v_2^2}{2 \times 9.81}$$

$V_2 = 16.0$m /s

ii) Rate of discharge, $Q = A_1 V_1$ or $A_2 V_2$

$= 0.0314 \times 4$

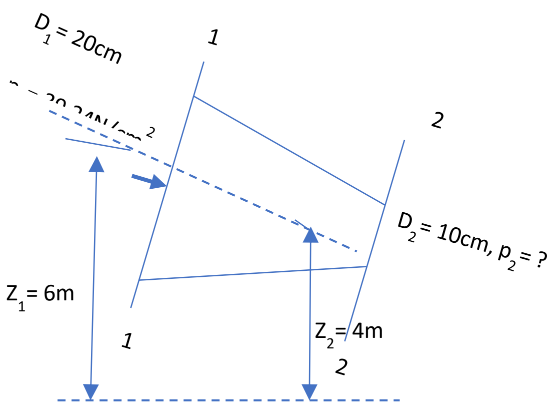
$= 0.1256$ m^3/sec

Q = 125.6 litres/sec

3. A pipe through which water is flowing is having diameters, 20cm and 10cm at the cross section 1 & 2 respectively. The rate of flow through pipe is 35liters/sec. The section 1 is 6m above datum line and section 2 is 4m above datum. If the pressure at section 1 is 39.24N/cm^2, find the pressure at section 2

Solution:

Given,



$D_1 = 20cm = 0.2m$ $\qquad$ $D_2 = 10cm = 0.1m$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314m^2$ $\qquad$ $A_2 = \frac{\pi}{4} \times 0.1^2 = 0.00785m^2$

$p_1 = 39.24N/cm^2$ $\qquad$ $p_2 = ?$

$= 39.24 \times 10^4$ N/m^2

$Z_1 = 6m$ $\qquad$ $Z_2 = 4m$

Rate of flow, discharge Q= 35lt/sec = 35×10^{-3} m^3 /sec

$$Q = A_1 V_1 = A_2 V_2$$

$V_1 = \frac{\mathbf{0.035}}{\mathbf{0.0314}} = 1.114$ m/s $\qquad$ $V_2 = \frac{\mathbf{0.035}}{\mathbf{0.00785}} = 4.456$ m/s

Applying Bernoulli's equation at section (1) and (2)

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + 6 + \frac{1.114^2}{2 \times 9.81} = \frac{p_2}{1000 \times 9.81} + 4 + \frac{4.456^2}{2 \times 9.81}$$

$$46.063 = \frac{p_2}{9810} + 5.012$$

$$p_2 = 41.051 \times 9810 \ N/m^2$$

$$= \frac{41.051 \times 9810}{10^4} \ N/cm^2$$

p_2= 40.27 N/cm^2

4. Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40lt/sec

Solution: Given,

D_1= 300mm = 0.3m D_2= 200mm = 0.2m

$A_1 = \frac{\pi}{4} \times 0.3^2 = 0.070\text{m}^2$ $A_2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\text{m}^2$

p_1 = 24.525 N/cm^2 p_2= 9.81 N/cm^2

$= 24.525 \times 10^4$ N/m^2 $= 9.81 \times 10^4$ N/m^2

$Z_2 - Z_1 = ?$

Rate of flow, discharge Q= 40lt/sec = 40×10^{-3} m^3 /sec

$$Q = A_1 V_1 = A_2 V_2$$

$V_1 = \frac{\mathbf{0.04}}{\mathbf{0.070}} = 0.57$ m/s $V_2 = \frac{\mathbf{0.04}}{\mathbf{0.0314}} = 1.27$ m/s

Applying Bernoulli's equation at section (1) and (2)

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{24.525\times10^4}{1000\times9.81} + Z_1 + \frac{0.57^2}{2\times9.81} = \frac{9.81\times10^4}{1000\times9.81} + Z_2 + \frac{1.27^2}{2\times9.81}$$

$$25 + Z_1 + 0.32 = \mathbf{10} + Z_2 + 1.623$$

$$\mathbf{Z_2 - Z_1} = 25.32 - 11.623$$

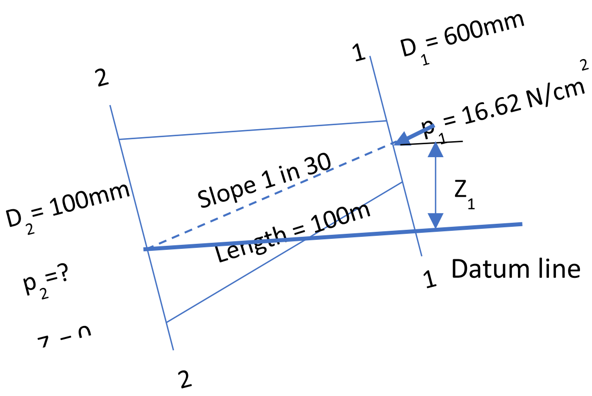
$$\mathbf{= 13.7\ m}$$

5. The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at the lower end, at the rate of 50lt/sec. The pipe has a slope 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 16.62 N/cm^2

