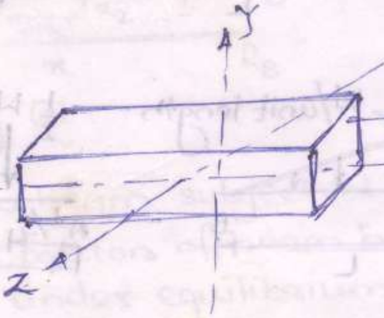
UNIT 5 :- Bending Moment and Shear Force in Beams

Definition :- A beam is a long structural member having relatively small cross-sectional dimensions. It is subjected to bending by the transverse forces acting on it.

Note :-



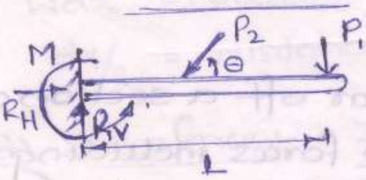
[Z and Y-axis are, $\perp^{ar}$ to axis is referred as radial, lateral or transverse]

→ cross-section

→ axis [along the axis is referred as axial, longitudinal. Parallel to axis]

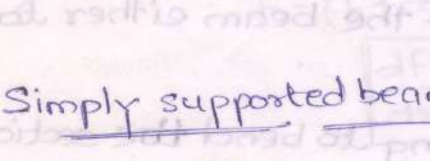
Types of beams

(1) Cantilever beams (fixed support) :- A beam having one end fixed is called a cantilever.



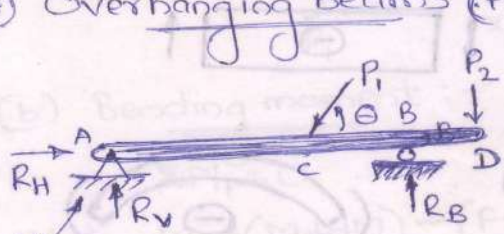
Fixed support constraints both translation and rotation of beam. Reactive force and reactive moment are exerted by fixed support.

(2) Simply supported beams :-



A beam just placed on simple supports is called simply supported beam. Rotation of beam is not arrested by a simple support and hence it does not exert any reactive moment.

(3) Overhanging beams (Hinged support) :-

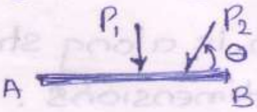


Translation motion is arrested in all directions. Rotation is not arrested. Hence reaction forces are exerted in the plane of loading.

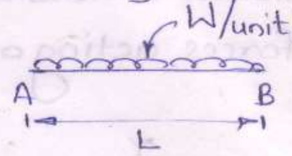
Types of loads :-

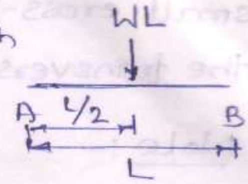
Beams are subjected to transverse loads.

(1) Concentrated loads :-

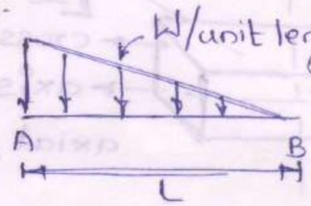


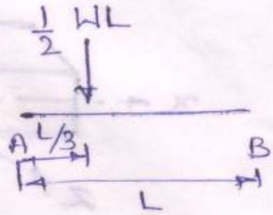
(2) Uniformly distributed loads :-



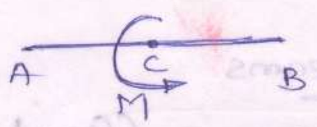


(3) Uniformly varying loads :-





(4) Externally applied moments :-



Definitions

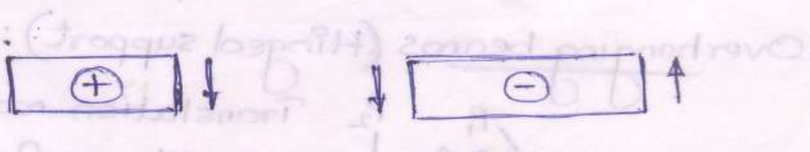
(1) Shear force :- Force that is trying to shear off a section and is obtained as the algebraic sum of all the forces including the reactions acting normal to the axis of the beam either to the left or to the right of the section.

(2) Bending moment :- moment that is trying to bend the section and is obtained as the algebraic sum of the moments about the section of all the forces (including the reactions) acting on the beam either to the left or to the right of the section.

