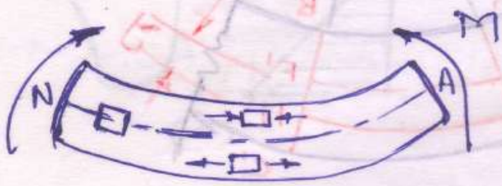
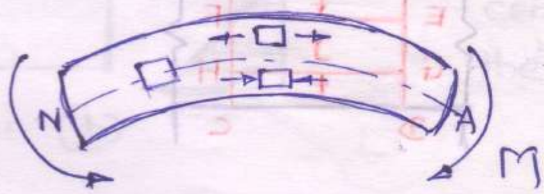
STRESSES IN BEAMS

Bending is usually associated with shear. For simplicity shear effect is neglected and only moment is considered.

" Theory that deals with stresses in a section of beam due to pure moment is called simple bending theory".



* Sagging moment



* Hogging Moment

(a) In sagging moment, the fibres on upper side of NA compressed and are in compression. the fibres on bottom side are stretched and are in tension.

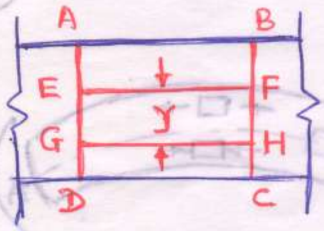
(b) In hogging moment, fibres are in tension on upper side of NA and are in compression on the lower side of NA.

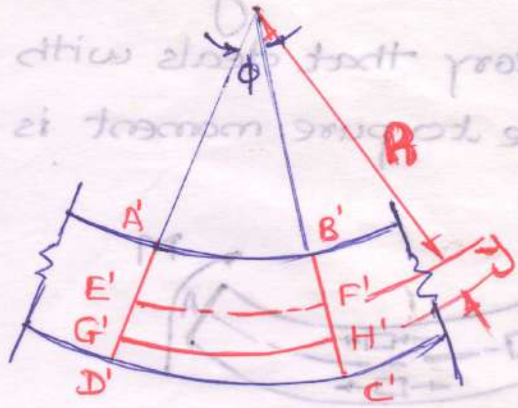
(c) The fibre along Neutral Axis (NA) remain unchanged.

Assumptions:-

(1) The beam is initially straight and every layer of it is free to expand or contract.
(2) The material is homogeneous and isotropic.
(3) Young's modulus is same in tension and compression.
(4) Stresses are within elastic limit.
(5) Plane section remains plane even after bending.
(6) The radius of curvature is large compared to depth of beam.

Relationship between Bending stresses and Radius of Curvature :-





Consider a section ABCD of a beam, subjected to bending moment.

Let, EF = Neutral Axis

GH = Element at a distance "y" from EF

R = Radius of curvature.

ϕ = angle subtended at centre by A'D' and B'C'.

Now, Bending stress, $\sigma_b = E\epsilon$ — ①

Strain in element GH, $\epsilon = \frac{\sigma_b}{E} = \frac{\text{Final length} - \text{initial length}}{\text{initial length}}$

$$\therefore \epsilon = \frac{G'H' - GH}{GH} \quad \text{— (a)}$$

Final length

$$G'H' = (R+y)\phi \quad \text{— (b)}$$

$$GH = EF = R\phi \quad \text{— (c)} \quad [\text{Neutral axis no stresses}]$$

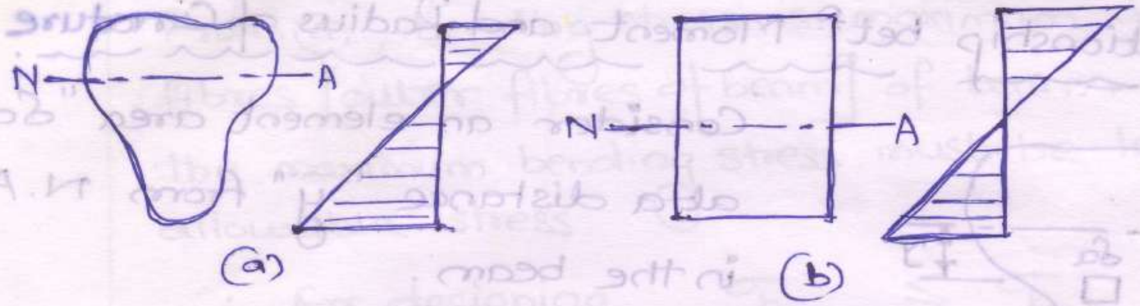
Substitute (b) and (c) in (a).

$$\therefore \epsilon = \frac{(R+y)\phi - R\phi}{R\phi} = \frac{y}{R} \quad \text{— (d)} \quad \text{Put in eq}^{n} \text{ ①}$$

$$\therefore \boxed{\frac{\sigma_b}{E} = \frac{y}{R}} \quad \text{— (e)}$$

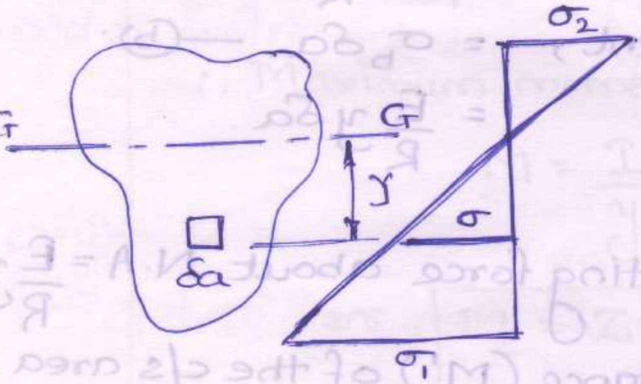
From equation (e), $\sigma_b = \left(\frac{y}{R}\right) E = \left(\frac{E}{R}\right) y$ —— (1)

Thus bending stress is proportional to depth of section from NA.