Solution: Given

D_1= 600mm = 0.6m D_2= 100mm = 0.1m

$A_1 = \frac{\pi}{4} \times 0.6^2 = 0.28\text{m}^2$ $A_2 = \frac{\pi}{4} \times 0.1^2 = 0.07068\text{m}^2$



$p_1 = 16.62$ N/cm^2 $\quad\quad\quad\quad$ $p_2 = ?$

$= 16.62 \times 10^4$ N/m^2

Rate of flow, discharge Q = 50 lt/sec = 50×10^{-3} m^3 /sec

$Z_2 = 0$

$Z_1 = \frac{1}{30} \times 100 = \frac{10}{3}$

$$Q = A_1 V_1 = A_2 V_2$$

$V_1 = \frac{0.05}{0.28} = 0.17$m/s $\quad\quad\quad\quad$ $V_2 = \frac{0.05}{0.007068} = 0.708$m/s

Applying Bernoulli's equation at section (1) and (2)

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{16.62 \times 10^4}{1000 \times 9.81} + 0 + \frac{0.17^2}{2 \times 9.81} = \frac{p_2}{1000 \times 9.81} + \frac{10}{3} + \frac{0.708^2}{2 \times 9.81}$$

$$\mathbf{p_2 = 19.85\ N/cm^2}$$

Bernoulli's Equation for Real fluid

- Bernoulli's equation was derived on the assumption that fluid is non-viscous and therefore frictionless.
- But all real fluids are viscous and hence offer resistance to flow.
- Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation these losses have been taken into consideration

Bernoulli's Equation for Real fluid is given as:

$$\boxed{\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + H_L}$$

Where, H_L is the loss of energy between point (1) and (2)

Loss of head, $H_L = E_A - E_B$

NUMERICALS

1. A pipe of diameter 400mm carries water at a velocity of 25m/sec. The pressure at the point A and B are given as 29.43 N/cm^2 and 22.563 N/cm^2 respectively while the datum head at A and B are 28m and 30m. Find the loss of head between A and B.

Solution: Given,

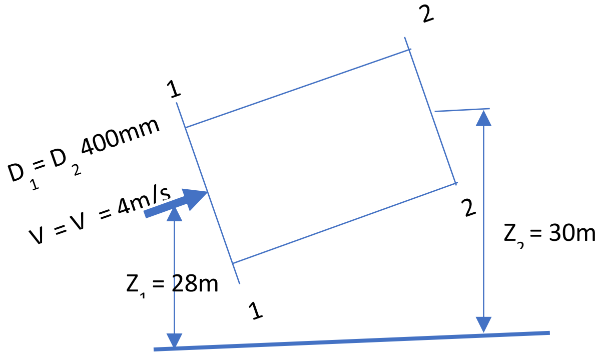
$D_A = D_B = 400$mm $= 0.4$m

$p_A = 29.43\ N/cm^2$ $\quad$ $p_B = 22.563\ N/cm^2$

$= 29.43 \times 10^4\ N/m^2$ $\quad$ $= 22.563 \times 10^4\ N/m^2$

$Z_A = 28m$ $\quad$ $Z_B = 30m$

$v_A = v_B = 25m/s$



Total energy at A, $E_A = \frac{p_A}{\rho g} + Z_A + \frac{V_A^2}{2g}$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + 28 + \frac{25^2}{2 \times 9.81}$$

$= 89.85m$

Total enegy at B , $E_B = \frac{p_B}{\rho g} + Z_B + \frac{V_B^2}{2g}$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + 30 + \frac{25^2}{2 \times 9.81}$$

$= 84.85\ m$

Loss of Energy, $E_A - E_B = 89.85 - 84.85 = 5.0m$

2. A conical tube of length 2.0m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5m/s while at the lower end is 2 m/s. The pressure head at the smaller end is 2.5m of liquid. The loss of head in the tube is $\frac{0.035(v_1 - v_2)^2}{2g}$ where v_1 and v_2 are velocity at upper end and lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.

Solution: Let the smaller end is represented by (1) and lower end by (2)

Given, L=2m $\quad$ $v_1 = 5\ m/s$

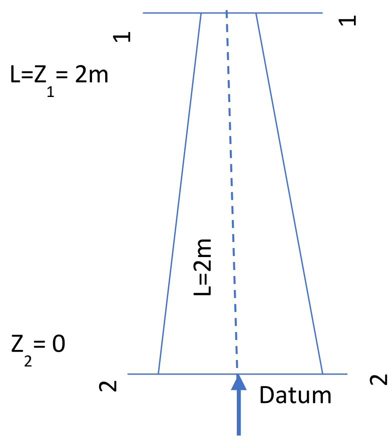
Pressure head at section (1) , $\frac{p_1}{\rho g} = 2.5m$