Sign Convention

(1) Shear force

(2) Bending moment :-

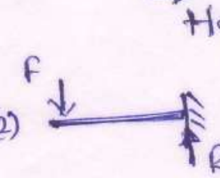


Note :- for SFD sign convention

(1)

Reaction on left
i.e moving from left to right

(2)



Reaction on right
i.e moving from right to left.

Relation Between load intensity, shear force and bending moment.

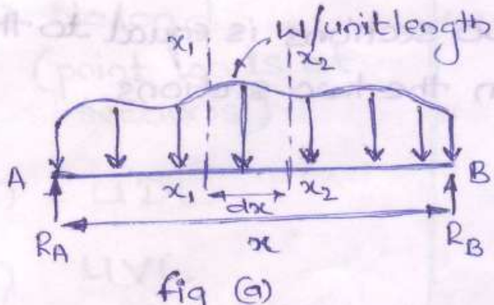


fig (a)

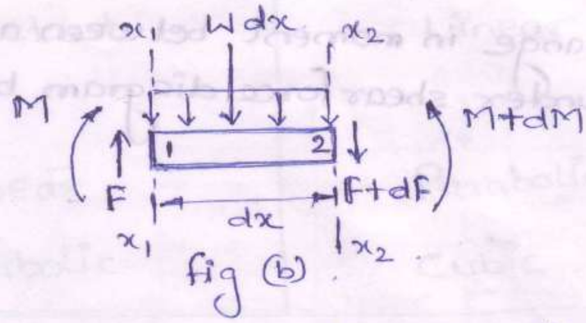


fig (b)

Consider a beam subjected to load distribution (w.x). Now, consider a portion of beam between sections x_1-x_1 and x_2-x_2. This portion is under equilibrium condition because of

F and M = shear force and bending moment on x_1-x_1.

(F+dF) and (M+dM) = ——— " ——— on x_2-x_2.

wdx = load intensity on the portion (resultant load).

$dx/2$ = distance from x_1-x_1, where load (wdx) acts.

(a) Shear force :- $\Sigma F_Y = 0$ — (a)

$$F-(F+dF)-w\,dx=0$$

$$\therefore \boxed{\frac{dF}{dx}=-w} \quad — ①$$

∴ Slope of shear force diagram is equal to intensity of load.
-ve sign indicates, shear force decreases with increasing "x" when load acts downwards.

Integrating eqⁿ ① between limits 1 and 2.

$$\int_{F_1}^{F_2} dF = \int_{x_1}^{x_2} w\,dx = F_2-F_1 \quad — (1a)$$

Change in shear force betⁿ any two sections is equal to area under the load diagram betⁿ the two sections.

(b) Bending moment :-

$\Sigma M_1 = 0$

$$\therefore -M-(M+dM)-(F+dF)dx-w\,dx\,\frac{dx}{2}=0 \quad — (b)$$

Ignoring higher order differentials, dFdx and $dx\left(\frac{dx}{2}\right)$

$$\therefore F=\frac{dM}{dx} \quad — ②$$

shear force at any section is equal to the slope of BMD at that section

Integrating eqn (2) ;

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} F\,dx = M_2 - M_1 \quad \text{— (2a)}$$

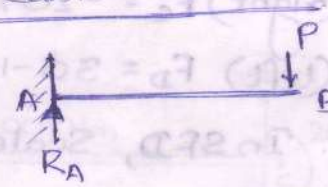
Change in moment between any two sections is equal to the area under shear force diagram between the two sections.

Shear force and Bending moment variations.

Load	Shear force	Bending Moment
(1) No load (point loads at sections)	Constant	Linear
(2) UDL	Linear	Parabolic
(3) UVL	Parabolic	Cubic

Problems on Cantilever beam.

→ Note :- (1)



Reaction force at fixed end. On section "AB" Shear force is constant.

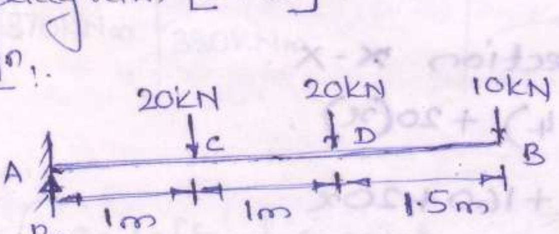
(2) Moment at free end is "zero". Moment is maximum at fixed end. Calculate moment from fixed end (moment of point load) and move towards free end (only for simplicity).