Thus NA lies on the centroid of the section.

Neutral axis coincides with centroid of the c/s



Consider a beam with arbitrary cross-section.

Let,

δa = elemental area

y = distance of "δa" from NA.

σ_b = stress on area.

$\therefore$ Force on the area $= \sigma_b \delta a$

Total force on c/s of beam $= \Sigma \sigma_b \delta a$ — (a)

Stress in beam, $\sigma_b = \left(\frac{E}{R}\right) y$ — (b) Put in (a).

$\therefore$ Total force on c/s of beam $= \Sigma \left(\frac{E}{R}\right) y \delta a$.

$= \frac{E}{R} \Sigma y \delta a$ — (c)

Since on beam there is no axial force and the above force is in axial direction, from equilibrium condition

$$\frac{E}{R} \Sigma y \delta a = 0$$

$$\therefore \Sigma y \delta a = 0$$

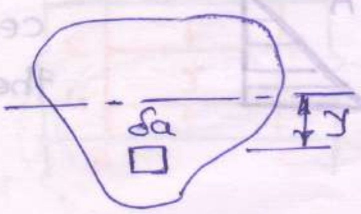
i.e $\frac{\Sigma y \delta a}{A} = 0$ where, A = Total Area.

$\Sigma y\delta a$ = moment of area about NA.

$\therefore \frac{\Sigma y \delta a}{A}$ = distance of centroid from N.A.

Thus N.A. coincides with centroid of the c/s.

Relationship betn Moment and Radius of Curvature



Consider an element area "δa" at a distance "y" from N.A. in the beam.

Stress on this element, $\sigma_b = \frac{E}{R} y$ — (a)

Force on this element, $= \sigma_b \delta a$ — (b)

$= \frac{E}{R} y \delta a$

~~∴ Total force~~

Moment of this resisting force about N.A $= \frac{E}{R} y^2 \delta a$

$\therefore$ Total moment of resistance (M') of the c/s area.

$\therefore M' = \frac{E}{R} \Sigma y^2 \delta a$ — (c)

Moment of inertia [second moment of area], $I = \Sigma y^2 \delta a$

$\therefore M' = \frac{E}{R} I$ — (d)

For equilibrium, Resisting moment = Applied moment

$M' = M$

$\therefore$ Equation (d) becomes, $\boxed{M = \frac{E}{R} I}$ — (2)

$\therefore$ Bending equation, from (1) and (2).

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R} \quad \text{— (3)}$$

Moment carrying capacity of a section

From bending equation, $\frac{M}{I} = \frac{\sigma_b}{y}$

$$\therefore \sigma_b = \frac{M}{I} y \quad \text{— (a)}$$

From (a), bending stress is maximum at extreme fibres [outer fibres of beam] of beam. For designing the maximum bending stress must be less than allowable stress.

$\therefore$ For designing, $\sigma_{b_{max}} \leq \sigma_{b_{allowable}}$

from eq^n (a) $\frac{M}{I} y_{max} \leq \sigma_{b_{allowable}}$

$\therefore$ Maximum moment carrying capacity of section.

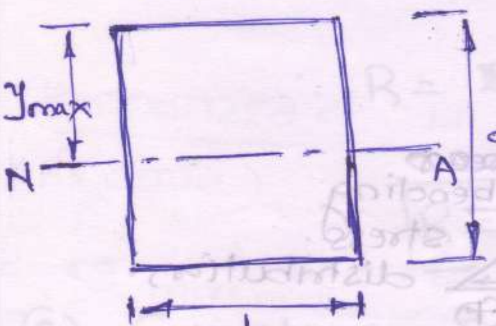
$$M = \frac{I}{y_{max}} \sigma_{b_{allowable}}$$

or $\boxed{M = Z \sigma_{b_{allowable}}}$ — (4)

where, $Z = \frac{I}{y_{max}}$ section modulus [unit mm^3]

Section modulus for

(a) Rectangular section $\quad$ b = width
$\quad$ d = depth.