$v_2 = 2\ m/s$

Loss of head, $H_L = \frac{0.035(v_1 - v_2)^2}{2g}$

$$= \frac{0.035(5-2)^2}{2 \times 9.81} = 0.16m$$

Pressure head at section (2) , $\frac{p_2}{\rho g} = ?$



Applying Bernoulli's equation at section (1) and (2)

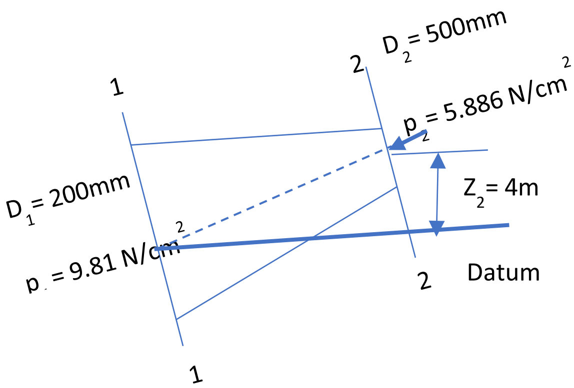
$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\mathbf{2.5} + 2 + \frac{5^2}{2\times 9.81} = \frac{p_2}{\rho g} + 0 + \frac{2^2}{2\times 9.81}$$

$$\frac{p_2}{\rho g} = (2.5+1.27+2.0)-(0.203+1.6)$$

$$= 5.407 \text{ m of fluid}$$

3. A pipe line carrying oil of specific gravity 0.87 changes in diameter from 200m diameter at position A to 500mm diameter at a position B which is 4m at a higher level, if the pressure at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and discharge is 200lt/sec. Determine the loss of head in direction of flow.



Solution: Given

$D_1 = 200\text{mm} = 0.2\text{m}$ $\qquad D_2 = 500\text{mm} = 0.5\text{m}$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314\text{m}^2$ $\qquad A_2 = \frac{\pi}{4} \times 0.5^2 = 0.\ 1963\text{m}^2$

$p_1 = 9.81 \text{ N/cm}^2$ $\qquad p_2 = 5.886 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$ $\qquad = 5.886 \times 10^4 \text{ N/m}^2$

$Z_1 = 0$ $\qquad Z_2 = 4\text{m}$

Rate of flow, discharge Q= 200 lt/sec = 200×10^{-3} m^3 /sec

$$Q = A_1 V_1 = A_2 V_2$$

$V_1 = \frac{\mathbf{0.2}}{\mathbf{0.0314}} =$ ___m/s $\qquad V_2 = \frac{\mathbf{0.2}}{\mathbf{0.1963}} =$ ____ m/s

Applying Bernoulli's equation at section (1) and (2)

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + H_L$$

$H_L =$ __________ m

Application of Bernoulli's equation

Bernoulli's equation is applied all problems of incompressible fluid flow where energy consideration are involved.

The devices commonly used in measuring flow are:

- Venturimeter
- Orifice meter
- Pitot-tube

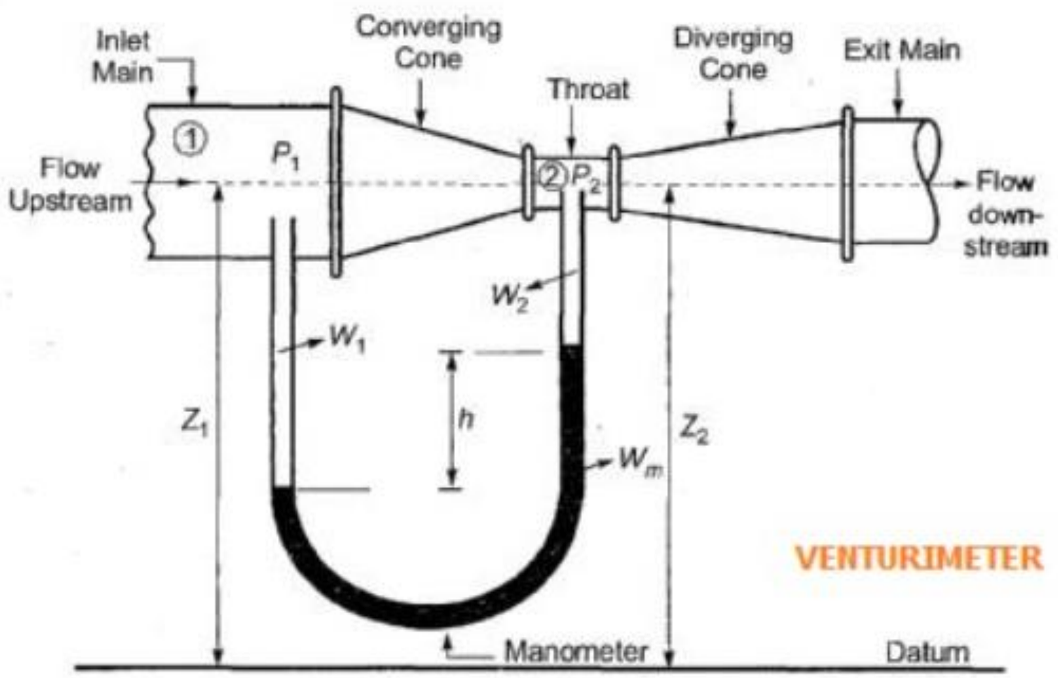
VENTURIMETER

The Venturimeter consists essentially of a convergence in a pipeline followed by a short parallel sided 'throat' and then a divergence. Consists of flowing three parts