(3) External moments at any point A — C — B. Calculate moments on left and right sections of that point (exclude moment at "C").

→

(1) Draw the bending moment diagram [BMD] and shear force diagram [SFD] for the cantilever beam shown.

Sol^n:



(a) SF :-

$F_A = R_A = 20+20+10 = 50\,KN$.

(left) $F_C = R_A = 50\,KN$.

(Right) $F_C = 20+10 = 30\,KN$.

(left) $F_D = 50-20 = 30\,KN$.

(Right) $F_D = 10\,KN$.

(left) $F_B = 50-20-20 = 10\,KN$.

SFD

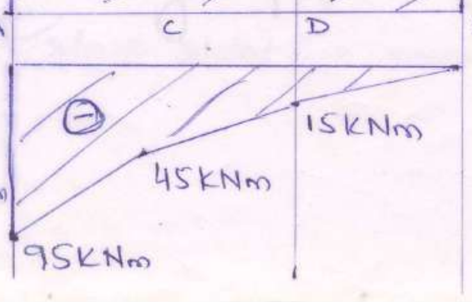
Scale

10KN = 1cm

BMD

Scale

10KNm = 1cm



(b) Bending Moment

$M_A = -[20\times1]-[20\times2]-[10\times3.5] = -95\,KNm$

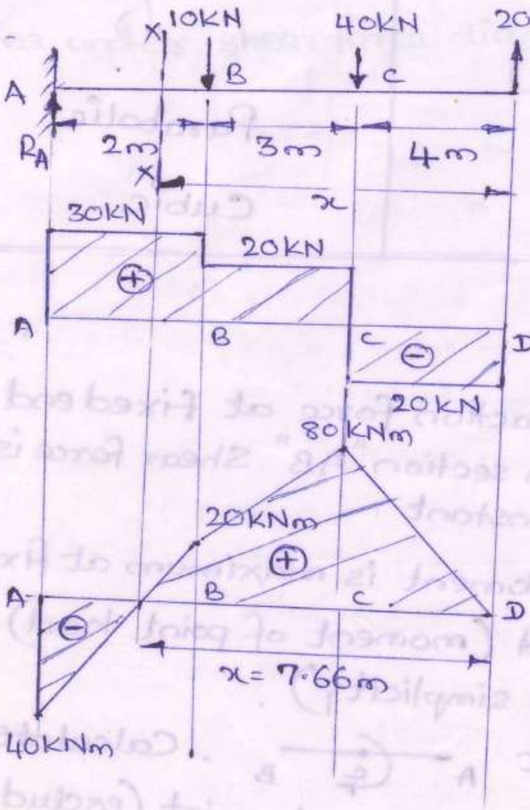
$M_C = -[20\times1]-[10\times2.5] = -45\,KNm$.

$M_D = -(10 \times 1.5) = -15$ kNm.

$M_B = 0$

② Draw the SFD and BMD for a cantilever shown.

Soln:-



(a) Shear Force :-

$F_A = R_A = 10 + 40 - 20 = 30$ KN.

(left) $F_B = R_A = 30$ KN.

(right) $F_B = 40 - 20 = 20$ KN

(left) $F_C = 30 - 10 = 20$ KN

(right) $F_C = -20$ KN.

(left) $F_D = 30 - 10 - 40 = -20$ KN.

∴ In SFD, Scale 10KN = 1cm

(b) Bending moment

$M_A = -(10 \times 2) - (40 \times 5) + (20 \times 9) = -40$ kNm

$M_B = -(40 \times 3) + (20 \times 7) = 20$ KNm.

$M_C = 20 \times 4 = 80$ KNm.

$M_D = 0$.

Scale :- 10kNm = 1cm.