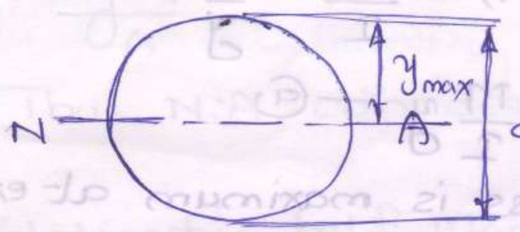


$$I = \frac{bd^3}{12}, \quad y_{max} = \frac{d}{2}$$

$$\therefore Z = \frac{I}{y_{max}} = \frac{bd^3}{12} \times \frac{2}{d}$$

$$\therefore \boxed{Z = \frac{bd^2}{6}} \quad \text{— (a)}$$

(b) Circular section :-



d = diameter

$I = \frac{\pi d^4}{64}$, $y_{max} = \frac{d}{2}$

$Z = \frac{I}{y_{max}} = \frac{\pi d^4}{64} \times \frac{2}{d}$

$\therefore \boxed{Z = \frac{\pi d^3}{32}}$ — (B)

Definitions :.

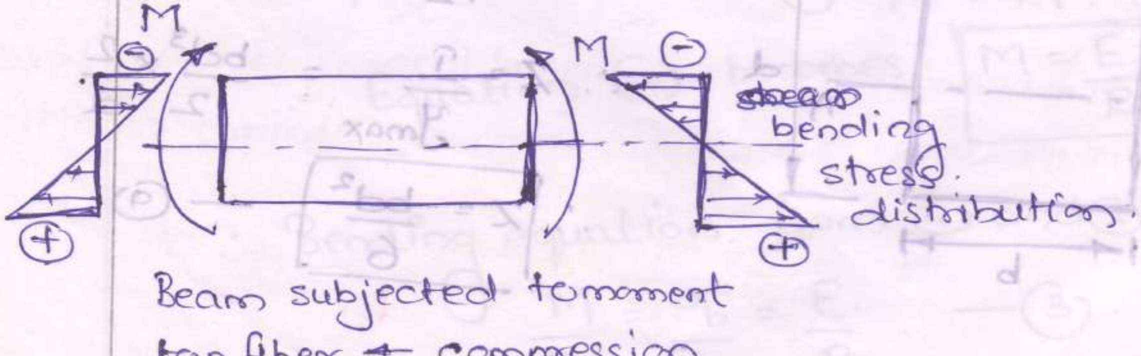
(1) Sagging moment :- Bending moment which causes a beam to bend into concave shape on its top surface is known as sagging moment.

(2) Hogging moment :- Bending moment that causes a beam to bend into convex shape on its top surface is known as hogging moment.

(3) Maximum bending moment that a beam can withstand without failure is called moment of resistance.

i.e. $M' = \frac{I}{y_{max}} \sigma_{b\,allowable}$.

(4)

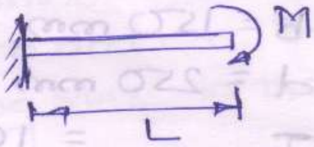


Beam subjected to moment
top fiber → compression
bottom fiber → tensile

Problems:

(1) A 1 meter long cantilever with a rectangular section of depth 75 mm and width 38 mm is subjected to a bending moment 2 KNm at its free end. Determine (a) Maximum bending stress (b) radius of circular arc and curvature of beam. Take, E = 200 GPa.

Soln Data Given:-



$L = 1m = 1000mm$

$d = 75\ mm$

$b = 38\ mm$

$M = 2\ KNm = 2\times10^6\ Nmm$

$E = 200\ GPa = 200\times10^3\ N/mm^2$

(a) Max bending stress is induced at extreme fibres

$\therefore$ From bending equation, $\frac{M}{I} = \frac{\sigma_b}{Y_{max}}$ — (a)

$$Y_{max} = \frac{d}{2} = \frac{75}{2} = \underline{37.5\ mm}$$

for rectangular section, $I = \frac{bd^3}{12} = \frac{38\times75^3}{12} = \underline{1.34\times10^6\ mm^4}$

$$\therefore (\sigma_b)_{max} = \frac{2\times10^6}{1.34\times10^6}\times 37.5 = \underline{56\ MPa} \text{ — (1)}$$

(b) Radius of arc, $\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$.

$$\therefore R = \frac{EI}{M} = \frac{200\times10^3\times1.34\times10^6}{2\times10^6}$$

$$\therefore \boxed{R = 1.34\times10^5}\ mm \text{ — (2)}$$

(c) Curvature, $= \frac{1}{R} = \frac{1}{1.34\times10^5} = \underline{0.75\times10^{-2}\ m.}$ — (3)

JUNE 2012 (10 mks)

(2) A simply supported beam of span 5 m has a c/s 150 mm × 250 mm. If the permissible stress is 10 N/mm², find.
(a) max. intensity of udL it can carry.
(b) max. concentrated load "P" applied at 2 m from one end it can carry.