- A short converging part
- Throat
- Diverging part
-

Expression for rate of flow through Venturimeter

Consider a Venturimeter fitted in a horizontal pipe through which a fluid flowing (say water) as shown in fig below:



Let, d_1= dia. at inlet or at section (1)

p_1= pressure at section (1)

v_1= velocity at section (1)

a_1= area at section (1)

And d_2 p_2 v_2 a_2 are corresponding values at throat section (2)

Applying Bernoulli's equation, we get,

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

As pipe is horizontal $Z_1 = Z_2$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{p_1\text{-}p_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} \text{ --------------- (1)}$$

$$\frac{p_1\text{-}p_2}{\rho g} = \text{pressure head at section (1) and (2)} = h$$

Substitute in Eq. (1), $h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$ -----------(2)

Apply continuity equation at section (1) and (2)

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

Substitute value of V_1 in eq, (2)

$$h = \frac{V_2^2}{2g} - \frac{(\frac{a_2 v_2}{a_1})^2}{2g} = \frac{v_2^2}{2g}(1 - \frac{a_2^2}{a_1^2})$$

or

$$v_2^2 = 2gh.\frac{a_1^2}{a_1^2 - a_2^2}$$

$$v_2 = \sqrt{2gh.\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

Discharge. $Q = a_2 v_2$

$$= a_2 \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh} \text{ ------------(3)}$$

Eq (3) gives under ideal conditions------Theoretical Discharge

But the Actual discharge will be less than Theoretical Discharge

Therefore, $Q_{act} = C_d. \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

Where, C_d= co-efficient of discharge and the value is <1

Value of 'h' is given by differential U-tube monometer

Case(i) When monometer consists of liquid heavier than the liquid through pipe

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

Case(ii) When monometer consists of liquid lighter than the liquid through pipe

$$h = x\left[1 - \frac{S_h}{S_o}\right]$$

Case(iii) Inclined Venturimeter with differential monometer

a) Differential monometer consists of heavier liquid

$$h = \left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + Z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

Case(iv) Inclined Venturimeter with differential monometer

b) Differential monometer consists of lighter liquid

$$h = \left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + Z_2\right) = x\left[1 - \frac{S_h}{S_o}\right]$$

NUMERICALS

1. A horizontal Venturimeter with inlet and throat diameters 30cm and 15cm respectively is used to measure the flow of water. The reading of differential monometer connected to the inlet and the throat is 20cm of mercury. Determine the rate of flow. Take C_d= 0.98

Solution:

Given, C_d= 0.98

Dia at inlet, d_1= 30cm; $a_1 = \frac{\pi}{4} \times 30^2 = 706.85cm^2$

Dia at throat, d_2=15cm $a_2 = \frac{\pi}{4} \times 15^2 = 176.7cm^2$

Differential monometer reading, x = 20cm of mercury

Difference in pressure head. $h = x\left[\frac{S_h}{S_o} - 1\right]$

S_h= Sp. Gravity of mercury = 13.6

S_o = Sp. Gravity of water = 1

$$h = x\left[\frac{S_h}{S_o} - 1\right] = 20\left[\frac{13.6}{1} - 1\right] = 252.0\text{cm}$$

The discharge through Venturimeter, $Q_{theo} = C_d.Q_{act}$

Therefore, $Q_{act} = C_d.\frac{a_1a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$

$$= 0.98.\frac{706.85 \times 176.7}{\sqrt{706.85^2 - 176.7^2}}\sqrt{2 \times 9.81 \times 252.0}$$

$$= 125756 \text{ cm}^3/\text{sec} = 125.756\text{lt/sec}$$

2. A horizontal Venturimeter with inlet dia of 20cm and throat dia 10cm is used to measure the flow of oil of Sp. gravity 0.8.The discharge of oil through Venturimeter is 60 lt/sec. Find the reading of oil-mercury differential monometer. Take C_d= 0. 8

Solution:

Given, C_d= 0. 8

Dia at inlet, d_1= 20cm; $a_1 = \frac{\pi}{4} \times 20^2 = 314.16cm^2$

Dia at throat, d_2=10cm $a_2 = \frac{\pi}{4} \times 10^2 = 78.5cm^2$

Differential monometer reading, $x =$?