Note:- Section x-x the bending moment changes sign. This point at a distance "x" from free end is called "point of contraflexure"

∴ (c) Distance "x" :-

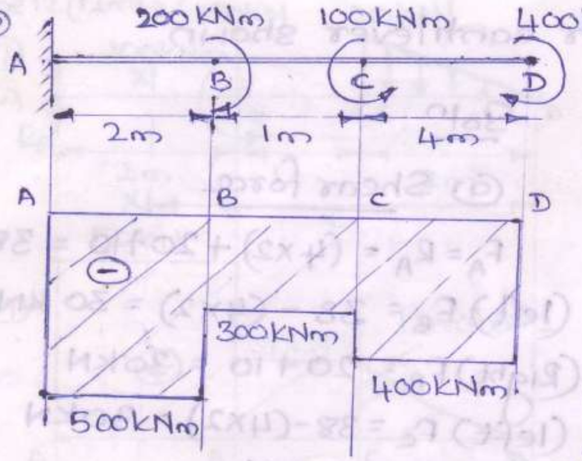
Taking moment about section x-x.

$\therefore M_{x-x} = -10(x-7) - 40(x-4) + 20(x)$

$M_{x-x} = 0 = -10x + 70 - 40x + 160 + 20x$

$\boxed{x = 7.66\text{ m}}$

(3)



Soln:-

(a) Shear force:- Beam subjected to pure bending does not have any shear force.

(b) Bending Moment:-

$M_A = -200 + 100 - 400 = -500\,KNm$

(left) $M_B = -500\,KNm$.

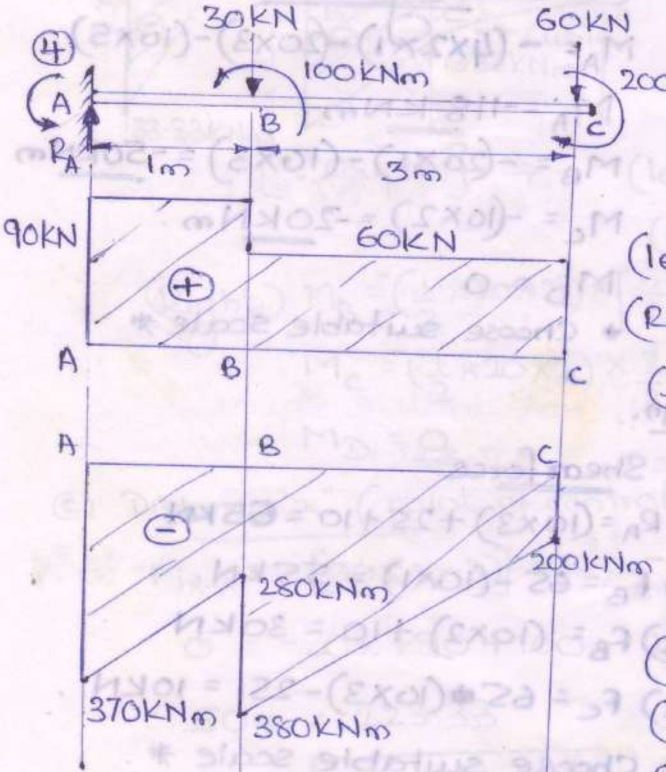
(Right) $M_B = 100 - 400 = -300\,KNm$.

(Left) $M_C = -500 + 200 = -300\,KNm$

(Right) $M_C = -400\,KNm$.

(left) $M_D = -500 + 200 - 100 = -400\,KNm$.

(4)



Soln:-

(a) Shear Force:-

$F_A = R_A = 30 + 60 = 90\,KN$

(left) $F_B = R_A = 90\,KN$

(Right) $F_B = 60\,KN$.

(left) $F_C = 90 - 30 = 60\,KN$.

(b) Bending moment

$M_A = -(30\times1) + 100 - (60\times4) - 200$

$\therefore M_A = -370\,KNm$.

(left) $M_B = -370 + (90\times1) = -280\,KNm$.

(Right) $M_B = -(60\times3) - 200 = -380\,KNm$.