Soln: Data:

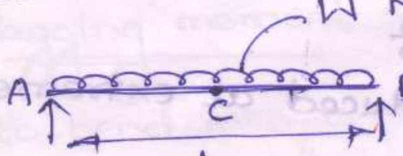
$L = 5\,m = 5000\,mm$

$b = 150\,mm$

$d = 250\,mm$

$\sigma_{b_{allowable}} = 10\,N/mm^2$

(a)



From bending equation,

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R} \quad \text{—(a)}$$

Max. bending moment, $M = \frac{I}{y_{max}}\,\sigma_{b_{allowable}}$.

$$\therefore M = \frac{bd^3}{12} \times \frac{\sigma_{b_{all}}}{(d/2)}$$

$$\therefore M = \frac{150 \times 250^3}{12} \times \frac{2}{250} \times 10 = \underline{\underline{15625000\ N\,mm}} \quad \text{—(b)}$$

Now, $M_A = M_B = 0$

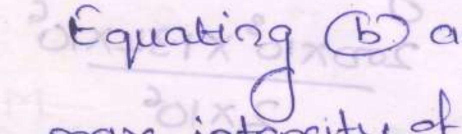
moment due to udL, $= \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{wL^2}{8}$ —(c)

Equating (b) and (c).

max. intensity of udL, $W = \frac{15625000 \times 8}{(5000)^2}$

$$\therefore \boxed{W = 5}\ N/mm \text{ or } \underline{\underline{5\ KN/m}} \quad \text{—(i)}$$

(b)



$M_C = R_B \times 3 = \underline{\underline{\frac{2}{5}P \times 3\ .\ Nm}}$ —(d)

$\begin{cases} \Sigma F_Y = R_A + R_B = P \\ \Sigma M_A = -2P + 5R_B = 0 \quad \therefore R_B = \frac{2}{5}P \end{cases}$

Equating (b) and (d).

$$\frac{2}{5}P\times3\times1000 = 15625000$$

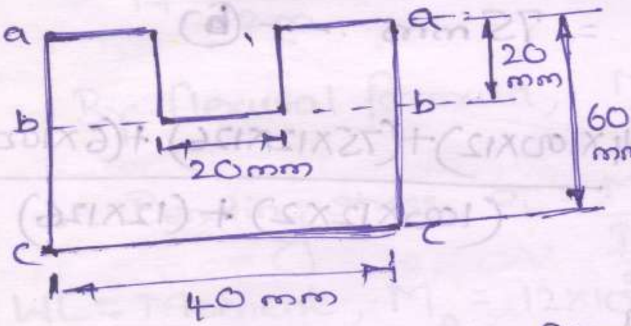
$$\left\{\begin{array}{l}\frac{2}{5}P\times3\ \underline{Nm} \\ \frac{2}{5}\times P\times3\times1000\ \underline{Nmm}\end{array}\right.$$

$$\therefore P = 13020.83\ N$$

$$\therefore \boxed{P = 13.02083\ kN} \quad — (2)$$

(3) The section of a beam shown is subjected to a hogging bending moment 800 Nm. Determine the bending stress induced along the fibers a-a, b-b and c-c. Draw a sketch showing the variation of bending stress in the section.

Soln.



Data :- $M = 800$ Nm.

By bending equation.

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R} \quad — (a)$$

(a) NA from bottom fibre,

$$\bar{y} = \frac{\Sigma y_i a_i}{A} = \frac{(40\times60\times30)-(20\times20\times50)}{(40\times60)-(20\times20)} = \underline{26\ mm} \quad — (a)$$

(b) Moment of Inertia.

$$I = \Sigma\left[\frac{bd^3}{12}+Ay^2\right] = \left\{\frac{40\times60^3}{12}+(40\times60)\times(30-26)^2\right\} - \left\{\frac{20\times20^3}{12}+(20\times20)(50-26)^2\right\}$$

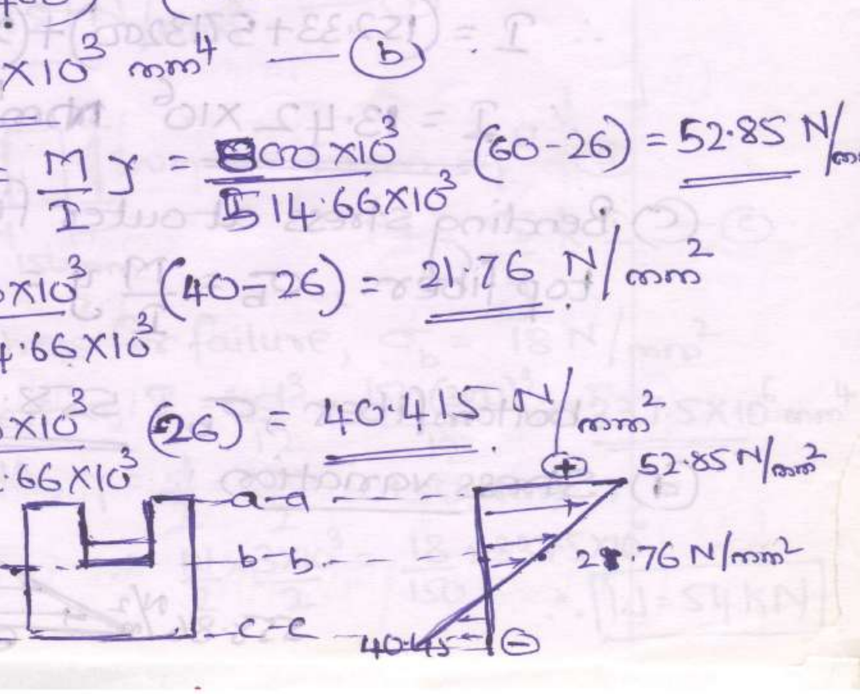
$$\therefore I = (720000+38400) - (13333.33+230400).$$

$$\therefore \underline{I = 514.66\times10^3\ mm^4} \quad — (b)$$

(c) for a-a, $\sigma_b = \frac{M}{I}y = \frac{800\times10^3}{514.66\times10^3}(60-26) = \underline{52.85\ N/mm^2}$