discharge of oil, Q= 60 lt/sec = 60 × 1000= cm^3/sec

We have, $Q_{act} = C_d.\frac{a_1a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$

$$60 \times 1000 = 0.8.\frac{314.16 \times 78.5}{\sqrt{314.16^2 - 78.5^2}}\sqrt{2 \times 9.81 \times h}$$

$$\sqrt{h} = 17.029; \quad h = 279.98 \text{ cm of oil}$$

Difference in pressure head. $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$279.98 = x\left[\frac{13.6}{0.8} - 1\right]$$

$$x = 17.49 \text{ cm}$$

3. A horizontal Venturimeter with inlet and throat diameters 20cm and 10cm respectively is used to measure the flow of water. The pressure at inlet is 17.658 N/cm^2 and vacuum pressure at throat is 30cm of mercury. Find the discharge of water through Venturimeter. Take C_d= 0.98

Solution:

Given, C_d= 0. 98

Dia at inlet,d_1= 20cm; $a_1 = \frac{\pi}{4} \times 20^2 = 314.16\text{cm}^2$ Dia at throat, d_2=10cm $a_2 = \frac{\pi}{4} \times 10^2 = 78.5\text{cm}^2$

pressure at inlet. p_1= 17.658 N/cm^2 = 17.658 × 10^4 = N/m^2, ρ = 1000 kg/ m^3

Pressure at @ inlet = $\frac{p_1}{\rho g}$

$$= \frac{17.658 \times 10^4}{1000 \times 9.81} = 18\text{m of water}$$

Given, Vacuum Pressure at @ throat = $\frac{p_2}{\rho g}$ = -30cm of mercury

$$= \frac{p_2}{1000 \times 9.81} = -0.3\text{m of mercury}$$

$$p_2 = -4.08 \text{ N/cm}^2$$

Differential head, h = $\frac{p_1}{\rho g} - \frac{p_2}{\rho g}$

$$= 18-(-4.08) = 22.08\text{m of water} = 2208\text{cm of water}$$

Discharge, $Q_{act} = C_d. \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

$$= 0.98. \frac{314.16 \times 78.5}{\sqrt{314.16^2 - 78.5^2}} \sqrt{2 \times 9.81 \times 2208}$$

$$= 165555 \text{ cm}^3/\text{sec}$$

$$= 165.335 \text{ lt/sec}$$

4. A Venturimeter is installed in a pipeline carrying water and is 30cm in dia. The throat dia is 12.5cm. The pressure in pipe line is 140kN/m^2 and vacuum in throat is 37.5cm of Hg. Assume 4% of differential head lost b/w the gauges. Find the flow rate in the pipe line. Assume the Venturimeter to be horizontal.

Given: C_d= 0. 98

Dia at inlet,d_1= 30cm= 0.3m; $a_1 = \frac{\pi}{4} \times 0.3^2$

Dia at throat, d_2=12.5cm=0.125m $a_2 = \frac{\pi}{4} \times 0.125^2$

pressure at inlet p_1= 140 kN/m^2 = 14×10^4 = kg/m^2

To determine, $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$

$v_1 = ?$

Q= ?

Step: 1 Pressure head @ throat = $\frac{p_2}{\rho g}$ = -37.5cm of mercury

$= \frac{-37.6 \times 13.6}{100}$ = -5.1m of water

Pressure head @ inlet = $\frac{14 \times 10^4}{1000 \times 9.81}$ = 14.27 m of mercury

Differential head, $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$

= 14.27-(-5.1) = 19.37m of water

Head lost = 4% of head = $\frac{4}{100} \times 19.37$ = 0.775m of water

Step: 2 Applying Bernoulli's equation for real fluid at inlet and throat

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + h_f$$

For horizontal Venturimeter, $Z_1 = Z_2$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{p_2}{\rho g} - \frac{p_1}{\rho g} + h_f$$

$$\frac{V_1^2}{2g}\left[1 - \frac{V_2^2}{V_1^2}\right] = -5.1 - 14.27 + 0.775 = -18.59$$

$$\frac{V_1^2}{2g}\left[1 - \frac{V_2^2}{V_1^2}\right] = -18.59 \quad \ldots\ldots\ldots\ldots (1)$$

Apply continuity equation at inlet and throat

$a_1 v_1 = a_2 v_2$

$$\frac{v_2}{v_1} = \frac{a_1}{a_2} = \frac{\frac{\pi}{4}\times d_1^2}{\frac{\pi}{4}\times d_2^2} = \frac{0.3^2}{0.125^2} = 5.76$$

Substitute value of $\frac{v_2}{v_1}$ in Eq. (1)

$$\frac{v_1^2}{2g}[1 - 5.76^2] = -18.59$$

$$v_1 = 3.367\text{m/sec}$$

Step: 3 Discharge, $Q = a_1 v_1$

$$= \frac{\pi}{4} \times 0.3^2 \times 3.367 \times 10^3 \text{ lt/sec} = 238 \text{ lt/sec}$$

