Fluid Mechanics and Hydraulics

21CV42

MODULE -1

Fluids and its Measurements

Introduction

Mechanics is the oldest physical science that deals with both stationary and moving bodies under the influence of forces.

The branch of mechanics that deals with bodies at rest is called Statics, while the branch that deals with bodies in motion is called Dynamics.

The subcategory fluid mechanics is defined as the science that deals with the behaviour of fluids at rest (*fluid statics)* or in motion (*fluid dynamics), and the interaction of fluids with solids* or other fluids at the boundaries.

The study of fluids at rest is called **Fluid statics**.

The study of f1uids in motion, where pressure forces are not considered, is called **Fluid kinematics** and if the pressure forces are also considered for the fluids in motion. that branch of science is called **Fluid Dynamics**.

Fluid mechanics itself is also divided into several categories.

- The study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **Hydrodynamics**.
- A subcategory of hydrodynamics is **Hydraulics**, which deals with liquid flows in pipes and open channels.

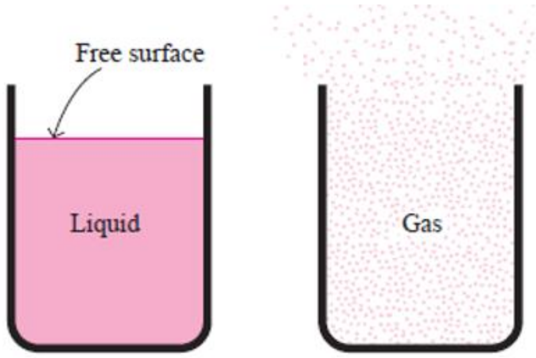
Fluid

A substance exists in three primary phases viz solid, liquid and gas.

A substance in the liquid or gas phase is referred to as a fluid or a substance which is capable to flow is termed as fluid

- In a **liquid**, molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules. As a result, a liquid takes the shape of the container it is in, and it forms a free surface in a larger container in a gravitational field.

- A **gas**, on the other hand, expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. Unlike liquids, gases cannot form a free surface



<u>Differences between liquid and gases</u>

Liquid	Gases
Difficult to compress and often regarded as incompressible	Easily to compress – changes of volume is large, cannot normally be neglected and are related to temperature
Occupies a fixed volume and will take the shape of the container	No fixed volume, it changes volume to expand to fill the containing vessels
A free surface is formed if the volume of container is greater than the liquid.	Completely fill the vessel so that no free surface is formed.

PROPERTIES OF FLUIDS

- Any characteristic of a system is called a **property.**
- Some familiar properties are pressure P, temperature T, volume V, and mass m.
- Other less familiar properties include viscosity, thermal expansion, coefficient, electric resistivity, and even velocity and elevation.
- Properties are considered to be either ***intensive or extensive***.
- ***Intensive*** properties are those that are independent of the mass

- of a system, such as temperature, pressure, and density.
- **Extensive** properties are those whose values depend on the size—or extent—of the system. Total mass, total volume *V*, and total momentum are some examples of extensive properties.

1. **Density or mass density (ρ)**

   Density or mass density of a fluid is defined as the ratio of the mass of a f1uid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho).

   The unit of mass density in SI unit is kg per cubic meter,i.e ., kg/m^3.

   Mathematically mass density is written as

$$\rho = \frac{\text{Mass of the fluid}}{\text{Volume of the fluid}}$$

   Unit of the mass density is kg/m^3

   Density of water $\rho_{water} = 1000\ kg/m^3$ or $1\ gm/cm^3$

2. **Specific Volume**

   Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

   Mathematically, it is expressed as,

$$Specific\ volume = \frac{\text{Volume of the fluid}}{\text{Mass of the fluid}}$$

$$= \frac{1}{\frac{\text{Mass of the fluid}}{\text{Volume of the fluid}}} = \frac{1}{\rho}$$

   Thus, specific volume is the reciprocal of mass density and expressed as m^3/kg.

3. **Specific weight**

   Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.

   It is represented by symbol *w*

Mathematically. $w = \frac{\text{Weight of the fluid}}{\text{Volume of the fluid}}$

$$= \frac{\text{Mass of fluid x Acceleration due to gravity}}{\text{Volume of the fluid}}$$

$$= \frac{\text{Mass of fluid x g}}{\text{Volume of the fluid}}$$

$$w = \rho \times g$$

Unit of the mass density is N/m^3

Density of water, $w_{water} = 1000 \times 9.81 = 9810\ N/m^3$

4. **Specific gravity**

Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid.

For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol *S*.

$$S(\text{for liquids}) = \frac{\text{Density of the liquid}}{\text{Density of the standard fluid (Water)}}$$

$$S(\text{for gases}) = \frac{\text{Density of the gas}}{\text{Density of the standard fluid (air)}}$$

Thus, density of a liquid = S x density of water

$= S \times 1000\ kg/m^3$

- If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water.
- For example, the specific gravity of mercury is 13.6,
density of mercury= 13.6 x 1000 = 13600 kg/m^3.

NUMARICAL

1. Calculate the specific weight, density and specific gravity of 1litre of liquid which weighs 7N.

Solution:

Given,

Volume of the liquid = 1lt = $\frac{1}{1000}$ m^3

Weight of the liquid = 7 N

Density ρ =? Specific weight, w =? Specific gravity, S =?

a) Specific weight

$$w = \frac{\text{Weight of the substance}}{\text{Volume of the substance}}$$

$$w = \frac{7}{\frac{1}{1000}} = 7000 \text{ kg/m}^3$$

b) Density

We have. Specific weight, $w = \rho \times g$

$$\rho = \frac{w}{g}$$

$$= \frac{7000}{9.81} = 713.5 \text{ N/m}^3$$

c) Specific gravity

$$S = \frac{\text{Density of the liquid}}{\text{Density of the standard liquid (Water)}}$$

$$S = \frac{713.5}{1000} = 0.7135$$

2.Calculate the density, specific weight and weight of 1lt of petrol having specific gravity 0.7.

Solution:

Given,

Volume of the petrol = 1 lt = 10^{-3} m^3

Specific gravity = S =0.7

Weight of the liquid =? Density ρ =? Specific weight ω =?

a) Density ρ

$$S = \frac{\text{Density of the petrol}}{\text{Density of the standard liquid(Water)}}$$

$$0.7 = \frac{\rho_{petrol}}{\rho_{water}}$$

$$0.7 = \frac{\rho_{petrol}}{1000}$$

$$\rho_{petrol} = 0.7x1000 = 700 \text{ kg/m}^3$$

b) Specific weight, w

$$w = \rho g = 700 \text{ x } 9.81 = 6867 \text{ N/m}^3$$

c) Weight, W

$$w = \frac{\text{Weight of the liquid}}{\text{Volume of the liquid}}$$

$$W = w \text{ x Volume of the substance}$$

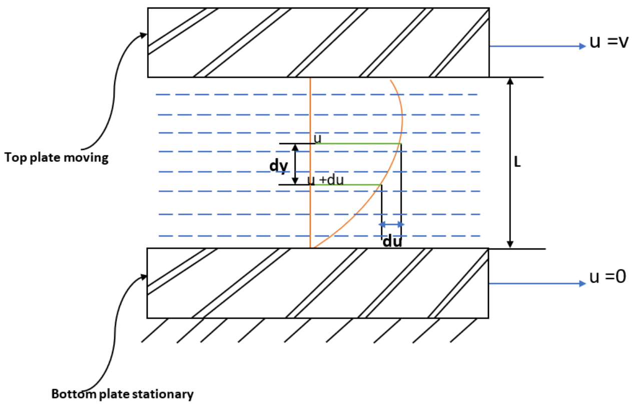
$$= 6867 \text{ x } 10^{-3} = 0.6867\ N$$

5. Viscosity

Viscosity is defined as the property of the fluid which offers the resistance of the movement of one layers of the fluid over another adjacent layer.

Consider a fluid filled between two plates separated by a distance 'L' as shown in above figure. Consider two layers of fluid separated by a distance **dy** moving with the velocity u and **u+du**. The bottom plate is stationary and top plate is moving with a velocity **v**, parallel to the bottom plate. The top layer causes shear stress on the adjacent bottom layer, while the lower layer causes stress on the adjacent top layer. This shear stress is proportional to rate of change of velocity wit

It is denoted by symbol, τ (tau)



Mathematically, $\tau \propto \frac{du}{dy}$

$$\tau = \mu \frac{du}{dy}$$

$$\mu = \frac{\tau}{\frac{du}{dy}}$$

where, τ = Shear stress

$\frac{du}{dy}$ = rate of change of shear strain or velocity gradient

μ = constant of proportionality, known as co-efficient of Dynamic Viscosity

Viscosity is also defined as shear stress requires to produce unit rate of shear strain.

The standard unit of viscosity is Poise

$$1 \text{ Poise} = \frac{1}{10} \text{ N-s/m}^2$$

6. **Kinematic Viscosity**

It is defined as the ratio of the dynamic viscosity to the density of the fluid.

It is denoted by symbol $\boldsymbol{\upsilon}$ (nu)

$$\text{Kinematic Viscosity} = \frac{\text{Dynamic viscosity}}{\text{Density}}$$

$$\upsilon = \frac{\mu}{\rho}$$

The standard unit of kinematic viscosity is stoke

$$1 \text{ stoke} = 1 \text{ cm}^2/\text{sec}$$

$$1 \text{ stoke} = \frac{1}{10^4} \text{ m}^2/\text{sec}$$

Newton's Law of Viscosity

It states that "the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain". The constant of proportionality is called the co-efficient viscosity.

Mathematically, it is expressed as given by equation, $\tau \propto \frac{du}{dy}$

Types of Fluids

The fluids may be classified as:

a) Ideal Fluid
b) Real Fluid
c) Newtonian Fluid
d) Non-Newtonian Fluid
e) Ideal Plastic Fluid

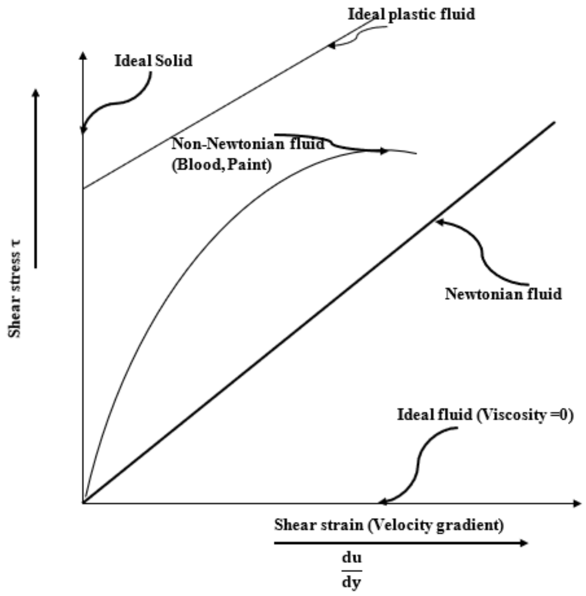
Ideal Fluid - A fluid having zero viscosity is known as ideal fluid. Ideal fluid is an imaginary fluid.

Real Fluid - A fluid which possess viscosity. All fluids are real fluid

Newtonian Fluid A fluid in which shear stress is directly proportional to the rate of change of shear strain Example: Water, Mercury, air etc

Non-Newtonian Fluid A fluid in which shear stress is not directly proportional to shear strain. Example Blood, paint, tooth paste

Ideal Plastic Fluid It is a fluid having shear stress more than yield value and shear stress is directly proportional to rate of change of shear strain



NUMARICAL

1. If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is velocity in meter per second at a distance y meter above the plate, determine the shear stress at y = 0 and y= 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.

Solution: given viscosity, μ = 8.63 poises = 8.63/10 =0.863 Ns/m^2

$$u = \frac{2}{3}y - y^2 \text{ (differentiate w.r.t 'y')}$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

at y = 0 , $\frac{du}{dy} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$

at y = 0.15 , $\frac{du}{dy} = \frac{2}{3} - 2(0.15) = \frac{2}{3} = 0.367$

$$\text{Shear stress, } \tau = \mu \frac{du}{dy}$$

$$\tau_0 = 0.863 * 0.667 = 0.5756 \text{ N/m}^2$$

$$\tau_{0.15} = 0.863 * 0.367 = 0.3167 \text{ N/m}^2$$

2. A Plate at a distance 0.0254mm from a fixed plate moves at 0.61m/s and requires a force of $1.962 N/m^2$ area of plate. Determine dynamic viscosity of liquid between the plates.

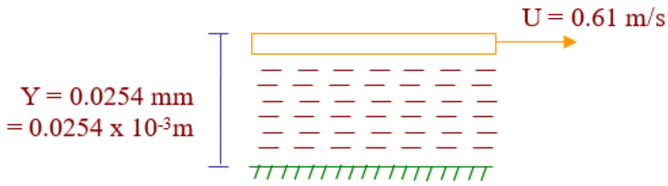
Solution:

Given, shear stress $\tau = 1.962 N/m^2$

velocity $u = 0.61 m/s$

distance $y = 0.0254mm = 0.0254$:

dynamic viscosity $\mu =$ __________



We have

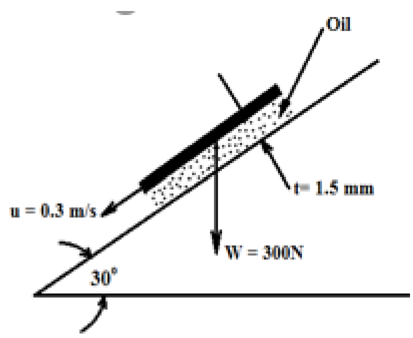
$$\text{shear stress, } \tau = \mu \frac{du}{dy}$$

$$1.962 = \mu \frac{0.61}{0.0254 \times 10\text{-}3}$$

$$\mu = 8.17 \times 10^{-5} Ns/m^2$$

$$= 8.17 \times 10^{-5} \times 10 = 8.17 \times 10^{-4} \text{ poise}$$

3. Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.