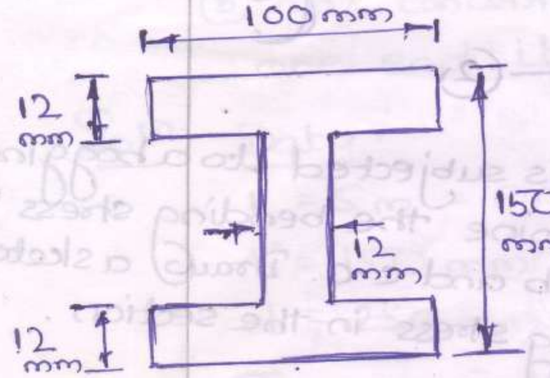
for b-b, $\sigma_b = \frac{800\times10^3}{514.66\times10^3}(40-26) = \underline{21.76\ N/mm^2}$

for c-c, $\sigma_b = \frac{800\times10^3}{514.66\times10^3}(26) = \underline{40.415\ N/mm^2}$



JUNE/JULY '14 (10 marks)

(4) At a given position in a beam of uniform I-section is subjected to a bending moment of 100 KNm. Plot the variation of bending stress across the section.



Soln :- $M = 100\ KNm$
$= 100 \times 10^6\ Nmm.$

From bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R} \quad \text{— (a)}$$

(a) NA from bottom fibre. [Since section is symmetric]

$$\bar{y} = \frac{d}{2} = \frac{150}{2} = 75\ mm \quad \text{— (b)}$$

(b) or $\bar{y} = \frac{\Sigma y_i a_i}{A} = \frac{(144\times100\times12)+(75\times12\times126)+(6\times100\times12)}{(100\times12\times2)+(12\times126)} = 75\ mm$

(b) Moment of Inertia,

$$I = \Sigma\left(\frac{bd^3}{12} + Ah^2\right) = \left\{\frac{100\times12^3}{12} + 100\times12(144-75)^2\right\}$$
$$+\left\{\frac{12\times126^3}{12} + 12\times126(75-75)^2\right\}$$
$$+\left\{\frac{100\times12^3}{12} + 100\times12(6-75)^2\right\}$$

$\therefore I = (152.33 + 5713200) + (2000376) + (152.33 + 5713200)$

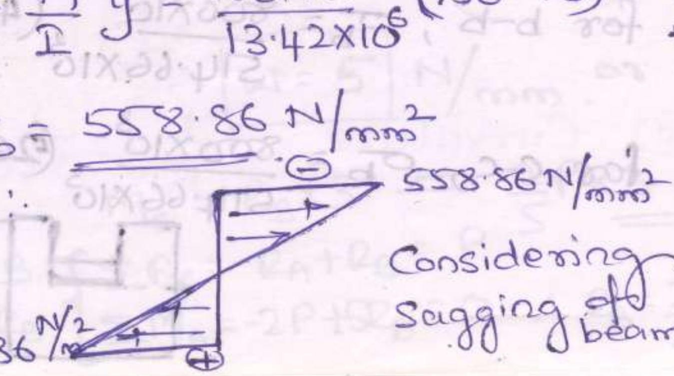
$\therefore I = 13.42 \times 10^6\ mm^4$ — (b)

(c) Bending stress at outer fibres,

top fiber, $\sigma_b = \frac{M}{I} y = \frac{100\times10^6}{13.42\times10^6}(150-75) = 558.86\ N/mm^2$

bottom fiber, $\sigma_b = 558.86\ N/mm^2$

(d) Stress variation :-



Considering sagging of beam

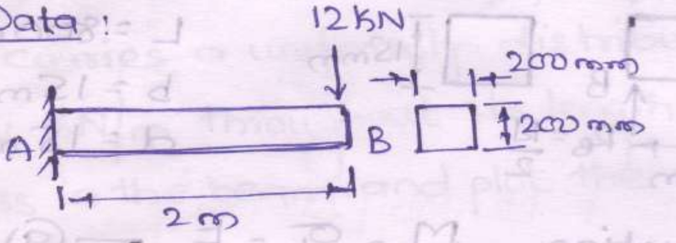
JUNE'13 (08 marks)

6/3

(5) A cantilever of square section 200 mm × 200 mm, 2 m long just fails in flexure when a load of 12 KN is placed at it's free end. A beam of the same material and having a rectangular c/s 150 mm wide and 300 mm deep is simply supported over a span of 3 m. Calculate the minimum central concentrated load required to break the beam.

Soln:- Data:-

(a)



$W = 12\,KN = 12 \times 10^3\,N$

$b = 200\,mm$

$d = 200\,mm$

$L = 2\,m = 2 \times 10^3\,mm$.