(left) $M_C = -370 + (90\times4) - (30\times3) - 100$

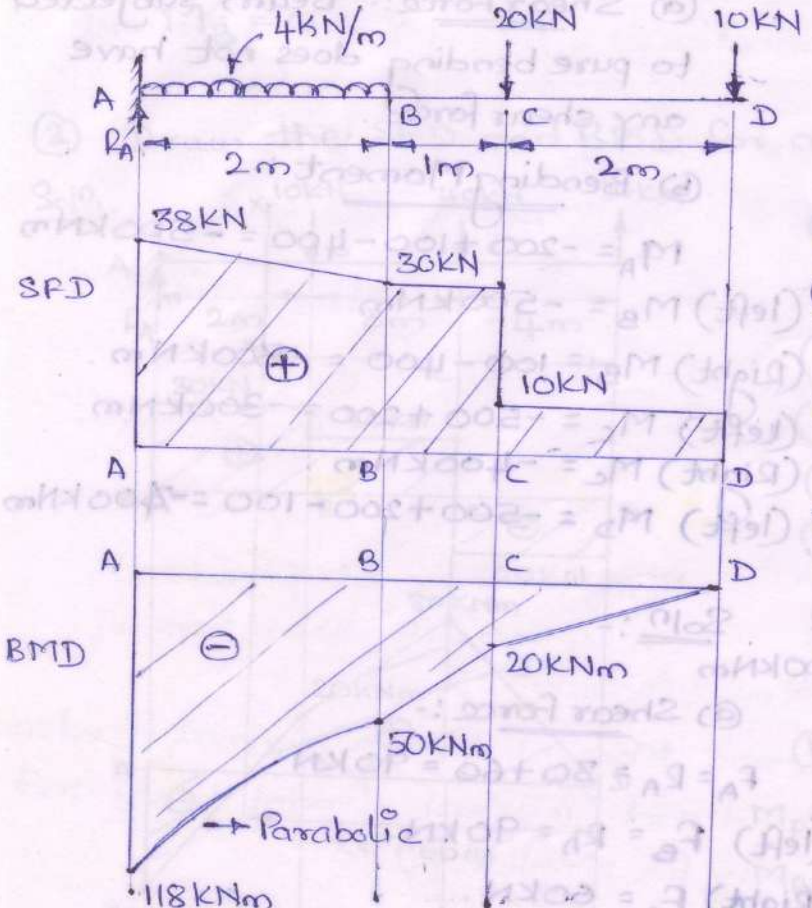
$M_C = -200\,KNm$.

Note:-

On left of a point → CCW -ve
CW +ve

On right of a point → CCW +ve
CW -ve

(5) Draw the SFD and BMD for cantilever shown.



Soln:

(a) Shear force

$F_A = R_A = (4\times2) + 20 + 10 = 38KN$

(left) $F_B = 38 - (4\times2) = 30\ kN$

(Right) $F_B = 20 + 10 = 30kN$

(left) $F_C = 38 - (4\times2) = 30KN$

(Right) $F_C = 10KN$

(left) $F_D = 38 - (4\times2) - 20 = 10kN$

* Choose suitable scale *

(b) Bending Moment

$M_A = -(4\times2\times1) - (20\times3) - (10\times5)$

$M_A = -118\ KNm$

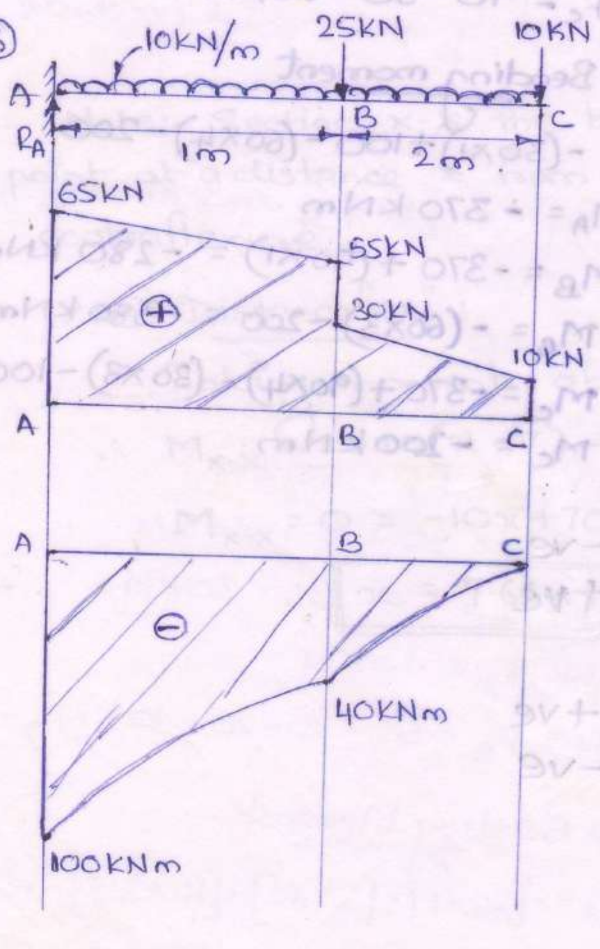
$M_B = -(20\times1) - (10\times3) = -50KNm$