Solution:

Given area of plate $= 0.8 \times 0.8 = 0.64 m^2$

velocity $u = 0.30 m/s$

$\theta = 30^o$

Weight $W = 300N$

distance $dy = 1.5mm = 1.5 \times 10^{-3} m$

Component of weight W, along inclined plane = Wcos (90-θ) = 300 cos60 = 150N

$$\text{Shear stress} = \frac{force}{area} = \frac{150}{0.64}$$

We have, shear stress $\tau = \mu \frac{du}{dy}$

$$\frac{150}{0.64} = \mu \frac{0.30}{1.5 \times 10\text{-}3}$$

$$\mu = 1.17 Ns/m^2 = 11.7 \text{poise}$$

4. A plate of area 1.5×10^6 mm^2 is pulled with a speed of 0.4 m/s, relative to another plate located at a distance of 0.15mm from it. Find the force located at a distance of 0.15mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity of 1 poise

Solution: area of plate = 1.5×10^6 mm^2 = 1.5 m^2

velocity u = 0.40 m/s

distance dy = 0.15mm = 0.15×10^{-3} m

viscosity μ = 1 poise = 1/10 Ns/ m^2

We have, shear stress, $\tau = \mu \frac{du}{dy}$

$= 0.1 \frac{0.4}{0.15\times 10\text{-}3}$

= 266.66 N/m^2

Shear force, F = τ x area

= 266.66 x 1.5

= 400N

Power required = F x u

= 400 x 0.4

= 160W

5. The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 meter per sec requires a force of 98.1 N to maintain the speed.

Determine :

i. the dynamic viscosity of the oil, and
ii. the kinematic viscosity of the oil if the specific gravity of the oil is 0.95.

Solution. Given:

Each side of a square plate = 60 cm = 0.6 m

Area, A= 0.6 x 0.6 = 0.36 m^2

Thickness of oil film, dy = 12.5 mm = 12.5×10^{-3} m

Velocity of upper plate, du = 2.5 m/s

Force required, F = 98.1 N

i) dynamic viscosity of the oil

Shear stress = $\frac{force}{area} = \frac{98.1}{0.36}$

shear stress $\tau = \mu \frac{du}{dy}$

$$\frac{98.1}{0.36} = \mu \frac{2.5}{12.5 \text{ x } 10\text{-}3}$$

$$\mu = 1.363 \text{ Ns/ m}^2 = 13.63 \text{ poise}$$

ii) kinematic viscosity of the oil (ν)

$$\nu = \frac{\mu}{r}$$

Density of oil, r = S x 1000 = 0.95 x 1000 = 950 kg/m^2

$$\nu = \frac{1.3635}{950} = 0.001435 \text{ m}^2\text{/sec}$$

$$= 0.001435 \text{ x } 10^4 \text{ cm}^2\text{/sec} = 14.35\text{stokes}$$

6. Determine the intensity of shear of an oil having a viscosity of 1 poise. The oil is used for lubricating the clearance b/w a shaft of dia 10cm and bearing. The clearance is 1.5mm and shaft rotates at 150 rpm

Solution: viscosity μ = 1 poise = 1/10 = 0.1 Ns/ m^2

clearance, dy = 1.5 mm = 1.5 x 10^{-3} m

shaft speed , N = 150rpm

Dia of shaft , d = 10cm =0.1m

Tangential velocity of shaft, u = $\frac{\pi * d * N}{60} = \frac{\pi * 0.1 * 150}{60}$

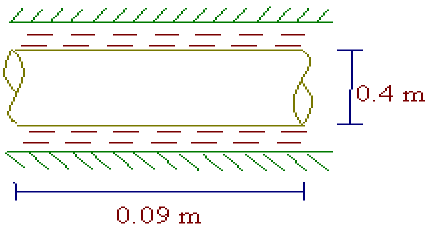
$$= 0.785 \text{ m/s}$$

Shear stress, $\tau = \mu \frac{du}{dy} = 0.1 \frac{0.785}{1.5 \text{ x } 10\text{-}3}$

$$= 52.33 \text{ N/m}^2$$

7. Dynamic viscosity of oil used for lubrication between a shaft and a sleeve is 6 Poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 0.09 m. Thickness of oil is 1.5mm.

Solution:



Given

dynamic viscosity μ = 6 poise = 6/10 = 0.6 Ns/ m^2

diameter, d = 0.4m

Length $L = 0.09m$

distance $dy = 1.5mm = 1.5\times 10^{-3}$ m

$N = 190$ rpm

shear stress τ = ________ Velocity of shaft, u = ________

Tangential velocity of shaft, $u = \frac{\pi dN}{60} = \frac{\pi \text{ x } 0.4 \text{ x } 160}{60} = 3.98$ m/s

Shear force, F= shear stress *area = $\mu \frac{du}{dy} \text{ x } \pi \text{ x } d \text{ x } L$

$= 0.6 \frac{3.98}{1.5\times 10\text{-}3} \text{ x } \pi \text{ x } 0.4 \text{ x } 0.09 = 180.05N$

Torque T= F x d/2

= 36.01 N-m

Power lost $= \frac{2\pi NT}{60} = \frac{2\pi \text{ x } 160 \text{ x } 36.01}{60}$

= 716.48 W

8. Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes

Solution:

Given, viscosity μ = 0.05 poise = 0.05/10 = 0.005 Ns/ m^2

Kinematic viscosity (ν) = 0.035 stokes = 0.035 x 10^{-4} m^2/s

$$\text{S(for liquids)} = \frac{\text{Density of the liquid}}{\text{Density of the standard fluid (Water)}}$$

kinematic viscosity , $\nu = \frac{\mu}{r}$

$0.035 \text{ x } 10^{-4} = \frac{0.005}{r}$

Density of fluid, $r = 1428.5$ Kg/ m^3

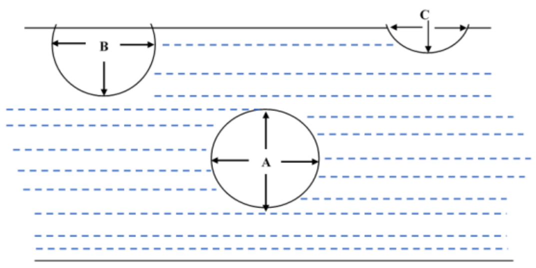
Specific gravity of fluid = $\frac{1428.5}{1000} = 1.428$

Surface Tension

A tensile force may be considered to be acting in the plane of the surface along any line in the surface. The intensity of the molecular attraction per unit length along any line in the surface is called the **Surface tension.**

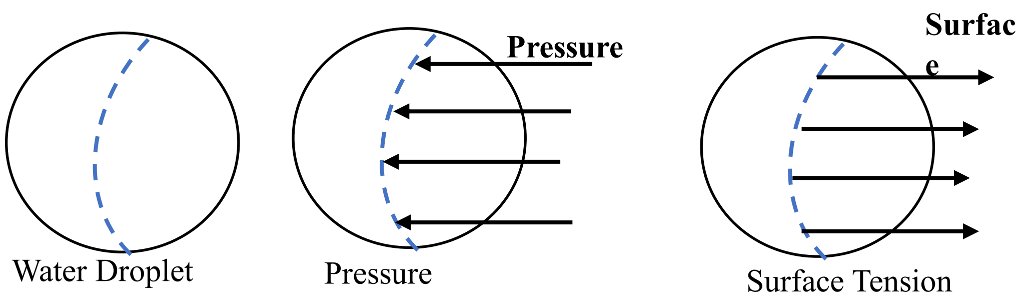
It is denoted by Greek letter σ (sigma).

The SI unit is N/m.



Consider three molecules A,B and C as shown in above figure. The molecule A subjected to internal and external force. The net resultant force acting on the molecule A is centre. The molecule B and C having the resultant force in the downward direction. But molecule B and C acting at the free surface due to surface tension force.

(i) Surface Tension of water droplet



Consider a small spherical droplet of a liquid of radius ‘r'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

d = diameter of droplet.

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

(i) tensile force due to surface tension acting around the circumference of the cut portion as shown and this is equal to = σ x Circumference

$= \sigma \text{ x } \pi d$

ii) pressure force on the area $(\pi/4)d^2$ and

$= p \text{ x } \frac{\pi}{4} d^2$

Water molecule is in equilibrium, that is equation (1) is equal to (2)

$p \text{ x } \frac{\pi}{4} d^2 = \sigma \text{ x } \pi d$

$$\mathbf{p = \frac{4\sigma L}{D}}$$

(ii) Surface Tension on hollow bubble

A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected surface tension.

In such case, $p \text{ x } \frac{\pi}{4} d^2 = 2(\sigma \text{ x } \pi d)$

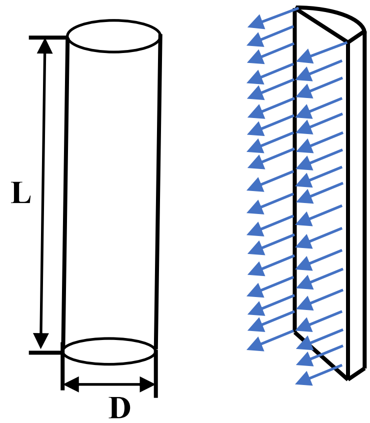
$$\mathbf{p = \frac{8\sigma}{d}}$$

(iii) Surface Tension on a liquid jet

Consider liquid jet od diameter ‘d’ and length ‘L’

Let σ = surface tension of the liquid

P = Pressure intensity inside the droplet



Force due to pressure = p x area of jet

$= p \text{ x } L \text{ x } d$

Force due to tension = $\sigma \text{ x } 2L$

Therefore, $\mathbf{p = \frac{2\sigma}{d}}$

NUMARICAL

1. Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Solution:

given, dia of bubble, d =40mm = 40 x 10^{-3} m

pressure outside the droplet, p = 2.5 N/m^2

For soap bubble (hollow bubble) $p = \frac{8\sigma}{d}$

$$\text{Surface Tension}, \sigma = \frac{8 \times 2.5}{40 \times 10^{-3}} = 500 \text{ N/ m}^2$$

2. The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Given: dia of bubble, d = 0.04 mm = 0.04x 10^{-3}

pressure out side the droplet = 10.32 N/cm^2 = 10.32 x 10^4 N/m^2

Surface Tension , σ = 0.0725 N/ m^2

Pressure inside the droplet in excess of outside pressure,

$$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = 0.725 \text{ N/cm}^2$$

Pressure inside the droplet = p + pressure outside the droplet

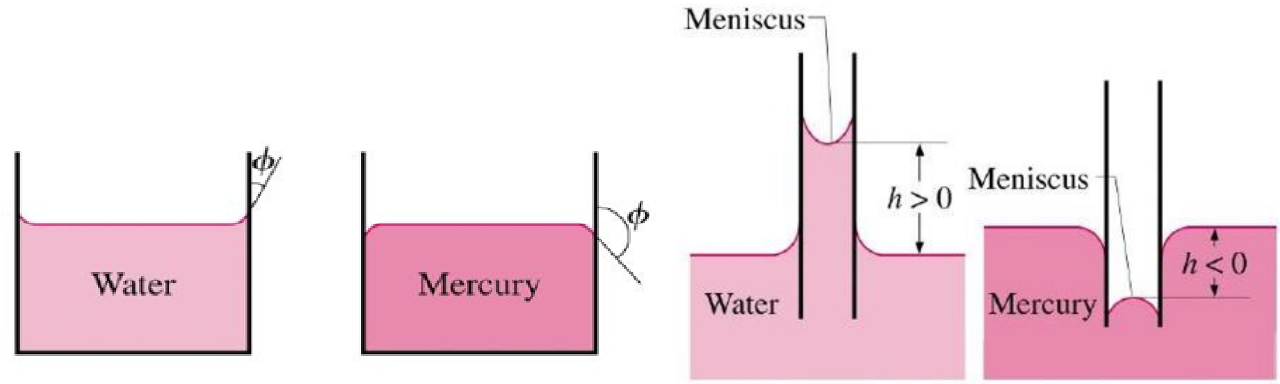
= 0.7250+ 10.32 N/cm^2

Capillarity

It is the phenomenon of rise or fall of the liquid in a glass tube with respect to adjacent free surface of the liquid, when the glass tube held in vertical directions.

If there is a rise of level of water with respect to adjacent free surface in a glass tube then it is known as **capillary rise**. If there is a fall of level of water with respect to adjacent free surface in a glass tube then it is known as **capillary fall**.

The attraction (adhesion) between the wall of the tube and liquid molecules is strong enough to overcome the mutual attraction (cohesion) of the molecules and pull them up the wall. Hence, the liquid is said to *wet the solid surface.*



Wetting fluid Non wetting fluid Capillary rise Capillary depression

Fig: Capillarity

It is expressed in terms of **cm or mm** of liquid.