By flexural formula, $\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$ — (a)

$\therefore$ Bending stress, $\sigma_b = \frac{M}{I} y$ — (b)

Put in (b)

WL = Moment, $M_A = 12 \times 10^3 \times 2 \times 10^3 = \underline{24 \times 10^6\,N\,mm}$.

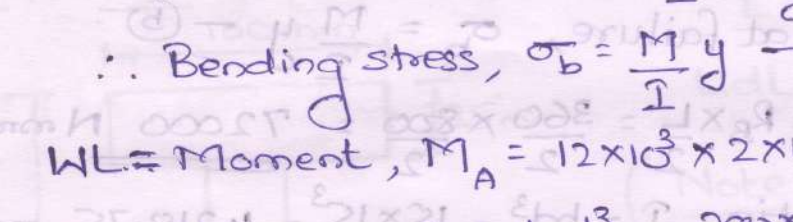
Inertia, $I = \frac{bd^3}{12} = \frac{200 \times 200^3}{12} = \underline{\underline{133.33 \times 10^6\,mm^4}}$

Outer fiber from NA, $y = \frac{d}{2} = \frac{200}{2} = \underline{\underline{100\,mm}}$

$\therefore \sigma_b = \frac{24 \times 10^6}{133.33 \times 10^6} \times 100 =$

$\therefore \boxed{\sigma_b = 18}\,N/mm^2$ — (1)

(b)



From eq^n (a)

$M = \frac{\sigma_b}{y} \times I$ — (c)

Put in (c)

$\therefore$ Max bending stress for failure, $\sigma_b = 18\,N/mm^2$

Moment of Inertia, $I = \frac{bd^3}{12} = \frac{150(300)^3}{12} = \underline{337.5 \times 10^6\,mm^4}$

outer fiber from NA, $y = \frac{d}{2} = \frac{300}{2} = 150\,mm$.

$\therefore M_c = R_B \times \frac{L}{2} = \frac{\sigma_b}{y} \times I \quad \therefore \frac{W}{2} \times \frac{3 \times 10^3}{2} = \frac{18}{150} \times 337.5 \times 10^6$

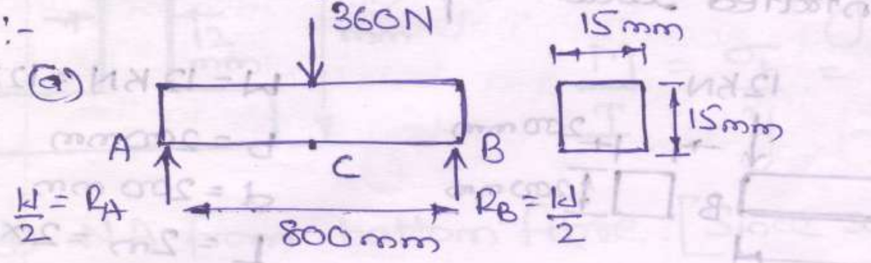
$\therefore \boxed{W = 54\,KN}$

Dec'13/JAN'14 (08 marks)

(6) A simply supported CI square beam of 800 mm length and 15 mm × 15 mm in section fails on applying a load of 360 N at the mid span. Find the maximum udl that can be applied safely to a 40 mm wide, 75 mm deep and 1.6 m long cantilever made of the same material.

Soln:-

(a)



$W = 360\ N$

$L = 800\ mm$

$b = 15\ mm$

$d = 15\ mm$

By Bending Equation, $\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$ — (a)

Bending stress at failure, $\sigma_b = \frac{M}{I} y$ — (b)

Put in (b):

Moment, $M_C = R_B \times \frac{L}{2} = \frac{360}{2} \times \frac{800}{2} = 72000\ N\,mm$

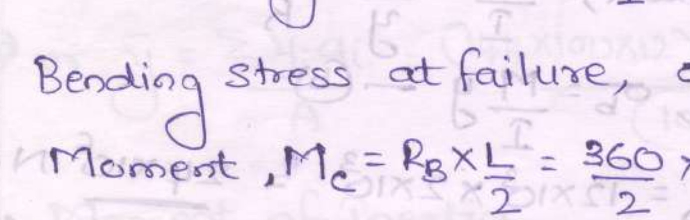
Moment of Inertia, $I = \frac{bd^3}{12} = \frac{15 \times 15^3}{12} = 4218.75\ mm^4$

Outer fiber distance from N.A, $y = \frac{d}{2} = \frac{15}{2} = 7.5\ mm$

$\therefore\ \sigma_b = \frac{72000}{4218.75} \times 7.5$

$\therefore \boxed{\sigma_b = 128}\ N/mm^2$ — (1)

(b)



$L = 1.6m = 1.6 \times 10^3\ mm$

$b = 40\ mm$

$d = 75\ mm$

$\sigma_b = 128\ N/mm^2$

From eqn (a), $M = \frac{\sigma_b}{y} \times I$ — (c)

Put in (c):

Bending moment, $M_A = (WL)\frac{L}{2} = \frac{W(1.6 \times 10^3)^2}{2} = 1.28 \times 10^6\ W$