$M_C = -(10\times2) = -20KNm$

$M_D = 0$

* Choose suitable scale *

(6)



Soln:

(a) Shear force

$F_A = R_A = (10\times3) + 25 + 10 = 65KN$

(left) $F_B = 65 - (10\times1) = 55KN$

(right) $F_B = (10\times2) + 10 = 30KN$

(left) $F_C = 65 - (10\times3) - 25 = 10KN$

* Choose suitable scale *

(b) Bending moment

$M_A = -(10\times1\times0.5) - (25\times1) - (10\times2\times2) - (10\times3)$

$\therefore M_A = -100KNm$

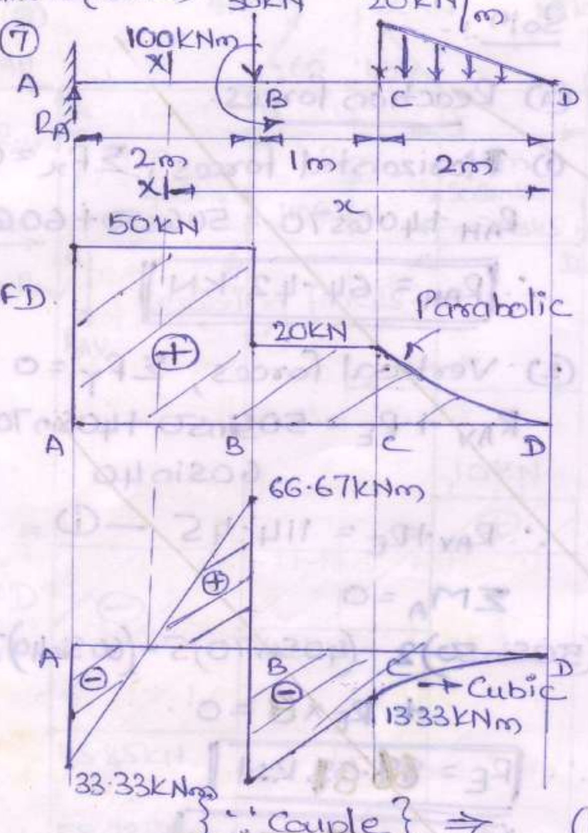
$M_B = -(10\times2\times1) - (10\times2) = -40KNm$

$M_C = 0$

* Choose suitable scale *

June'12 (12mks) Pg 5/5

(7)



Sol^n :-

(a) Shear Force.

$F_A = R_A = 30 + \left(\frac{1}{2}\times 20\times 2\right) = 50KN$.

(left) $F_B = 50KN$.

(Right) $F_B = \frac{1}{2}\times 20\times 2 = 20KN$

(left) $F_C = 50-30 = 20KN$.

(Right) $F_C = \frac{1}{2}\times 20\times 2 = 20KN$.

(left) $F_D = 50-30-\frac{1}{2}\times 20\times 2 = 0$.

* Choose scale *

(b) Bending moment.

$M_A = -(30\times 2) + 100 - \left[\frac{1}{2}\times 20\times 2\right]\times\left[\frac{2}{3}+3\right]$

$\therefore M_A = -33.33 \text{ KNm}$

$\{\because \text{Couple}\} \Rightarrow$ (left) $M_B = M_A + (R_A\times 2) = -33.33 + (50\times 2)$

(left) $M_B = 66.67$ KNm.

(Right) $M_B = -\left(\frac{1}{2}\times 20\times 2\right)\times\left[\frac{2}{3}+1\right] = -33.33$ KNm

$M_C = -\left(\frac{1}{2}\times 20\times 2\right)\times\frac{2}{3} = -13.33$ KNm.

$M_D = 0$

(c) Distance "x" (point of contraflexure):

$M_{x-x} = -[30(x-3)] + 100 - \left[\frac{1}{2}\times 20\times 2\right]\left[x-\left(1+\frac{2}{3}\right)\right] = 0$

$0 = -30x + 90 + 100 - 20x + \frac{100}{3}$

$\therefore 50x = 223.33$

$\therefore \boxed{x = 4.46m}$ from free end of cantilever beam.