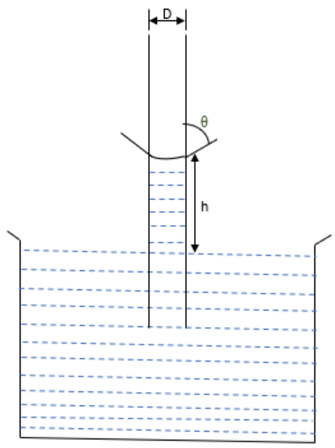
Capillarity depends upon:

a. Diameter of the glass tube

b. Specific weight of the liquid in the glass tube

c. Height of the liquid in the glass tube

Expression of Capillary rise



Consider a glass tube having a diameter ‘D’ placed in a beaker as shown in a above figure.

Let θ be the angle made between glass tube and free surface of the water. h be the height of water in the glass tube and σ is the surface tension of the water in the glass tube.

The weight of the liquid of height h in the tube

= (Area of the tube x h) x ρ x g

$= \frac{\pi}{4} d^2 \times h \times \rho \times g$ ………. (1)

Where, ρ = density of liquid

Vertical component of the surface tensile force

$= (\sigma \times circumference) \times cos\,\theta = \sigma \times \pi\, d \times cos\,\theta$ ……. (2)

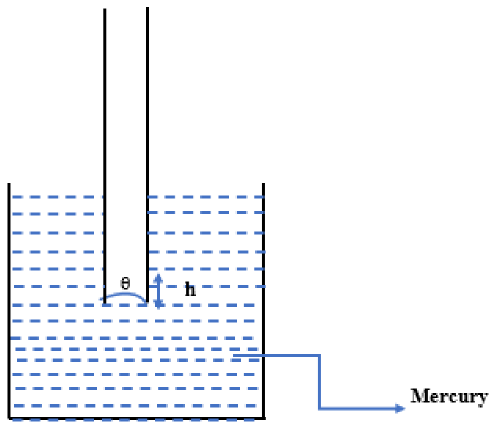
For equilibrium equate (1) and (2)

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi\, d \times cos\,\theta$$

$$h = \frac{\sigma \times \pi\, d \times cos\,\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma cos\,\theta}{\rho \times g \times d}$$

For water θ =0^0, $\boldsymbol{h} = \frac{\mathbf{4\sigma}}{\mathbf{\rho g D}}$

Capillary Fall



Where, θ for mercury = 128^0-130^0

$$\boldsymbol{h} = \frac{\mathbf{4\sigma cos(\theta)}}{\mathbf{\rho g D}}$$

NUMARICAL

1. Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = $130^{o.}$

Solution: Dia of tube, d=2.5mm = 2.5 x 10^{-3}m

Surface tension(σ), for water = 0.0725 N/m

For mercury = 0.52N/m

S_g= 13.6 θ = 130°

Density of mercury, $\rho_g = S_g \times 1000$ = 13.6 x 1000= 1360 kg/m^3

(i) Capillary rise in water, $h = \frac{4\sigma}{\rho \times g \times d}$

$$= \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \text{ x } 10^{-3}} = 0.0118\text{m} = 1.18\text{cm}$$

(ii) Capillary rise in mercury, $h = \frac{4\sigma\cos\theta}{\rho \times g \times d}$

$$= \frac{4 \times 0.52 \times \cos 130}{1360 \times 9.81 \times 2.5 \text{ x } 10^{-3}} = -0.4\text{m}$$

Negative sign indicates capillary depression

2. Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Solution: Surface tension(σ), for water = 0.073575 N/m

capillary rise h= 2mm = 0.002m

Density of water, ρ = 1000 kg/m^3

Dia of tube, d = ________

Capillary rise in water, $h = \frac{4\sigma}{\rho \times g \times d} = 0.002$

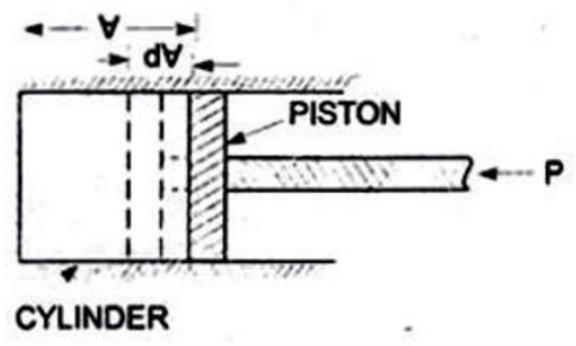
$$d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 0.002}$$

= 0.015m = 1.5cm

Compressibility and Bulk modulus

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of the compressive stress to the volumetric strain.

Consider a cylinder fitted with a piston as shown in fig:



Let V= volume of gas enclosed in the cylinder

p = pressure of gas, when volume is V

Let the pressure is increased to p+dp, the volume of gas decreased from V to V-dV.

Thus, increase in pressure = dp kg/m^2

Decrease in volume = dv

Therefore, Volumetric strain = $\frac{-\,dv}{v}$

Negative sign indicates, the decrease in volume with increase in pressure.

Bulk modulus, K = $\frac{\textit{Increase in pressure}}{\textbf{volumetric strain}}$

$$= \frac{dp}{\frac{-\,dv}{v}} = \frac{-\,dp}{dv} \cdot v$$

Compressibility is given by $\frac{1}{K}$

Fluid Continuum

A continuous and homogenous medium is called as Continuum

FLUID PRESSURE AND MEASUREMENT

Vapour pressure: It is the phenomenon in which liquid state is converted in to gaseous state is known as vaporization. It occurs due to escaping of the molecules into air. The pressure created to the liquid state to gaseous state is known as vapour pressure.

Pressure at a point

Consider an element having an area dA subjected to a force dF , then the ratio of force to the corresponding area of the element is known as pressure.

$$dp = \frac{dF}{dA}$$

If the force (F) is uniformly distributed over the area (A) , then pressure at any point is given by

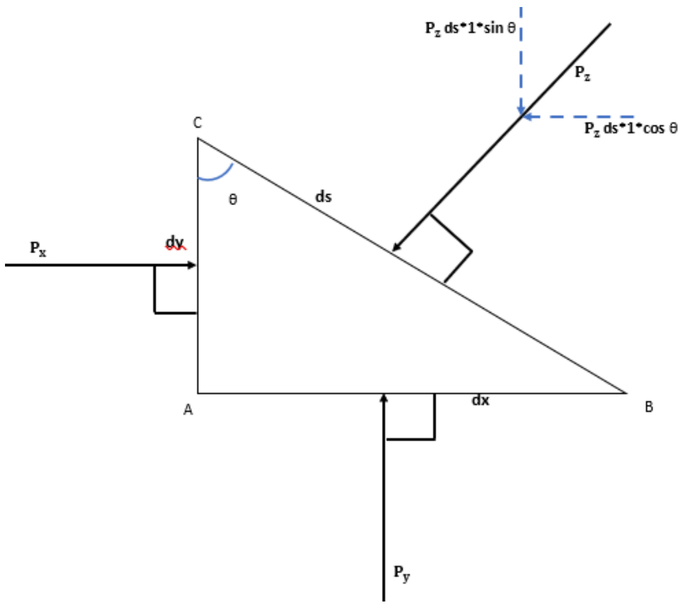
$$P = \frac{Force}{Area}$$

The unit of pressure is N/m^2 , standard unit is Pascal

1Pa = 1 N/m^2 , 1kPa = 1000 N/m^2, 1bar Pa = 10^5 N/m^2

PASCAL LAW

It states that "If a fluid is in state of static or rest, the pressure acting on the element in the fluid in all direction remains same"



Consider a triangular element of a fluid (in rest) at static condition, the triangle ABC as shown in above figure.

Let The triangular element having unit width. The sides of the triangular element are AB = dx, AC= dy and BC = ds.

p_x = Intensity of horizontal pressure on the liquid element

p_y = Intensity of vertical pressure on the liquid element

p_s = Intensity of pressure on the diagonal right angled triangular element

Consider a small element of fluid, vertical column of constant cross sectional area dA, and totally surrounded by fluid of mass density ρ.

Let p_x, p_y and p_s be the intensity of pressure intensity acting on AB, Ac and BC respectively

Let < ABC = θ, then the forces acting on elements are

1. Pressure force normal to the surface
2. Weight of the element in vertical direction

The forces on the faces are:

Force on the face AB = $p_x \times$ area of face AB

$= p_x \times dy \times 1$

Similarly, Force on the face AC = $p_y \times dx \times 1$

Force on the face BC = $p_z \times ds \times 1$

Weight of element = Mass of element × g

$$= (\text{Volume} \times \rho) \times g = (\frac{1}{2} \times AB \times AC \times 1) \times \rho\, g$$

Where ρ= density of liquid

Resolving the forces in x-direction,

$p_{x,}.dy . 1 - p_{s,}.ds . 1. \cos\theta = 0$

$p_{x,}.dy . 1 = p_{s,}.ds . 1. \cos\theta$

From fig $\cos\theta = (AB/Ac) = (\frac{dy}{ds})$

$dy = ds.\cos\theta$

$p_{x,}.dy . 1 = p_{s,}.dy$

$p_x = p_s$ …………………….. (1)

Similarly resolving the forces in Y direction

$p_y .dx .1 - p_{z,}.ds . 1. \sin\theta - \frac{dy.dx}{2}. 1.\rho g = 0$

$\sin\theta = \frac{dx}{ds}$. $\quad dx = ds. \sin\theta$

The element is very small and hence weight is negligible

$p_y .dx - p_y .dx = 0$

$p_y = p_z$ ………………….. (2)

From (1) and (2) $\quad \mathbf{p_x = p_y = p_z}$

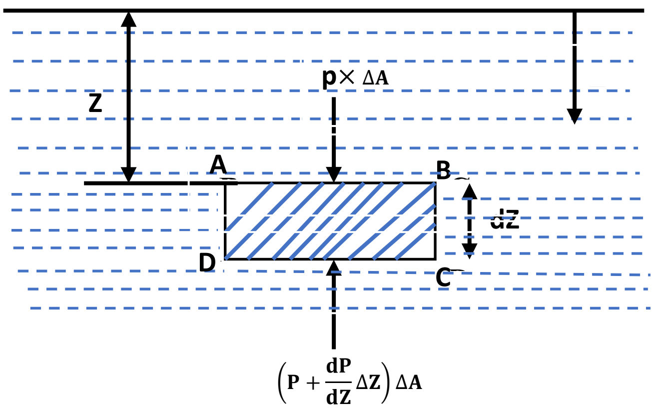
The above equations shows that the pressure at any point in x, y and z directions is equal

Pressure variation in static fluid (Hydrostatic Law)

The rate of increase of pressure in vertical downward direction when the fluid is in static condition gives the hydrostatic law.

Hydrostatic law states that "the rate of increase of pressure of an element in a fluid at static is equal to specific weight of the fluid at that point"

Consider a small element of fluid, vertical column of constant cross sectional area dA, and totally surrounded by fluid of mass density p



Let,

p_1 = Pressure on face AB

dA = Cross section area of the element

Z = distance of fluid element from surface

dZ = Height of fluid element

p_2 = Pressure on face DC

The forces acting on the fluid element are

- Pressure force on face AB = $p1 \times dA$ (downward direction)
- Pressure force on face CD = $p_2 \times dA$ (upward direction)
- Weight of the fluid element = weight density x volume

  =(density x g) x volume

  = $\rho\, g \times (dA \times dZ)$
- Pressure force on surface AC and BD are equal and opposite

Fluid is at rest the element must be in equilibrium and the sum of all vertical forces must be zero

$$p_1\, dA - p_2\, dA - W = 0$$

$$p_1\, dA - p_2\, dA - \rho\, g\, (dA \times dZ) = 0$$

$$(p_1 - p_2) - \rho\, g\, dZ = 0$$

$$dP = \rho\, g\, dZ$$

$$\boldsymbol{\frac{dp}{dz} = \rho\, g = W}$$

Where w = Weight density of fluid

Above equation states that "rate of increase of pressure in a vertical direction is equal to weight density of the fluid at the point" and this known as Hydrostatic law.

On Integrating above equation, we get

$$\int dp = \int \rho\, g\, dz$$

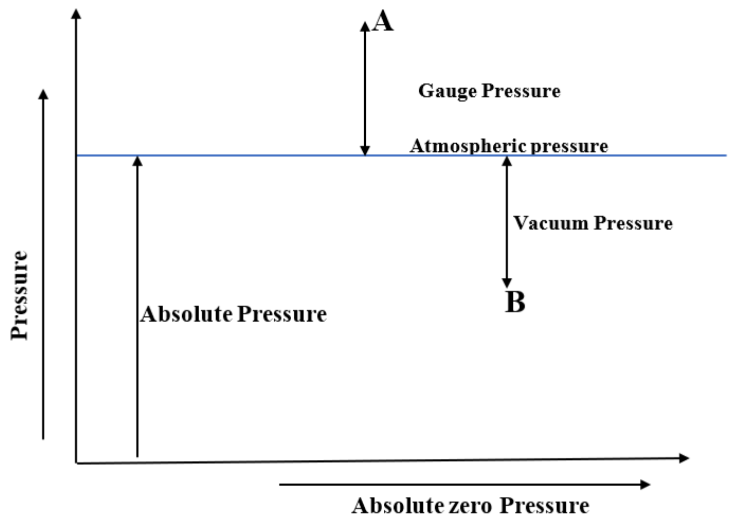
$$p = \rho\, g\, z \ \ldots\ldots\ldots\ldots.. (1)$$

Where p = pressure above the atmospheric pressure

$$z = \frac{p}{\rho\, g} = \frac{p}{W}$$

Where Z is known as **Pressure head**

TYPES OF PRESSURE



Atmospheric pressure: - It is pressure exerted by the air on the Surface of earth. The air is compressible fluid, the density of air vary from time to time due to changes in its temperature, therefore atmospheric pressure is not constant

It is measured with reference to absolute Vacuum Pressure

Gauge pressure: - Gauge Pressure is measured with the help of Pressure gauge. The atmospheric pressure is taken as datum. In this Pressure, atmospheric pressure is considered zero, and this pressure is above the atmospheric pressure.