Distance of outer fiber from N.A, $y = \frac{d}{2} = \frac{75}{2} = 37.5\ mm$

Moment of Inertia, $I = \frac{bd^3}{12} = \frac{40 \times 75^3}{12} = 1.40625 \times 10^6\ mm^4$

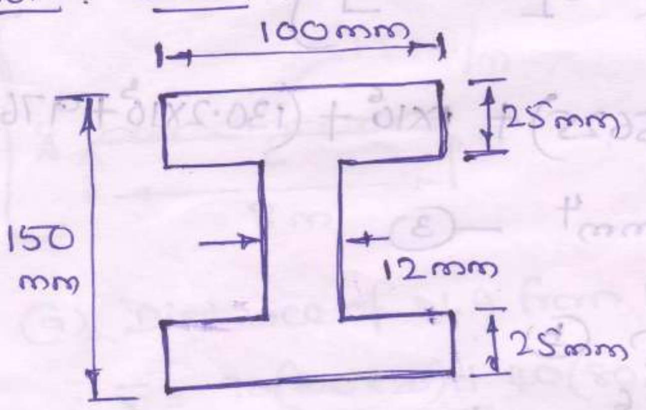
$\therefore$ UDL, $w = \frac{128}{37.5} \times \frac{1.40625 \times 10^6}{1.28 \times 10^6}$

$\therefore \boxed{w = 3.75}\ N/mm$

Dec'11 (15 mks)

(7) A uniform I-section is 100 mm wide and 150 mm deep with a flange thickness of 25 mm and web thickness of 12 mm. The beam is simply supported over a span of 5 m. It carries a uniformly distributed load of intensity 83.4 KN/m throughout its length. Determine the bending stress in the beam and plot the stress distribution across its c/s.

Soln: Data :



$L = 5m = 5 \times 10^3$ mm

UdL, $w = 83.4$ KN/m.

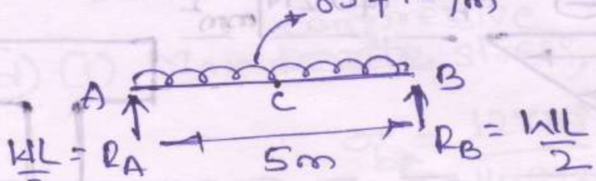
Note:- Section is symmetric. Hence find bending stress at top and bottom fibers.

$\sigma_{b_{top}} = \sigma_{b_{bottom}}$.

$\Rightarrow$ From Bending Equation, $\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$ — (a)

(a) At top fiber, bending stress, $\sigma_b = \frac{M}{I} y$ — (b)

(1). Moment of UDL, $M_c = -\frac{wL}{2}\left(\frac{L}{4}\right) + \frac{wL}{2}\left(\frac{L}{2}\right) = \frac{wL^2}{8}$



$\therefore M = M_c = \frac{83.4 \times 5^2}{8} = 260.625$ KN — (1)

(2) ~~Moment of Inertia, $I = \Sigma\left(\frac{bd^3}{12} + Ay^2\right)$~~

$= \cancel{\frac{100 \times 25^3}{12} + (100 \times 25 \times}$

(2) Distance of N.A from bottom fiber, $\bar{Y} = \frac{\sum y_i a_i}{A}$

$$\therefore \bar{y} = \frac{\{137.5(100\times25)\} + \{75(12\times100)\} + \{12.5(100\times25)\}}{(100\times25)2 + (12\times100)}$$

$$\therefore \bar{y} = \frac{465000}{6200} = \underline{\underline{75\,mm}} \quad \text{—(2)}$$

$$\text{or } \bar{y} = \frac{d}{2} = \frac{150}{2} = 75\,mm \quad \{\text{for symmetric section}\}.$$

(3) Moment of Inertia, $I = \sum\left\{\frac{bd^3}{12} + Ah^2\right\}$

$$\therefore I = \left\{\frac{100\times25^3}{12} + 100\times25\,(137.5-75)^2\right\}$$
$$+\left\{\frac{12\times100^3}{12} + 12\times100\,[75-75]^2\right\}$$
$$+\left\{\frac{100\times25^3}{12} + 100\times25\,[12.5-75]^2\right\}$$

$$\therefore I = (130.2\times10^3 + 9765625) + 1\times10^6 + (130.2\times10^3 + 9765625)$$