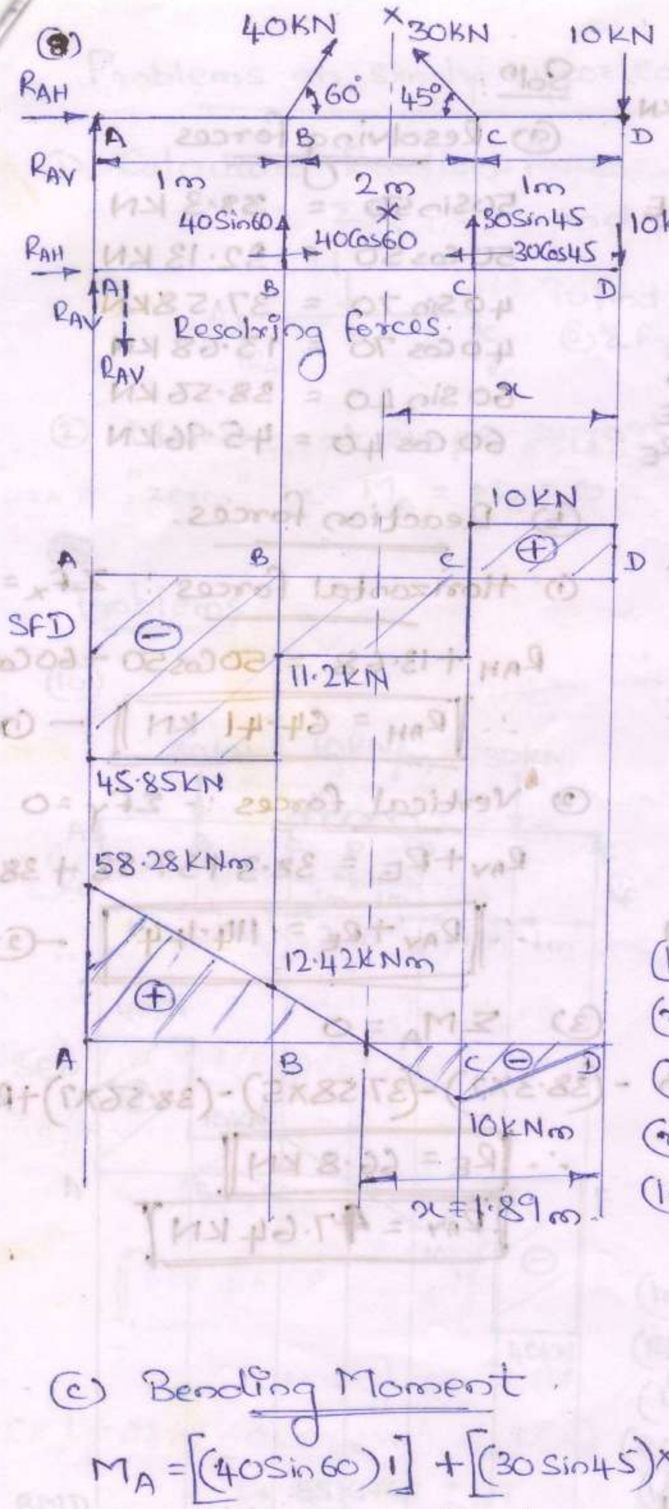
or $x = 5 - 4.46 = 0.54$ m from fixed end.

~~Note: The calculation for (c) is to be corrected~~

Calculation is wrong

Pb on Pg 6/5

(8)



Soln:

(a) Reaction forces

(1) Horizontal reaction forces

$\Sigma F_x = 0$

$R_{AH} + 40\cos 60 = 30\cos 45$

$\therefore \boxed{R_{AH} = 1.06\,KN}$

(2) Vertical reaction forces

$\Sigma F_Y = 0$

$R_{AV} + 40\sin 60 + 30\sin 45 = 10$

$\therefore \boxed{R_{AV} = -45.85\,KN}$

-ve sign indicates "R_{AV}" acts in downward direction.