Vacuum pressure: - When pressure is below the atmospheric pressure is called vacuum pressure. It is also known as negative gauge pressure. It is measured by vacuum gauge.

Absolute pressure:- It is pressure which is measured with reference to absolute vacuum pressure. It is independent of the change in atmospheric pressure. It is measured above the zero of pressure.

Mathematically,

- Absolute pressure = Atmospheric Pressure + Gauge pressure

$$p_{abs} = p_{atm} + p_{gauge}$$

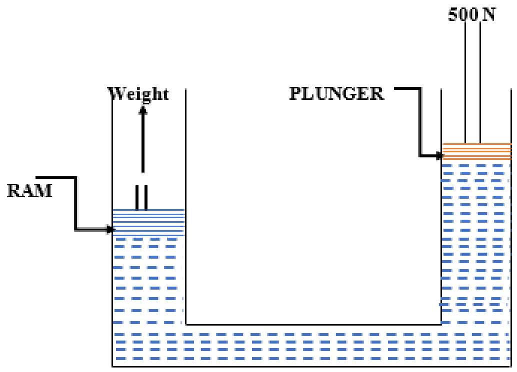
- Vacuum Pressure = Atmospheric Pressure - Absolute pressure

$$p_{vac} = p_{atm} - p_{abs}$$

NUMARICAL

1.A hydraulic pressure has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic pressure when the force applied at the plunger is 500N.

Solution:



Given dia of ram, D = 30cm= 0.3m

dia of plunger, d = 4.5cm = 0.045m

force on plunger F = 500N

weight lifted by hydraulic press, W = ?

Area of ram, A = $\frac{\pi D^2}{4} = \frac{\pi \times 0.3^2}{4} = 0.07068m^2$

Area of plunger, a = $\frac{\pi d^2}{4} = \frac{\pi \times 0.045^2}{4} = 0.00159m^2$

pressure intensity due to plunger = $\frac{force\ on\ plunger}{area\ of\ plunger}$

$= \frac{500}{0.00159} = 3144654\ N/m^2$

As per Pascal's law, intensity of pressure will be equally transmitted in all directions. Hence pressure intensity due to plunger = pressure intensity at the ram

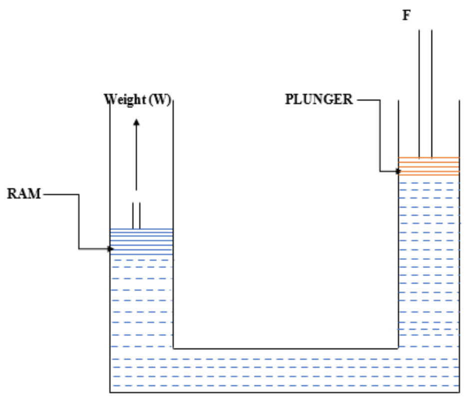
pressure intensity at the ram= $\frac{force\ on\ ram}{area\ of\ ram}$

$3144654 \quad = \frac{W}{0.07068}$

Weight lifted by hydraulic press, W = 22,222N = 22.22kN

2. A hydraulic press has a ram of 20cm diameter and a plunger of 3cm diameter. It is used to lifting a weight of 30kN. Find the force required at the plunger

Solution:



Given dia of ram, D = 20cm= 0.2m

dia of plunger, d = 3cm = 0.03m

weight lifted by hydraulic press, W = 30kN = 30,000 N

force on plunger F = ?

Area of ram, A = $\frac{\pi D^2}{4} = \frac{\pi \times 0.2^2}{4} = 0.0314m^2$

Area of plunger, a = $\frac{\pi d^2}{4} = \frac{\pi \times 0.03^2}{4} = 7.068 \times 10^{-4}m^2$

pressure intensity due to plunger = $\frac{force\ on\ plunger}{area\ of\ plunger}$

$= \frac{F}{7.068 \times 10^{-4}}$

pressure intensity at the ram = $\frac{force\ on\ ram}{area\ of\ ram}$

$$= \frac{30,000}{0.0314}$$

As per Pascal's law, intensity of pressure will be equally transmitted in all directions. Hence pressure intensity due to plunger = pressure intensity at the ram

$$\frac{F}{7.068 \times 10^{-4}} = \frac{30,000}{0.0314}$$

Force acting on ram, F = 675.2 N

3. Calculate the pressure due to a column of 0.3m of

(a) Water (b) Oil of sp. Gravity 0.8 (c) Mercury of Sp. Gravity 13.6

Take density of water $\rho = 1000$ kg/ m^3

Solution: Height of liquid column, Z = 0.3m

Pressure intensity at any point in a liquid is given by,

$$\mathbf{p = \rho g h}$$

(a) For water,

$$p = 1000 \times 9.81 \times 0.3$$
$$= 2943 \text{ N/m}^2 = 0.29 \text{ N/cm}^2$$

(b) For oil

Density of oil, $\rho = 1000 \times S = 1000 \times 0.8 = 800$ kg/ m^3

$$p = 800 \times 9.81 \times 0.3$$
$$= 2354.4 \text{ N/m}^2 = 0.2354 \text{ N/cm}^2$$

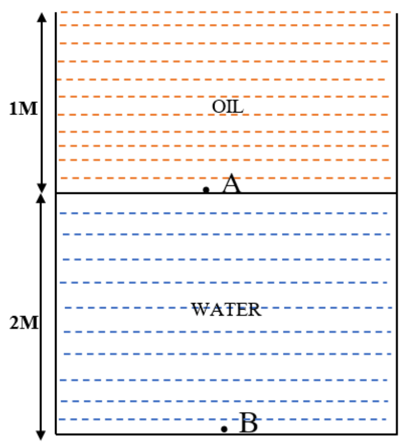
b) For mercury

Density of mercury, $\rho = 1000 \times S = 1000 \times 13.6 = 13600$ kg/ m^3

$$p = 13600 \times 9.81 \times 0.3$$
$$= 40025 \text{ N/m}^2 = 4.02 \text{ N/cm}^2$$

4. An open tank contains water upto a depth of 2m and above it an oil of Sp.gravity 0.96 for a depth of 1m. Find the pressure intensity (i) at the interface of two liquids (ii) at the bottom of the tank.

Solution:



Height of water, $Z_w = 2$m

$S_w = 1$

$\rho_w = 1000$ kg/m^3

Height of oil, $Z_o = 1$m

$S_o = 0.96$

$\rho_o = 0.96 \times 1000$

$= 960$ kg/m^3

Pressure intensity at any point in a liquid is given by,

$$\mathbf{p = \rho gh}$$

(i) at the interface of two liquids, i.e at A

$$p = \rho_w gh = 1000 \times 9.81 \times 2 = 19{,}620\ /m^2$$

ii) at the bottom of the tank i.e at B

$$p = \rho_o gh = 960 \times 9.81 \times 1 + 1000 \times 9.81 \times 2 = 94176\ N/m^2$$

5. What are the gauge pressure and absolute pressure at a point 3m below the free surface of a liquid having a density of 1.53×10^3 kg/m^3. If the atmospheric pressure is equivalent to 750mm of mercury? The sp. Gravity of mercury is 13.6 and density of water = 1000 kg/m^3

Solution:

Given Depth of liquid, $Z_1 = 3m$

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, Zo = 750mm of Hg = 0.75

Density of mercury, $\boldsymbol{\rho_o} = \mathbf{13.6} \times 1000 = 13600$ kg/m^3

Atmospheric pressure, $p_{atm} = \boldsymbol{\rho_o} \times \mathbf{g} \times Z_o$

$= 13600 \times 9.81 \times 0.75$

$= 100062$ N/m^2

Pressure at point, which is at a depth of 3m from free surface of the liquid is give by

$p = \boldsymbol{\rho_1} \times \mathbf{g} \times Z_1$

$= (1.53 \times 1000) \times 9.81 \times 3$

$= 45028$ N/m^2

Gauge pressure, p = 45028 N/m^2

Absolute pressure = Gauge pressure + Atmospheric pressure

$= 45028 + 100062$

$= 145090$ N/m^2

PRESSURE MEASUREMENT

The pressure of a fluid may be measured by the following devices.

- Monometers
- Mechanical devices

Manometers

Manometers are the devices in which pressure at a point can be measured by balancing the column of liquid by the same or different liquid.

Monometers are classified as:

1. Simple Manometers
- Piezometers
- U-tube monometers
- Single column monometers

2. Differential Manometers

Simple Manometers

A simple monometer consists of glass tube with one end connected to a point where pressure is to be measured and other end remains open to atmosphere

PIEZOMETER

- It is the simplest form of monometer used for measuring gauge pressure
- One end of this is connected to the point (A) where pressure isto be measured and other end is open to the atmosphere as shown in fig.
- The rise of liquid gives pressure head at point A
- The height of liquid say water is h in piezometer tube then pressure at Point A is given by

$$\mathbf{p_A = \rho\, g\, h} \quad N/m^2$$

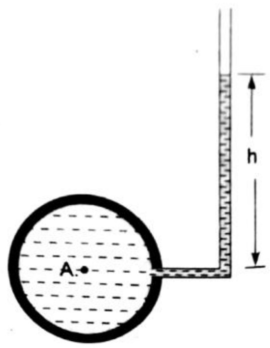
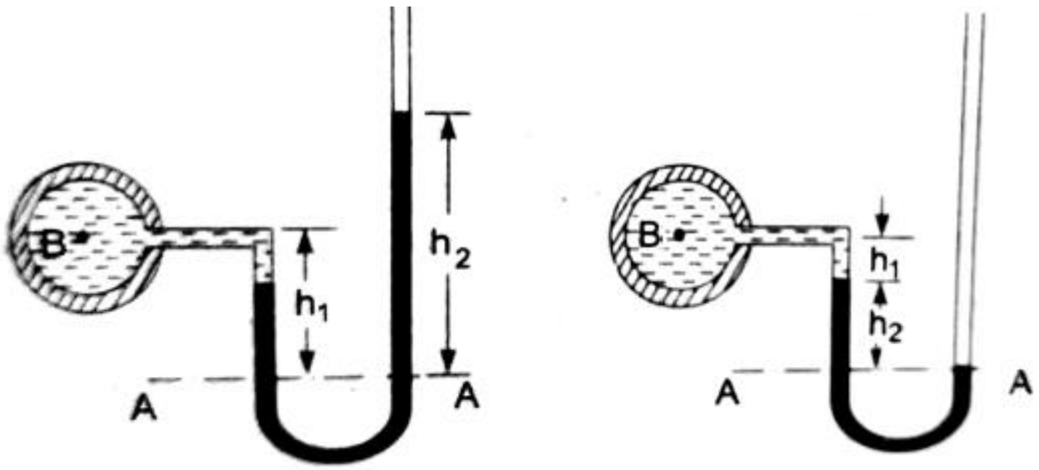


Fig: Piezometer

Simple U-tube manometer

- It consists of glass tube bent in U shape.
- One end of this is connected to the point where pressure is to be measured (A) and other end is open to the atmosphere

- The tube generally contains liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.
- Generally **Mercury** is used.



(a) For Gauge Pressure (b) For Vacuum Pressure

Fig: U-tube Monometer

(a) Gauge Pressure measurement:

Let B is the point at which pressure is to be measured (p)

The datum line is A-A

Let h_1 = height of the liquid above the datum line in left limb

h_2 = height of the liquid above the datum line in right limb

S_1= specific gravity of light liquid

S_2= specific gravity of heavy liquid

ρ_2 = density of heavy liquid = 1000 x S_2

Pressure above A-A in the left column = $p+ \rho_1 g h_1$….. (1)

Pressure above A-A in the right column = $\rho_2 g h_2$ …….. (2)

As pressure is same for horizontal surface, equating (1) and (2)

$$p+ \rho_1 g h_1 = \rho_2 g h_2$$

$$\boxed{\mathbf{p = \rho_2 g h_2 - \rho_1 g h_1}}$$

(a) Vacuum Pressure measurement:

Pressure above A-A in the left column = $p+ \rho_1 g h_1 + \rho_2 g h_2$ ….. (1)

Pressure above A-A in the right column = 0 ....(2)

As pressure is same for horizontal surface, equating (1) and (2)

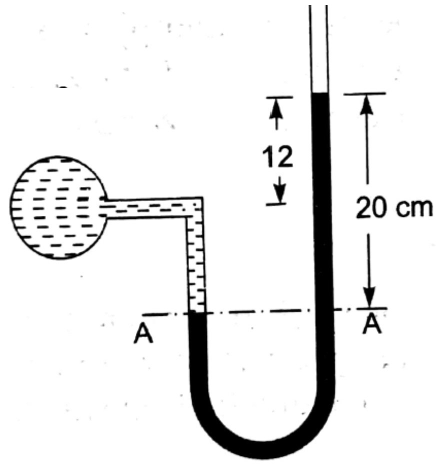
$$p + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$\boxed{\mathbf{p = -(\rho_2 g h_2 + \rho_1 g h_1)}}$$

NUMARICALS:

1. The right limb of a simple U-tube manometer containing mercury is open to atmosphere while the left is connected to a pipe in which a fluid of Sp.gravity 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Solution:



Given Sp. Gravity of fluid, $S_1 = 0.9$

Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000$

$= 900$ kg/m^3

Sp. Gravity of mercury, $S_2 = 13.6$

Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000$

$= 13600$ kg/m^3

Difference of mercury level, $h_2 = 20$cm$= 0.2$m

Height of fluid from A-A , $h_1 = 20-12 = 8$cm$=0.08$m

Equating the pressure above A-A, we get

$$\mathbf{p + \rho_1 g h_1 = \rho_2 g h_2}$$

$$p + 900 \times 9.81 \times 0.2 = 13600 \times 9.81 \times 0.08$$

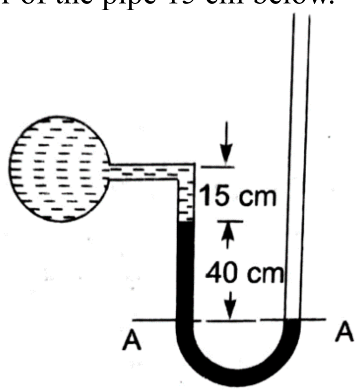
$$p = 26683-706 = 25977 \text{ N/m}^2$$

$$= 2.597 \text{ N/cm}^2$$

2. A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. Gravity 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and height of the fluid in the left from the center of the pipe 15 cm below.