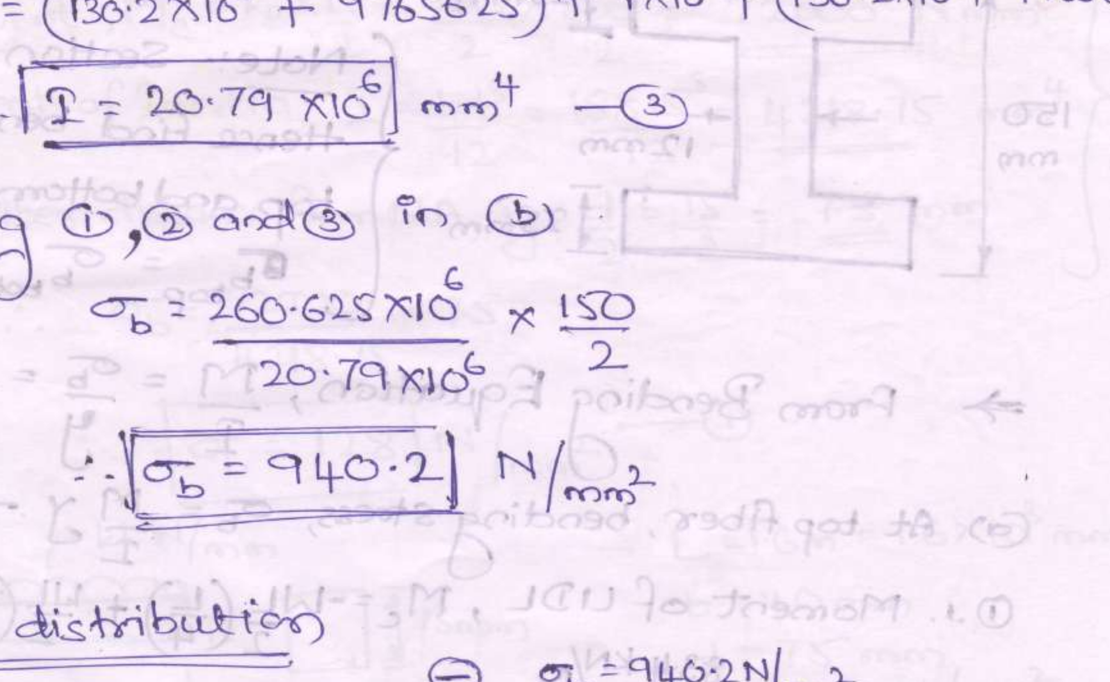
$$\therefore \boxed{I = 20.79\times10^6}\ mm^4 \quad \text{—(3)}$$

(b) Using (1), (2) and (3) in (b)

$$\therefore \sigma_b = \frac{260.625\times10^6}{20.79\times10^6}\times\frac{150}{2}$$

$$\therefore \boxed{\sigma_b = 940.2}\ N/mm^2$$

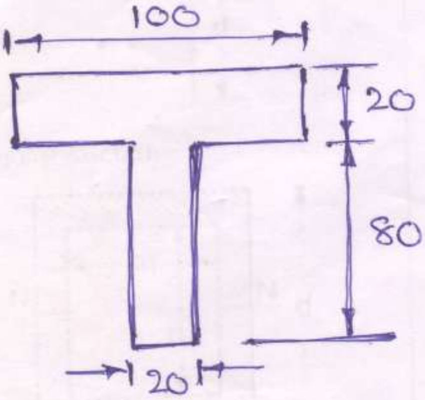
(c) Stress distribution



Dec'12 (10 marks)

(8) A CI beam is of T-section as shown. The beam is simply supported on span of 8 m. The beam carries a udl of 1.5 kN/m length on the entire span. Determine the maximum tensile and maximum compressive stresses.

Soln :-



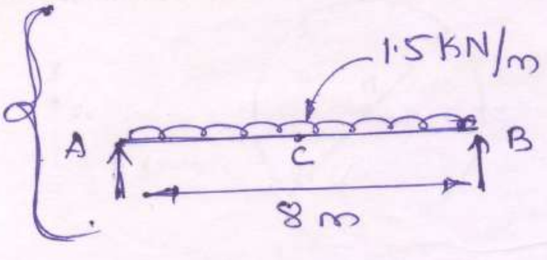
All dimensions in mm

$L = 8\,m = 8000\,mm$

$W = 1.5\,kN/m$

⇒ By bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R} \quad \text{— (a)}$$



This beam is subjected to sagging bending moment.

∴ Top fiber → compression $= \sigma_{bc}$

Bottom fiber → tensile stress $= \sigma_{bt}$

(a) Distance of N.A from bottom fiber, $\bar{Y} = \frac{\Sigma y_i a_i}{A}$

$$\therefore \bar{Y} = \frac{90(100\times20) + 40(80\times20)}{(100\times20)+(20\times80)} = 67.77\ mm \text{ from bottom fiber.}$$

(b) Moment of Inertia, $I = \cancel{\frac{100(100)^3}{12} - \frac{80(80)^3}{12} = 4.92\times10^6\,mm^4}$

$$I = \Sigma\left(\frac{bd^3}{12} + Ay^2\right) = 3.14\times10^6\,mm^4$$

(c) Bending moment for simply supported beam with udL, $M_c = \frac{WL^2}{8} = \frac{1.5\times8^2}{8} = 12\ kNm$

(d) (i) Max ~~tensile~~ compressive stress, $\sigma_{bc} = \frac{M}{I}y = \frac{M}{I}[90-\bar{y}]$

$$\therefore \sigma_{bc} = \frac{12\times10^6}{3.14\cdot92\times10^6}[100-67.77] = 123.17\ N/mm^2$$

(ii) Max tensile stress,

$$\sigma_{bt} = \frac{M}{I}[40-67.77] = \frac{12\times10^6}{3.14\cdot92\times10^6}[\text{[illegible]}-67.77] = 258.93\ N/mm^2$$