Inclined Venturimeter

5. A 30cmX 15cm Venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury ,monometer connected to the inlet and throat gives a reading of 20cm. Find the discharge. Take C_d= 0.98

given C_d= 0.98

Dia at inlet,d_1= 30cm; $a_1 = \frac{\pi}{4} \times 30^2 = 706.85\text{cm}^2$

Dia at throat, d_2=15cm $a_2 = \frac{\pi}{4} \times 15^2 = 176.7\text{cm}^2$

Differential monometer reading, x = 20cm of mercury

Difference in pressure head. $h = x\left[\frac{S_h}{S_o} - 1\right]$

S_h= Sp. Gravity of mercury = 13.6

S_o = Sp. Gravity of water = 1

$$h = x\left[\frac{S_h}{S_o} - 1\right] = 20\left[\frac{13.6}{1} - 1\right] = 252.0\text{cm}$$

The discharge through Venturimeter, $Q_{act} = C_d.Q_{theo}$

Therefore, $Q_{act} = C_d.\frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$

$$= 0.98.\frac{706.85\times176.7}{\sqrt{706.85^2 - 176.7^2}}\sqrt{2 \times 9.81 \times 252.0}$$

$$= 125756 \text{ cm}^3/\text{sec} = 125.756\text{lt/sec}$$

6. A 20cmX 10cm Venturimeter is inserted in a vertical pipe carrying oil of Sp.gravity 0.8, flow of oil is in the upward direction. The difference in level b/w throat and inlet is 50cm. The oil- mercury differential monometer reading of 30cm of mercury. Find the discharge of oil. Neglect losses.

given C_d= 1 (neglecting losses)

Dia at inlet, d_1= 20cm; $a_1 = \frac{\pi}{4} \times 20^2 = 314.16 cm^2$ S_o=0.8

Dia at throat, d_2=10cm $a_2 = \frac{\pi}{4} \times 10^2 = 78.5 cm^2$ S_h= 13.6

Differential monometer reading, x = 30cm of mercury

$$h = \left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + Z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

$$= 30\left[\frac{13.6}{0.8} - 1\right] = 480\text{cm of oil}$$

Discharge, $Q_{act} = C_d. \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

$$= 1 \times \frac{314.16 \times 78.5}{\sqrt{314.16^2 - 78.5^2}} \sqrt{2 \times 9.81 \times 480}$$

= 78725.75 cm^3/sec

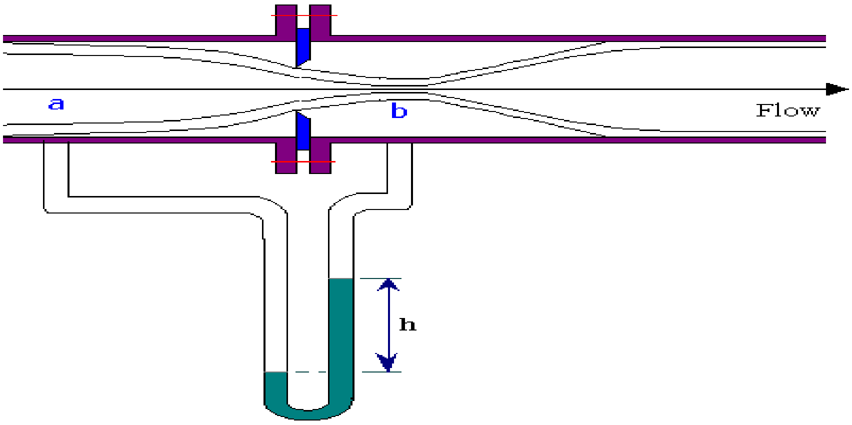
= 78.725 lt/sec

ORIFICEMETER

- An orifice plate is a device used for measuring flow rate, for reducing pressure or for restricting flow.
- An orifice plate is a thin plate with a hole in it, which is usually placed in a pipe.
- When a fluid (whether liquid or gaseous) passes through the orifice, its pressure builds up slightly upstream of the orifice. But as the fluid is forced to converge to pass through the hole, the velocity increases and the fluid pressure decreases.
- A little downstream of the orifice the flow reaches its point of maximum convergence, the *vena contracta* (see drawing to the right) where the velocity reaches its maximum and the pressure reaches its minimum. Beyond that, the flow expands, the velocity falls and the pressure increase
- By measuring the difference in fluid pressure across tappings upstream and downstream of the plate, the flow rate can be obtained from Bernoulli's equation using coefficients established from extensive research.

Principle

The principle of the orifice meter is identical with that of the venturi meter. The reduction of the cross section of the flowing stream in passing through the orifice increases the velocity head at the expense of the pressure head, and the reduction in pressure between the taps is measured by a manometer. Bernoulli's equation provides a basis for correlating the increase in velocity head with the decrease in pressure head



Discharge in orifice meter

Let d_1 = diameter at section 1

p_1 = pressure at section 1

v_1 = velocity at section 1

A_1 = area at section1

d_2, p_2, v_2, A_2 are the corresponding values at section 2.