Solution:

Given:



Sp. Gravity of fluid, $S_1 = 0.8$

Density of fluid, $\rho_1 = S_1 \times 1000 = 0.8 \times 1000$

$= 800$ kg/m^3

Sp. Gravity of mercury, $S_2 = 13.6$

Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000$

$= 13600\ kg/m^3$

Difference of mercury level, $h_2 = 40cm = 0.4m$

Height of fluid from A-A, $h_1 = 15cm = 0.15m$

Equating the pressure above A-A, we get

$$\mathbf{p + \rho_1 g h_1 + \rho_2 g h_2 = 0}$$

$$p + 800 \times 9.81 \times 0.15 = 13600 \times 9.81 \times 0.4$$

$$p = -(53366.4+1177.2) = -54543.6\ N/m^2$$

$$= -5.454\ N/cm^2$$

3. A U-tube manometer is used to measure the pressure of water in a pipe line. The right limb of the manometer contains mercury and is Open to atmosphere. The contact between water and mercury is in the left limb. If the difference in the mercury level is 10 cm and the free surface of mercury is in level with the center of pipe and if the pressure of water in pipe line is reduced to 9810 N/m2 calculate the difference in the mercury level. Sketch the arrangement

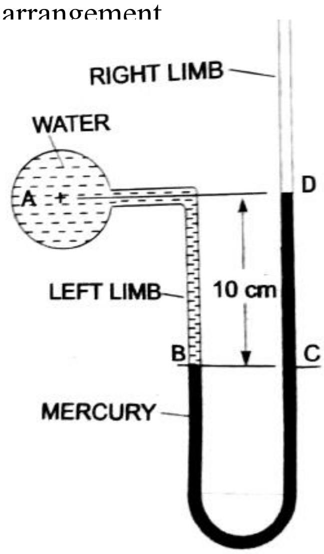
Solution:

Given,

Difference in mercury level = 10cm =0.1m

Take, $\rho_1 = 1000\ kg/m^3$ $\rho_2 = 13.6 \times 1000 = 13600\ kg/m^3$

Case(a): The arrangement is as shown in fig



case (a)

The point B and C are in line hence pressure at B should be equal to pressure at C

Pressure at B = pressure at A + pressure due to 10cm water column

$= p_A + \rho_1 g h$ ()

$= p_A + 1000 \times 9.81 \times 0.1 = p_A + 981$

1Pressure at C = pressure due to 10cm of mercury column

$= \rho_2 g h$

$= 13600 \times 9.81 \times 0.1 = 13341.6\ N/m^2$

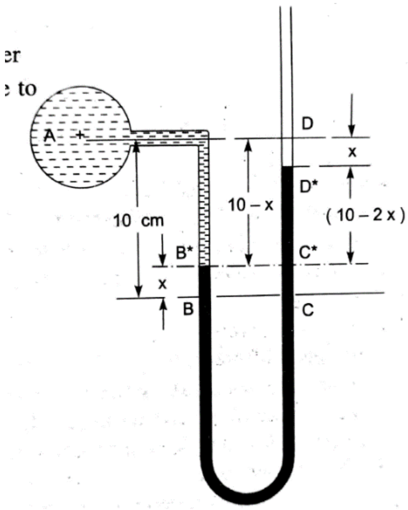
Equating the pressure at B and C

$$p_A + 981 = 13341.6$$

$$p_A = 12360.6\ N/m^2$$

Case(b): $p_A = 9810\ N/m^2$

New difference in mercury level =?



Arrangements are as shown in fig

In this case pressure 9810 N/m^2 is less than 12360.6 N/m^2

Hence mercury in left limb will rise. The rise in mercury in lest limb will be equal to fall in mercury level in right limb as the total volume of mercury remains same.

Let, x = rise of mercury in left limb in cm

then fall in mercury level in right limb = x cm

Then point B, C and D shows initial condition whereas B*, C* and D* shows the final condition

Pressure at B* = pressure at C*

pressure at A + pressure due to (10-x) cm water column = pressure due to (10-2x) cm of mercury

$$\mathbf{p_A + \rho_1 g h_1 = \rho_2 g h_2}$$

$$9810 + 1000 \times 9.81 \times \left(\frac{10-x}{100}\right) = 13600 \times 9.81 \times \left(\frac{10-2x}{100}\right)$$

Dividing by 9.81, we get

$1000 + 100\text{-}10x = 13600\text{-}1100$

$262x = 260$

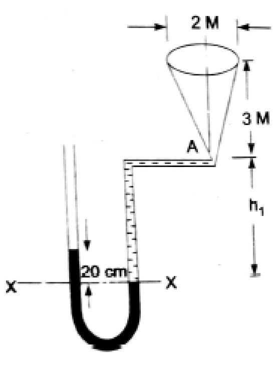
$x = 0.992\text{cm}$

New difference in mercury = 10-2x

= 10-2*0.992

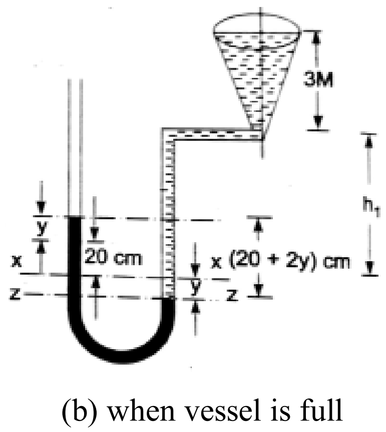
= 8.016cm

4. Fig shows a conical vessel having its outlet at A to which a U-tube monometer is connected. The reading of the monometer given in the fig shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.



(a) when vessel is empty

$h_1 = 2.72$ m



(b) when vessel is full

y = 11.45 cm

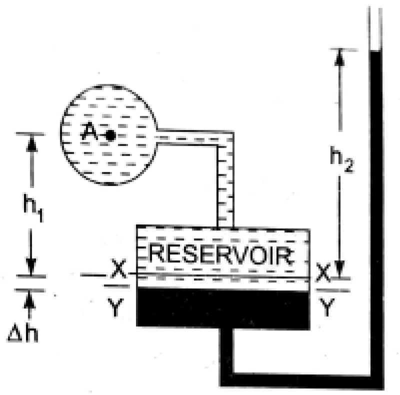
Monometer reading $= 20 + 2y = 42.90$ cm

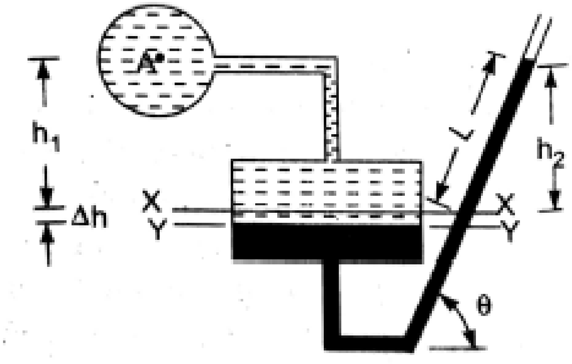
Single column manometer

- It is the modified form of a U-tube monometer, in which a reservoir having a large cross-sectional area about 100times as compared to the area of tube connected to one of the limb (say left limb) of the, monometer as shown in fig.
- Due to large cross –section area of reservoir, for any variation in pressure change in liquid level in the reservoir being very small and is neglected.
- Therefore, pressure is given by height of liquid in the other limb
- The other limb may be vertical or inclined. Thus, there are two types of single column monometer as

(i) Vertical single column manometer

(ii) Inclined single column manometer

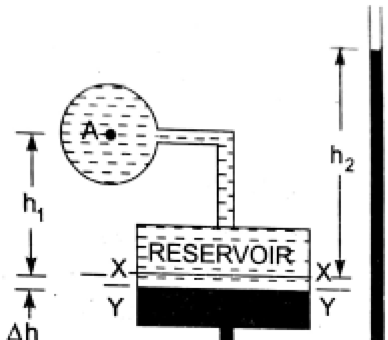




(a) Vertical single column manometer (b) Inclined single column manometer

Vertical Single Column Monometer

Fall of liquid in reservoir will cause a rise of heavy liquid



in the right limb

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

Consider the datum line y-y as shown in fig

Pressure in the right limb above y-y = $\rho_2 \times g\,(\Delta h + h_2)$ …….. (1)

Pressure in the right limb above y-y = $p_A + \rho_1 \times g\,(\Delta h + h_1)$ …… (2)

Equating the pressure, we have

$$\rho_2 \times g\,(\Delta h + h_2) = p_A + \rho_1 \times g\,(\Delta h + h_1)$$

$$p_A = \rho_2 \times g\,(\Delta h + h_2) - \rho_1 \times g\,(\Delta h + h_1)$$

$$p_A = \Delta h\,(\rho_2 g - \rho_1 g) + \rho_2 g h_2 - \rho_1 g h_1$$

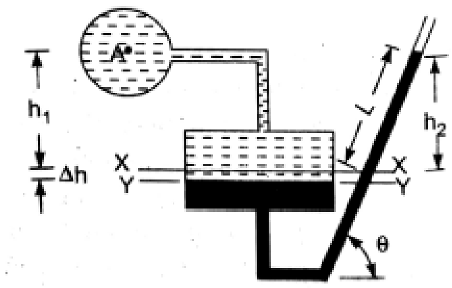
Substituting the value of Δh

$$\boxed{p_A = \frac{a \times h_2}{A}\,(\rho_2 g - \rho_1 g) + \rho_2 g h_2 - \rho_1 g h_1}$$

As the area A is very large compared to a, ratio (a/A) becomes very small and can be neglected

$$\mathbf{p_A = \rho_2 g h_2 - \rho_1 g h_1}$$

Inclined single column manometer



Let L = length of heavy liquid moved in right limb from x-x to y-y

θ = inclination of right limb with horizontal

h_2 = vertical rise of heavy liquid in the right limb from x-x = L $\sin\theta$

From equation, $\mathbf{p_A = \rho_2 g h_2 - \rho_1 g h_1}$

Substituting the value of h_2

$$\boxed{p_A = L \sin\theta\, g\, \rho_2 - \rho_1 g h_1}$$

NUMARICALS

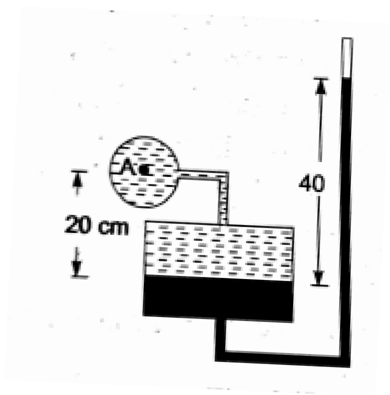
1. A single column monometer is connected to a pipe containing a liquid of sp. gravity 0.9 as shown in fig. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the monometer reading shown in fig. The specific gravity of mercury is 13.6

Solution:

Given Sp. Gravity of liquid in pipe, $S_1 = 0.9$

Density of liquid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000$

$= 13600\ kg/m^3$



Sp. Gravity of mercury, $S_2 = 13.6$

Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000$

$= 13600\ kg/m^3$

$$\frac{Area\ of\ reserviour}{area\ of\ right\ limb} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20cm = 0.2m$

Rise of mercury in right limb, $h_2 = 40cm = 0.4m$

p_A= pressure in pipe

$$p_A = \frac{a \times h_2}{A}(\rho_2\, g - \rho_1\, g) + \rho_2 g h_2 - \rho_1 g h_1$$

$$= 100 \times 0.2\,(13600 \times 9.81 - 900 \times 9.81) + (13600 \times 9.81 \times 0.4 - 900 \times 9.81 \times 0.2)$$

$\mathbf{p_A = 52134\ N/m^2}$

Differential Monometer

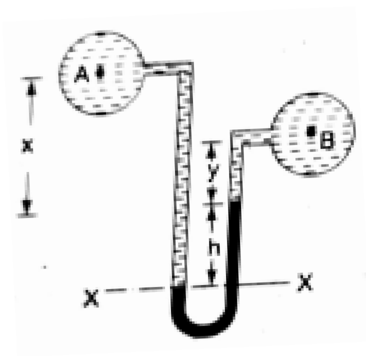
- Differential monometer is used for measuring the difference of pressure b/w two points in a pipe or in two different pipes
- Differential monometer consists of U-tube, containing a heavy liquid, whose ends are connected to the point where difference of pressure is to be measured.
- Most commonly used types are:

(i) U-tube Differential monometer

(ii) Inverted U-tube Differential monometer

U-tube Differential Monometer

Case (a) – A and B are at different level



Pressure in the left limb = $p_A + \rho_1.g.(h+x)$

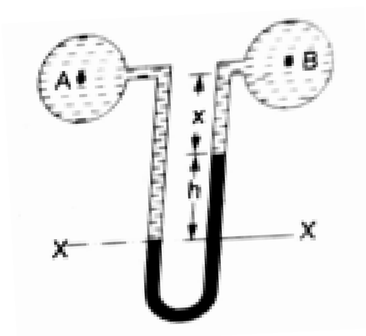
Pressure in the right limb = $p_B + \rho_2.g.y + \rho_g.\ g.h$

Pressure in the left limb = pressure in the right limb

$$p_A + \rho_1.g.(h+x) = p_B + \rho_2.g.y + \rho\, r_g.\ g.h$$

$$\boxed{\mathbf{p_A - p_B = g.h\,(\rho_g - \rho_1) + \rho_2.g.y - \rho_1.g.x}}$$

Case (b) – A and B are at same level



Pressure in the left limb = $p_A + \rho_1.g.(h+x)$

Pressure in the right limb = $p_B + \rho_1.g.x + \rho_g.g.h$

Pressure in the left limb = pressure in the right limb

$p_A + \rho_1.g.(h+x) = p_B + \rho_1.g.x + \rho_g.g.h$

$$\boxed{\mathbf{p_A - p_B = g.h.(\rho_g - \rho_1)}}$$

NUMARICALS

1. A pipe contains an oil of Sp. gravity 0.9. A differential monometer connected at the two points A and B shows a difference in mercury level as 15cm. Find the difference of pressure at the two points.

Solution:

Given Sp. Gravity of oil, $S_1 = 0.9$

Density of liquid, $\rho_1 = S_1 \times 1000 = 0.9\ 1000$

$= 900\ kg/m^3$

Sp. Gravity of mercury $S_2 = 13.6$

Density of mercury, $\rho_2 = S_2 \times 1000 = 13.6 \times 1000$

$= 13600\ kg/m^3$

Difference of mercury level, h = 15cm= 0.15m

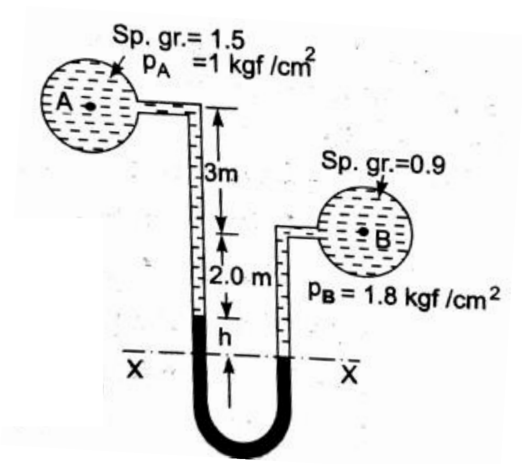
Difference in pressure is given by, $p_A - p_B = g.h\ (\rho_g - \rho_1)$

$= 9.81 \times 0.15\ (13600-100)$

$= 18688\ N/m^2$

2. A differential monometers is connected at the points A and B of two pipes as shown in Fig: The pipe A contains a liquid of Sp. Gravity = 1.5 while pipe B contains a liquid of sp. Gravity=0.9. The pressure at A and B are 1 kgf/cm^2 and 1.80 kgf/cm^2 respectively. Find the difference in mercury level in the differential monometer.

Solution:



Given,

Sp. Gravity of oil, $S_1 = 1.5$, $\rho_1 = S_1 \times 1000 = 1.5 \times 1000$

$= 1500\ kg/m^3$

Sp. Gravity of mercury, $S_2 = 0.9$, $\rho_2 = S_2 \times 1000 = 0.9 \times 1000$

$= 900\ kg/m^3$

Density of mercury $= 13.6 \times 1000 = 13600\ kg/m^3$

Pressure at A, $p_A = 1\ kgf/cm^2 = 1 \times 10^4\ kg/m^2$

$= 1 \times 10^4 \times 9.81\ N/m^2$

Pressure at A, $p_B = 1.8\ kgf/cm^2 = 1.8 \times 10^4\ kg/m^2$

$= 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$

Taking X-X as datum line.

Pressure above X-X in left limb $= 13600 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + p_A$

$= 13600 \times 9.81 \times h + 1500 \times 9.81 \times 5 + 1 \times 10^4 \times 9.81$

Pressure above X-X in right limb $= 900 \times 9.81 \times (2+h) + p_B$

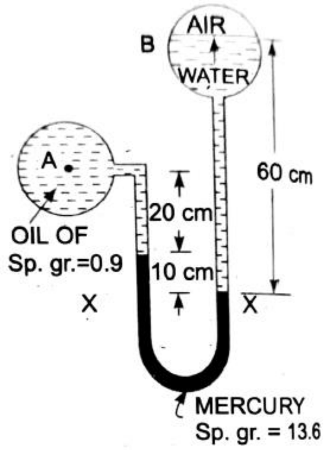
$= 900 \times 9.81 \times 2 + 900 \times 9.81 \times h + 1.8 \times 10^4 \times 9.81$

Equating Pressure above X-X in left limb = Pressure above X-X in right limb

$13600\times9.81\times\mathbf{h} + 1500\times9.81\times5 + 1\times10^4\times9.81 = 900 \times 9.81 \times 2 + 900 \times 9.81 \times h + 1.8 \times 10^4 \times 9.81$

$$\mathbf{h = 0.18m = 18.1cm}$$

3. A differential monometer is connected at the two points A and B as shown in fig. At B air pressure is 9.81 N/cm^2, find the absolute pressure at A



Solution:

Hint**

x= 20cm = 0.2m y=60cm = 0.6m h=10cm=0.1m

Pressure above X-X in left limb= $p_A + \rho_1.g.x + \rho_g.g.h$

Pressure above X-X in right limb= $p_B + \rho_2.g.y$

$p_A = 88876.8 \text{ N/m}^2$

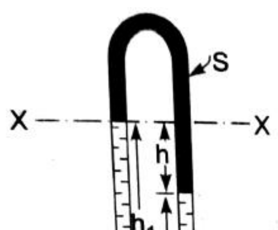
Inverted U-tube Differential Monometer

It consists of inverted U-tube, containing light liquid

Two ends of the tubes are connected to the point where difference of pressure is to be measured.

It is used for measuring low pressure

Figure shows inverted U-tube monometer connected to two points A and B. Let pressure at point A is more than the pressure at B



Pressure in the left limb = p_A - $\rho_1.g.h_1$

Pressure in the right limb = p_B - $\rho_2.g.h_2$- $\rho_3.g.h$

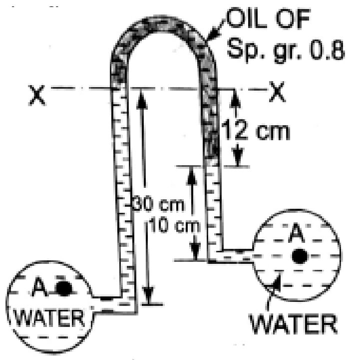
pressure in left limb = pressure in right limb

$$p_A - r_1.g.h_1 = p_B - \rho_2.g.h_2 - \rho_3.g.h$$

$$\mathbf{p_A - p_B = \rho_1.g.h_1 - \rho_2.g.h_2 - \rho_3.g.h}$$

NUMARICALS

1. Water is flowing through two different pipes to which an inverted differential monometer having an oil of Sp. Gravity 0.8 is connected. The pressure head in the pipe A is 2m of water, find the pressure in the pipe B for the monometer reading as show



Solution:

Hint**

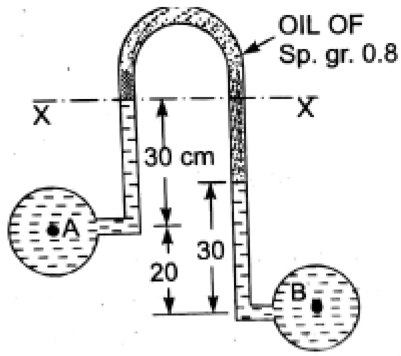
x= 30cm = 0.3m y=10cm = 0.1m h=12cm=0.12m

Pressure above X-X in left limb= p_A - $\rho_1.g.x$

Pressure above X-X in right limb= p_B - $\rho_2.g.y$ - $\rho_3.g.h$

$\mathbf{p_B = 18599.76\ N/m^2}$

2. In fig an inverted differential monometer is connected to two pipes A and B which convey water. The fluid in monometer is oil of Sp. Gravity 0.8. For the monometer reading shown in fig, find the pressure difference between A and B.



Solution:

Hint**

x= 30cm = 0.3m y=30cm = 0.3m h= (30+20-30)=20cm=0.2m

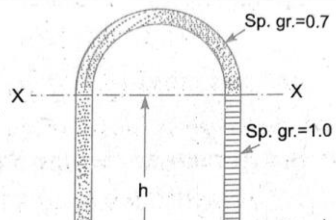
Pressure above X-X in left limb = p_A - $\rho_1.g.x$

Pressure above X-X in right limb = p_B - $\rho_2.g.y$ - $\rho_3.g.h$

$\mathbf{p_B - p_A = 1569.6\ N/m^2}$

3. Fig out the differential reading 'h' of an inverted U-tube monometer containing oil of Sp. Gravity 0.7 as the manometric fluid when connected across pipes A and B as shown in fig below, conveying liquids of specific gravities 1.2 and 1.0 and immiscible with manometric fluid. Pipes A and B are inverted are located at the same level and assume the pressure at A and B are equal.

Solution:



Hint**

$x = 30cm = 0.3m$ $y = (30+h)\ cm$ $h = ?$

$p_A = p_B$

Pressure above X-X in left limb= $p_A - r_1.g.x - r_3.g.h$

Pressure above X-X in right limb= $p_B - r_2.g.y$

h= 20cm

Total Pressure

Total pressure is defined as the force exerted by a static fluid on surface either plane or curved, when the fluid comes in contact with surfaces. This forces always acts normal to the surfaces.

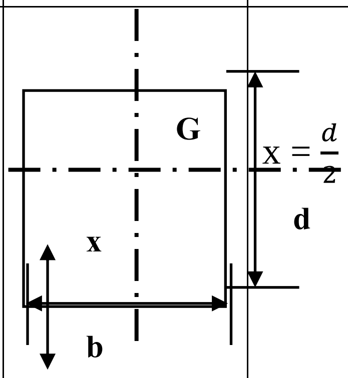
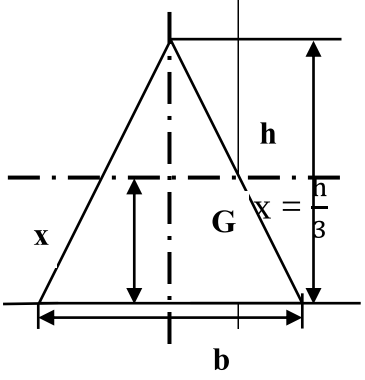
Centre of Pressure

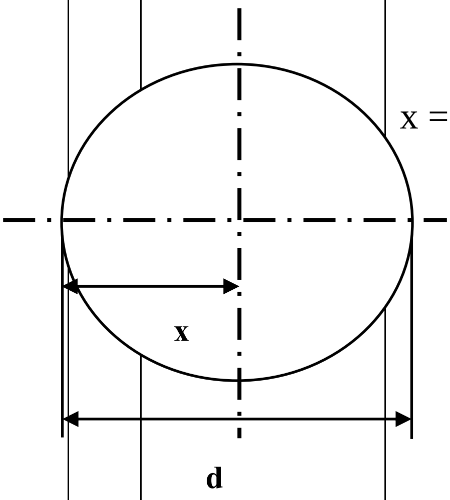
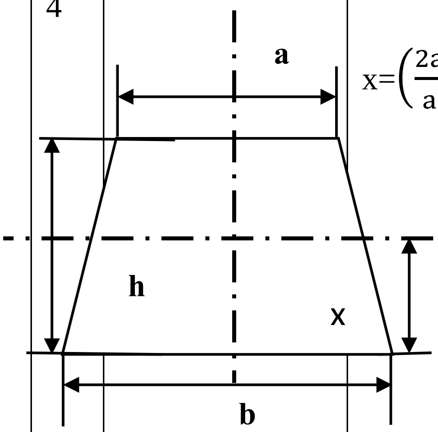
It is defined as the point of application of the total pressure on the surfaces

The following are the four cases of a submerged surfaces

- Vertical Plane surface
- Horizontal Plane surface
- Inclined Plane surface
- Curved Plane surface

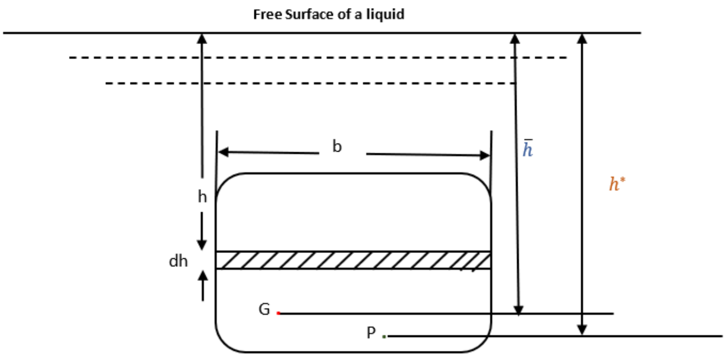
Moment of Inertia and other geometric properties of a plane surface

Sl No	Plane Surface	C.G. from the base	Area	MI about an axis passing through CG & parallel to the base	MI about an axis passing through CG & parallel to I_o
1		$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2		$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

3		$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	------------
4		$x=\left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\left(\frac{a+b}{2}\right)h$	$\left(\frac{a^2+4ab+b^2}{a+b}\right)\frac{h^3}{36}$	

Vertical plane surface submerged in liquid

a. Total Pressure Force



Consider a strip of thickness dh and width b at a depth of h from free surface of a liquid as shown in above figure

Pressure intensity of a strip, P = ρgh

Area of strip = dA= b×dh

Total pressure force on the whole surface,

dF = P*Area= ρgh × (b×dh)

There fore total pressure force on strip, dF = p× Area = ρgh × (b×dh)

$$F=\int dF = \int \rho gh \times (b \times dh)$$

$$= \rho g \int h \times (b \times dh)$$

$$= \rho g \int h \, dA$$

F= ρg A $\bar{h}$ - - - - - ►

for water the values of **ρ = 1000 kg/m^3 g =9.81 m/sec^2** **(1)_**

b. Centre of Pressure

Centre of pressure is calculated by using the 'principal of moments' , which states that the moment of the resultant force about an axis is equal to the sum of the moments of the components about the same axis

The resultant force F is acting at P, at a distance $h^{\times}$ from free surface of the liquid as shown in above

figure.