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + Z_2\right) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Difference in head, $h = \left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + Z_2\right)$

$$h + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$v_2 = \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)} \text{ ………….. } \mathbf{(1)}$$

Now section (2) is at vena contracta and A_2 represents the area of vena contracta. If A_o is the area of orifice, then

we have , $C_c = \frac{A_2}{A_o}$

Where, C_c = Co-efficient of contraction

$$A_2 = C_c.\ A_o$$

Apply continuity equation at section (1) and (2)

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2 V_2}{A_1}$$

$$V_1 = \frac{C_c.\ A_o.\ V_2}{A_1}$$

Substitute value of V_1 in eq, (1)

$$V_2 = \sqrt{2gh + \left(\frac{C_c.\ A_o.\ V_2}{A_1}\right)^2}$$

$$V_2^2 = 2gh + \left(\frac{A_o}{A_1}\right)^2.\ C_c^2.\ V_2^2$$

$$V_2^2\ [\ 1 - \left(\frac{A_o}{A_1}\right)^2.\ C_c^2\ ] = 2gh$$

$$V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{A_o}{A_1}\right)^2.\ C_c^2}}$$

Discharge. $Q = A_2 V_2 = A_o.C_c.V_2$

$$Q = A_o C_c.\frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{A_o}{A_1}\right)^2.\ C_c^2}}$$

If C_d is the co-efficient of discharge for orifice meter, which is defined as

$$C_d = \frac{C_c\sqrt{1 - \left(\frac{A_o}{A_1}\right)^2}}{\sqrt{1 - \left(\frac{A_o}{A_1}\right)^2.\ C_c^2}}$$

$$C_c = \frac{C_d\sqrt{1 - \left(\frac{A_o}{A_1}\right)^2.\ C_c^2}}{\sqrt{1 - \left(\frac{A_o}{A_1}\right)^2}}$$

Substituting the value of C_c

$$Q = A_o C_c . \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2 . C_c{}^2}}$$

$$= A_o . C_d . \frac{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2 . C_c{}^2}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2}} . \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2 . C_c{}^2}}$$

$$Q = A_o . C_d . \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2}} \ ,$$

$$\mathbf{Q = C_d . A_o . A_1 . \frac{\sqrt{2gh}}{\sqrt{A_1{}^2 - A_0{}^2}}}$$

NUMERICALS

1. The following data relate to the orifice meter

Dia. of pipe=240mm   Dia. Of orifice= 120mm C_d= 0.65

Sp. Gravity of oil=0.88   Reading of differential monometer = 400mm of Hg

Determine the rate of flow of oil.

Solution: Given, C_d= 0.65

Dia of pipe,D_1= 240mm=0.24m; $A_1 = \frac{\pi}{4} \times 0.24^2$ = 0.0452m^2

Dia of orifice, D_0 = 120mm= 0.12m; $A_o = \frac{\pi}{4} \times 0.12^2$ = 0.0113cm^2

Sp. Gravity of oil, S_o=0.88   Sp. Gravity of mercury, S_h=13.6

Differential monometer reading, x = 400mm of mercury = 0.4m of Hg

Difference in head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 0.4\left[\frac{13.6}{0.88} - 1\right] = 5.78\text{m of oil}$$

$$\mathbf{Q = C_d . A_o . A_1 . \frac{\sqrt{2gh}}{\sqrt{A_1{}^2 - A_2{}^2}}}$$

$$= 0.65 \times 0.0113 \times 0.0452 \times \frac{\sqrt{2\times 9.81 \times 5.78}}{\sqrt{0.0452^2 - 0.0113^2}}$$

$$= 0.08 \ m^3/sec$$

2. Water flows at the rate of $0.015m^3/sec$, through a 100m dia orifice used in 200mm pipe. What is the difference of pressure head b/w the upstream section and the vena contracta section. Take co-efficient of contraction C_c= 0.6 and C_v= 1.0

Solution:

Given Rate of flow, Q= $0.015m^3/sec$

Dia of pipe,D_1= 200mm=0.20m; $A_1 = \frac{\pi}{4} \times 0.20^2 = 0.03142m^2$

Dia of orifice, D_0 = 100mm= 0.01m; $A_o = \frac{\pi}{4} \times 0.01^2 = 0.007854cm^2$

C_c= 0.6 C_v= 1.0

Co-efficient of discharge, $C_d = C_c . C_v = 0.6 \times 1 = 0.6$

$$\mathbf{Q = C_d . A_o . A_1 . \frac{\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

$$0.015 = 0.6 \times 0.007854 \times 0.03142 \times \frac{\sqrt{2 \times 9.81 \times h}}{\sqrt{0.03142^2 - 0.007854^2}}$$

h = 0.484 m of water

3. An orifice meter with orifice dia 10cm is inserted in a pipe of 20cm dia. The pressure gauge fitted upstream and downstream of orifice meter gives reading of 19.62 N/cm^2 and 9.81 N/cm^2 respectively. C0-efficient of discharge is given as 0.6. Find the discharge of water through pipe.