Hence moment of the force F about free surface of liquid = $F\times h$ $\dashrightarrow$ (2)

Moment of force dF, acting on a strip about the free surface of liquid = $dF \times h^*$

$= [\rho gh \times (b\times dh)]\times h$

Sum of the moment of all such forces about free surface of liquid = $[\rho gh \times (b\times dh)] * h$

$= \rho gb\times h\times h\times dh$

$= \rho gh\times h\times dA$

$= \rho g I_o$ $\dashrightarrow$ (3)

Equating (2) & (3)

$F\times h^\times = \rho g I_o$ $\qquad F = \rho g A\,\bar{h}$

$\rho g A\,\bar{h} \times h^\times = \rho g I_o$

$h^\times = \frac{I_o}{A\,\bar{h}}$

$\boxed{h^* = \frac{I_o}{A\,\bar{h}}}$ $\dashrightarrow$ (4)

By theorem of parallel axis, we have

$I_o = I_G + A\overline{h^2}$

Where I_G = moment of inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of the liquid

Put I_o in (4)

$\boxed{\mathbf{h^* = \frac{I_G + A\overline{h}^2}{A\,\bar{h}}}}$ $\dashrightarrow$ (5)

NUMERICAL

1.A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and

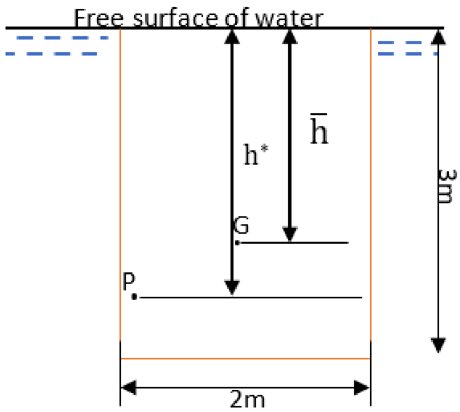
a. Coincides with water surface
b. 2.5m below the free surface of water

Solution:

Width of plane surface, b=2m

Depth of plane surface d=3m

a. Upper edge is coincides with water surface as shown in below figure



Total pressure is given by

F= ρg A $\bar{h}$

ρ = 1000 kg/m³ g =9.81 m/sec²

A = 3×2= 6 m²

$$\bar{h} = \frac{1}{2} * 3 = 1.5m$$

F = 1000×9.81×6×1.5

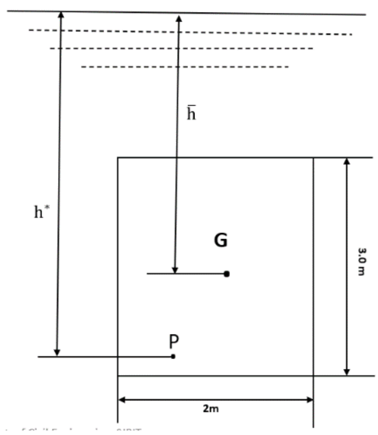
= 88.29 kN

Depth of the centre of pressure is given by

$$\mathbf{h}^{\times} = \frac{I_G + A\bar{h}^2}{A\,\bar{h}}$$

where I_G = Moment of Inertia = bd³/12 = 2×3³/12 =4.5m⁴ $h^{\times}$ = 2m

b. upper edge is 2.5m below the free surface of water



Total pressure force (F) is given by

F= ρg A $\bar{h}$

where $\bar{h}$ - is the distance of CG from free surface of water

= 2.5+(3/2) = 4.0m

F= 1000×9.81×6×4 = 235.44 kN

Centre of pressure is given by

$$\mathbf{h}^{*} = \frac{I_G + A\bar{h}^2}{A\,\bar{h}}$$

= 4.5/(6×4) +4

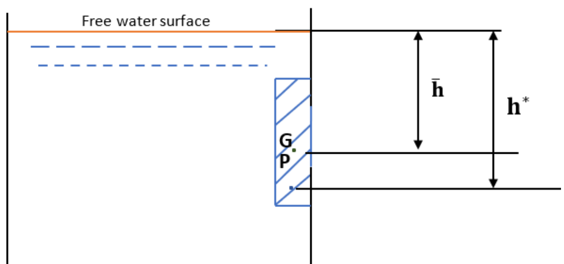
=4.1875m

2. A circular opening 3m diameter, in a vertical side of a tank is closed by a disc of 3m diameter which can rotate about horizontal diameter, calculate

a. The force on the disc and

b. The torque required to maintain the disc in equilibrium in the vertical position when the water above the horizontal diameter is 4m.

Solution:



d= 3m , A = $\pi/4 * 3^2$=7.0685 m^2,

Depth of C.G. = $\bar{h}$ = 4m

(i) Force on the disc is given by

F= ρg A $\bar{h}$

F= 1000×9.81×7.0685×4 = 277.368 kN

(ii) $\mathbf{h^*} = \frac{\mathbf{I_G + A\overline{h^2}}}{\mathbf{A\,\bar{h}}}$

$I_{GG} = \pi/64\, D^4 = \pi/64\, 3^4$

A= $\pi/4\, D^2 = \pi/4\, 3^2$

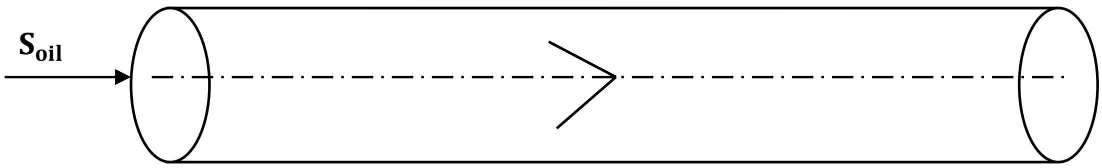
h^*= 4.14 m

(iii) Torque = F×($\mathbf{h^{\times}}$-$\bar{\boldsymbol{h}}$) = 277.368 ×10^3 (4.14-4) = 38831 Nm

3. A Pipeline which is 4m in diameter contains agate valve. The pressure at the centre of the pipe is 19.6 N/cm^2. If the pipe is filled with an oil of specific gravity 0.87, find the force exerted by the oil upon the gate and the position of centre of pressure

Solution:

D = Diameter = 4m



A= $\pi/4 * 4^2$, S=0.87, $\rho_{oil} = 870\ kg/m^3$

Pressure head at centre = p/$\rho_{oil} g$= 22.988m

The depth of C.G. of the gate valve from free oil surface from the centre of the pipe = 22.988m= $\bar{\boldsymbol{h}}$

i. Now the force exerted by the oil on the gate is given by

F= ρg A $\bar{h}$ = 870×9.81×4π =2.465 MN

ii. Position of centre of pressure (h^*)

iii. $\mathbf{h^*} = \frac{\mathbf{I_G + A\overline{h^2}}}{\mathbf{A\,\bar{h}}}$

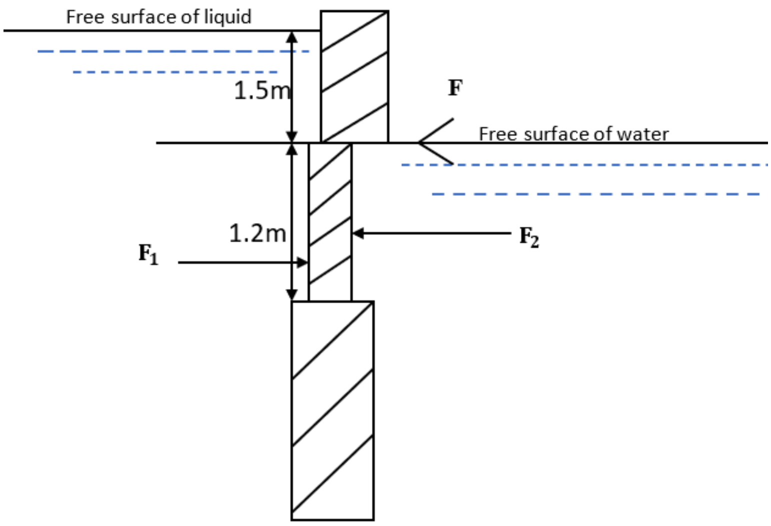
$I_{GG} = \pi/_{64}\, D^4 = \pi/_{64}\, 4^4$

$A = \pi/_4\, D^2 = \pi/_4\, 4^2$

$h^* = 23.031$ m

4. A Vertical sluice gate is used to cover an opening in a dam. The opening is 2m wide and 1.2m high on the upstream of the gate, the liquid of specific gravity 1.45, lies upto a height of 1.5m above the top of the gate, where as on the downstream side the water is available up to a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening on it. Assume that gate is hinged at the bottom.

Solution:



a. Resultant force and position of the resultant force

Let F_1 be the force acting on the gate due to liquid on upstream side

Similarly, F_2 be the force acting on the gate due to liquid on downstream side. F be the force required on the top of the gate to open it as shown in above Figure.

A= area of the gate $=2\times1.2 = 2.4$ m^2 , ρ_1 = density of liquid = $\rho_{water}\times$ S

=- 1000× 1.45 = 1450 kg/m^3

$\overline{h_1}$ = Vertical distance measured from free surface on the upstream side of the centroid of the gate

$\overline{h_1}$ = 1.5+1.2/2 = 2.1m

$F_1 = \rho_1 g\overline{h_1}A$= 1450×9.81×2.1×2.4 = 71.69 kN

$\overline{h_2}$ = Vertical distance measured from free surface on the downstream side of the centroid of the gate

$$\overline{h_2} = \frac{1.2}{2} = 0.6m$$

$F_2 = \rho_3 g\overline{h_2}A$= 1000×9.81×0.6×2.4 = 14.126 kN ρ_2 = density of water

Resultant force acting on the gate = F_1- F_2 = 71.69-14.126 = 57.564 kN

b. <u>Position of resultant force with respect to hinge</u>

$h^*{}_1$ = Centre of pressure measured from free surface at upstream side

$$\mathrm{h}^*{}_1 = \frac{\mathrm{I_G} + \mathrm{A}\overline{h_1}^2}{\mathrm{A}\,\overline{h_1}}$$