Solution: Given C_d= 0.6

Dia of pipe,D_1= 20cm=0.24m; $A_1 = \frac{\pi}{4} \times 0.20^2 = 314.16cm^2$

Dia of orifice, D_0 = 10cm= 0.10m; $A_o = \frac{\pi}{4} \times 0.10^2 = 78.54cm^2$

Pressure at upstream end, $p_1 = 19.62 \ N/cm^2 = 19.62 \times 10^4 \ N/m^2$

Pressure at downstream end, $p_2 = 9.81 \ N/cm^2 = 9.81 \times 10^4 \ N/m^2$

Difference in head, $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$

$$= \frac{19.62 \times 10^4}{\mathbf{1000 \times 9.81}} - \frac{9.81 \times 10^4}{\mathbf{1000 \times 9.81}} = 20\text{-}10 = 10\text{m of water}$$

= 1000cm of water

Discharge, $\mathbf{Q = C_d . A_o . A_1 . \frac{\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$

$$= 0.6 \times 78.54 \times 314.16 \times \frac{\sqrt{2\times9.81\times1000}}{\sqrt{314.16^2 - 78.54^2}}$$

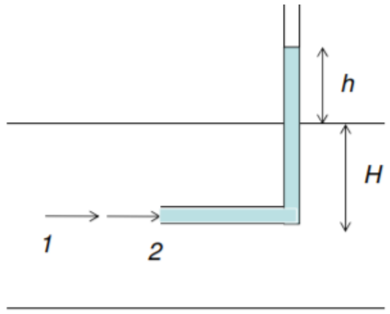
$$= 68213.28 \text{ cm}^3/\text{sec}$$

$$= 68.21 \text{ lt/sec}$$

PITOT TUBE

- Pitot tube is a device used for measuring the velocity of flow at any point in a pipe or a channel.
- Principle: If the velocity at any point decreases, the pressure at that point increases due to the conservation of the kinetic energy into pressure energy.

In simplest form, the pitot tube consists of a glass tube, bent at right angles



Point 2 is just at the inlet of the Pitot-tube
Point 1 is far away from the tube

Let, p_1 = pressure at section 1

P_2 = pressure at section 2

v_1 = velocity at section 1

v_2 = velocity at section 2

H = depth of tube in the liquid

h = rise of liquid in the tube above free surface

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

But $Z_1 = Z_2$, and $v_2 = 0$

$$\frac{p_1}{\rho g} = \text{pressure heat at 1} = H$$

$$\frac{p_2}{\rho g} = \text{pressure heat at 2} = h + H$$

Substituting the values, we get

$$H + \frac{V_1^2}{2g} = h + H$$

$$v_1 = \sqrt{2gh}$$

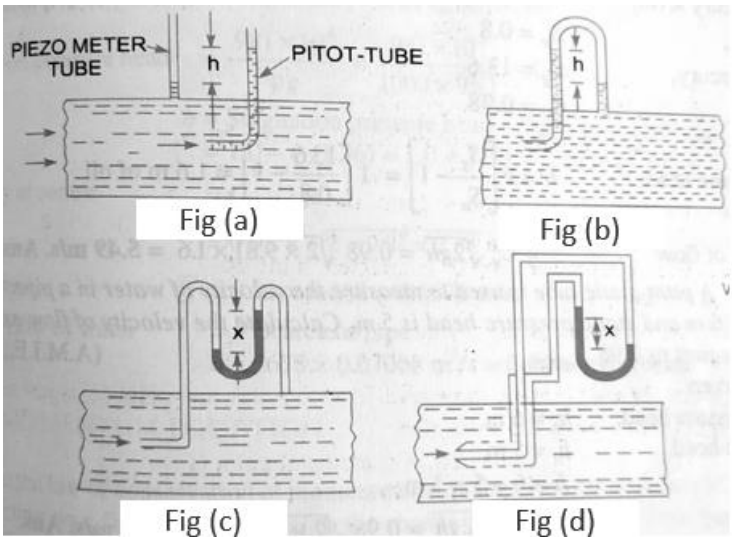
This is theoretical velocity, actual velocity is given by,

$$(v_1)_{act} = C_v\sqrt{2gh}$$

Where, C_v = co-efficient of pitot tube

Velocity of flow in a pipe by pitot tube:

The following arrangements are adopted to find velocity at any point in a pipe using Pitot tube.



1. Pitot tube along with vertical piezometer tube (fig. a)
2. Pitot tube connected with piezometer tube (fig. b)
3. Pitot tube and vertical piezometer tube connected with differential U-tube monometer (fig. c)
4. Pitot static tube, consists of two circular concentric tubes one inside the other with some annular space between (Fig.d)

   The out let of these two is connected to the differential monometer, where difference in pressure head is measured as, $h = x\left[\frac{S_h}{S_o} - 1\right]$