I_G = Moment of Inertia = $bd^3/12 = 2\times1.2^3/12 = 0.288m^4$

$$\mathrm{h}^*{}_1 = 2.157\ \mathrm{m}$$

The position of F_1 from hinge = 1.5+1.2- $h^*{}_1$ = 2.7×-2.157 = 0.543m

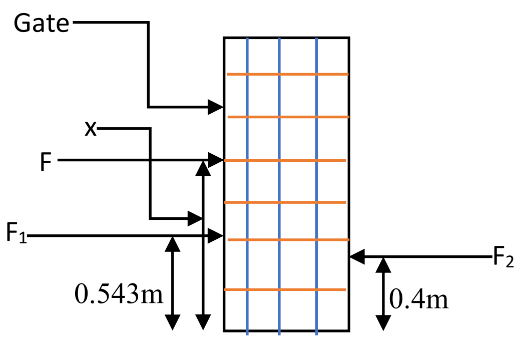
$h^*{}_2$ = Centre of pressure measured from free surface at upstream side

$$\mathrm{h}^*{}_2 = \frac{\mathrm{I_G} + \mathrm{A}\overline{h_2}^2}{\mathrm{A}\,\overline{h_2}}$$

$$\mathrm{h}^*{}_2 = 0.8m$$

The position of F_2 from hinge = 1.2- $h^*{}_2$ = 1.2-0.8 = 0.4m

Applying Varignons theorem with respect to hinge



a. Force acting on the top of the gate which makes the gate open

$\sum M_{hinge} = 0$

$F_1 \times 0.543 - F \times 1.2 - F_2 \times 0.4 = 0$

$71.69 \times 0.543 - 14.126 \times 0.4 = 1.2F$

F= 27.731 kN

Inclined plane surface

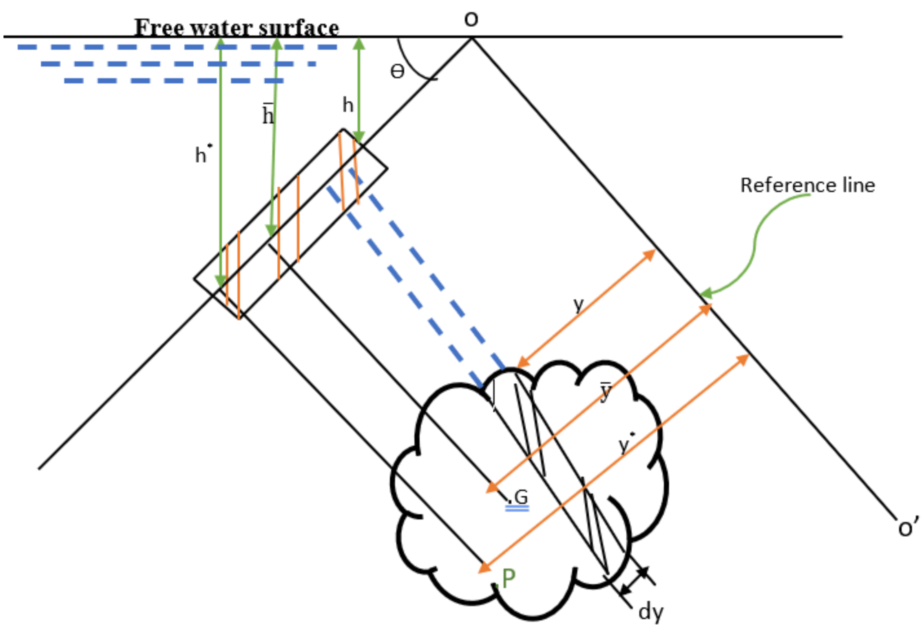
Consider a inclined body submerged in a inclined plane having an area A as shown in above figure.

Consider an elementary strip of thickness dy located at a distance y from reference line oo'.

$\bar{h}$- vertical distance measured from free surface of water to the centroid G

h- vertical distance measured from free surface of water to strip

$h^{\times}$- vertical distance measured from free water surface to total pressure P.



Total pressure force (F)

Let $y^{\times}$, $\bar{y}$ and y are the corresponding distance measured from reference line.

$$\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$$

Area of the strip =dA, Pressure = ρgh

Pressure force on a strip = dF= PdA

Total pressure force acting on the bottom = F

F =$\int dF$ dA

F =$\int \rho g h \, dA$

$F = \rho g \int h \, dA$

$\mathbf{F = \rho g} \, \boldsymbol{A\bar{h}}$

Centre of pressure (h^*)

Applying Varignons theorem,

Moment produced by resultant force = Sum of the moment produced by induvial force

Moment produced by resultant force = $F \times y^*$ ………… (1).

Sum of the moment produced by induvial force = $\int y \, dF$

Force acting on strip =dF

$dF = \rho gh \, dA$

Moment produced by strip about oo' = $dF \times y$

Sum of the moment produced by strip = $dF \times y$

$$= (\rho gh \, dA) \times y$$

$$= (\rho g (y \sin\theta) \; dA) \times y$$

$$= \int (\rho g y^2 \sin\theta \;) \; dA)$$

$$= \rho g \sin\theta \int y^2 \; dA$$

$$= \rho g \sin\theta \, I_o \text{ ……………………….(2).}$$

Equating equation (1) & (2)

$$F \times y^{\times} = \rho g \sin\theta \, I_o$$

$$y^* = \frac{\rho g \sin\theta \, I_o}{F} \text{ …………………..(3).}$$

$$I_0 = I_g + A\bar{h}^2 \text{ ………………… (4)}$$

$$F = \rho g A\bar{h} \text{ …………………..(5)}$$

$$y^* = \frac{h^*}{\sin\theta}$$

Put (4) & (5) in (3)

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta \, I_o}{F}$$

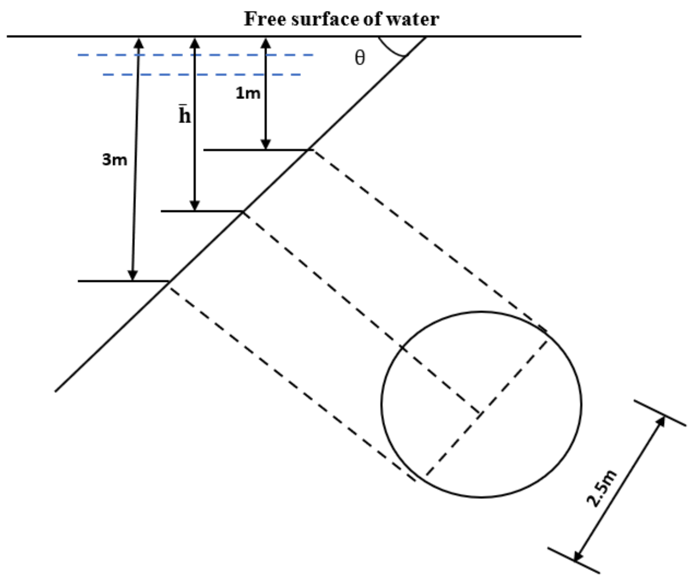
$$h^* = \frac{\rho g \sin\theta \;{}^2 \theta \, I_o}{\rho g \, A\bar{h}}$$

$$\mathbf{h^* = \left[\frac{\sin\theta \;{}^2 I_G}{A\bar{h}} + \bar{h}\right]}$$

NUMERICAL

1.A circular plate is 2.5m diameter is immersed in water its greatest and least depth below the free surface being 3m and 1m respectively. Find the (i) Total pressure force on the plate (ii) position of centre of pressure.

Solution:



$\theta = 53.13^{o}$

$\bar{h} = 2m$

d = diameter = 2.5m

Area $= \frac{\pi}{4} d^2$ = 4.908 mm^2

$I_G = \frac{\pi}{64} d^4$= 1.92 mm^4

a)Total pressure force

F = $\rho g A\bar{h}$ = 1000×9.81×4.908×2 = 96.29 kN

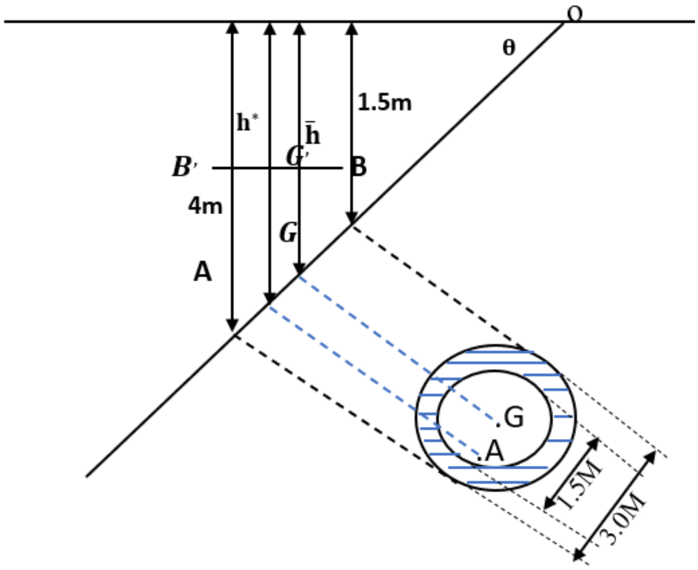
b) Position of pressure force

$$\mathbf{h^* = \left[\frac{\sin\theta^{\;2}\, I_G}{A\bar{h}} + \bar{h}\right]}$$

$$\mathbf{h^* = 2.125}\boldsymbol{m}$$

2.A hollow circular of inner diameter 1.5m and external diameter 3m is immersed in water in such way that its greatest and least depth of flow from free surface are 4m and 1.5m respectively. Determine the total pressure on one of the plane surface and position of the total pressure force from free surface.

Solution:



$\text{Sin}\theta = \frac{AB'}{AB} = \frac{4-1.5}{3}$

$\Theta = 56.44^{O}$

$\text{Sin}\theta = \frac{GG'}{BG} = \frac{GG'}{1.5}$

$GG' = 1.25m$

$\bar{h} = 1.5 + GG' = 1.5+1.25 = 2.75m$

a) Total pressure force

$A = \frac{\pi}{4}\left(d_e^{\ 2} - d_i^{\ 2}\right) = \frac{\pi}{4}(3^2 - 1.5^2) = 5.3 \text{ m}^2$

$I = \frac{\pi}{64}\left(d_e^{\ 4} - d_i^{\ 4}\right) = \frac{\pi}{4}(3^4 - 1.5^4) = 3.73 \text{ m}^4$

$F = \rho g\, A\bar{h} = 1000\times9.81\times5.3\times2.75 = 143 \text{ kN}$

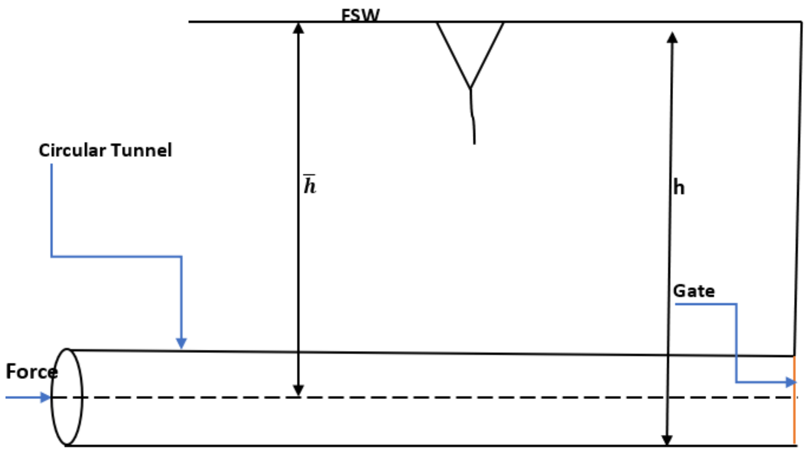
b) Position of Total pressure force

$h^* = \left[\frac{\sin\theta\ {}^2 I_G}{A\bar{h}} + \bar{h}\right] = \frac{\sin 56.44\ {}^2 * 3.73}{5.3*2.75} + 2.75$

$h^* = 2.93m$

3. A vertical gate closes a circular tunnel of 5m diameter running full of water. The pressure at the bottom of the tank is 1MPa. Determine the hydrostatic pressure force and centre of pressure force of the gate.

Solution:



Diameter of the tunnel = 5m , Pressure of the tunnel = 1MPa

Let h be the depth tunnel measured from free surface of water

P =wh

$h = \frac{P}{w} = \frac{1*10^6}{9810} = 101.9\text{m}$

$\bar{h} = h - {}^{d}/_{2} = 101.9\text{-}5/2 = 99.4\text{m}$

A = area of tunnel = 19.63 m^2 , Moment of Inertia of tunnel = 30.68 m^4

a) Total pressure force

$F = \rho g\, A\bar{h} = 1000\times 9.81\times 19.63\times 99.4 = 19.14\times 10^3$ kN

b) Position of Total pressure force

$$\mathbf{h^* = \left[\frac{I_G}{A\bar{h}} + \bar{h}\right] = \frac{30.68}{19.63*99.4} + 99.4}$$

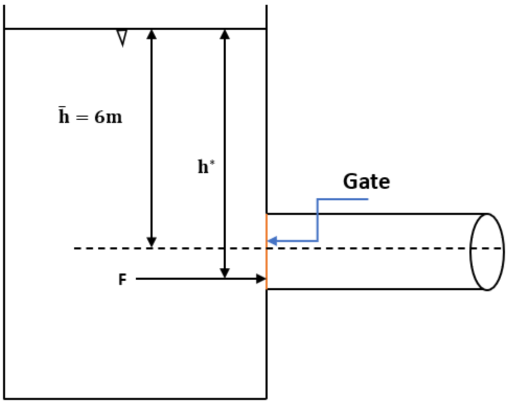
$$\mathbf{h^* = 299.64}\boldsymbol{m}$$

4. A circular opening 3m diameter in the vertical side of a tank is closed by a disc of 3m diameter which can rotate about horizontal diameter. Calculate the

i. The force of the disc

ii. The torque required to calculate the maintain the dics in equilibrium in vertical position. When the head of water above the horizontal plane is 6m.

Solution:



ρ =1000 kg/m^3 , A= A= $\frac{\pi}{4}(D^2)$ = 7.06 m^2 , I_G = 3.98 m^4

a) Total pressure force

F = ρg $A\bar{h}$ = 1000×9.81×7.06×6 = 4.14 kN

b) Position of Total pressure force

$$\mathbf{h^* = \left[\frac{I_G}{A\bar{h}} + \bar{h}\right] = \frac{3.98}{17.06*6} + 6}$$

$$\mathbf{h^* = 6.1}\boldsymbol{m}$$

c) Moment created by force with respect to centre of disc

= Force× perpendicular distance = 415×(6.1-6) = 41.5 kNm (Anticlock wise)

For equilibrium 41.5 kN-m (Clock wise direction)