(b) Shear force

$F_A = R_{AV} = -(40\sin 60) - (30\sin 45) + 10$

$\therefore R_{AV} = -45.85\,KN$

(left) $F_B = -45.85\,KN$

(right) $F_B = (-30\sin 45) + 10 = -11.2\,KN$

(left) $F_C = -45.85 + (40\sin 60) = -11.2\,KN$

(right) $F_C = 10\,KN$

(left) $F_D = -45.85 + (40\sin 60) + (30\sin 45) = 10\,KN$

(c) Bending Moment

$M_A = [(40\sin 60)1] + [(30\sin 45)\times 3] - (10\times 4) = 58.28\,KNm$

$M_B = [(30\sin 45)\times 2] - (10\times 3) = 12.42\,KNm$

$M_C = -(10\times 1) = -10\,KNm$

$M_D = 0$

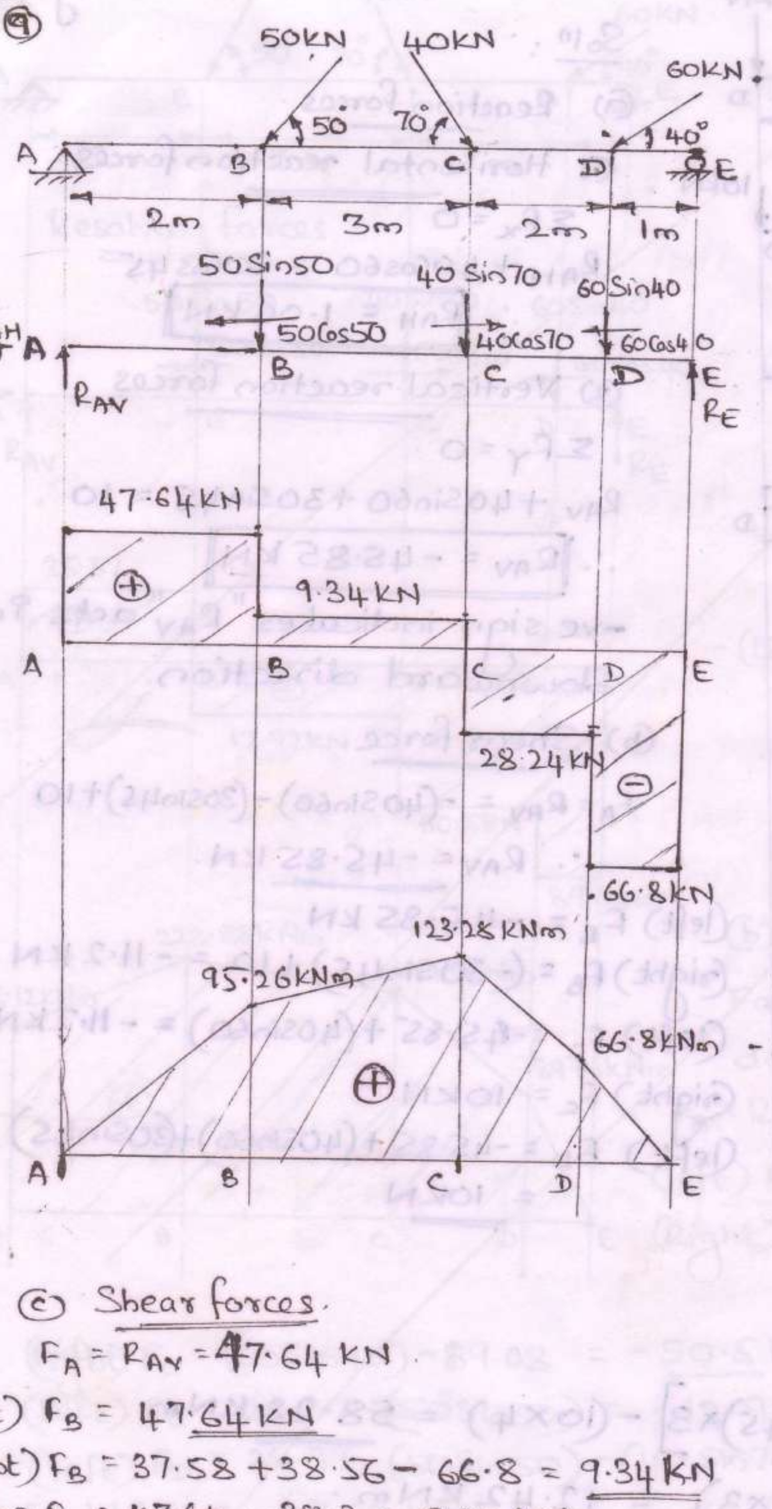
(d) Point of Contraflexure

$M_{x-x} = [(30\sin 45)(x-1)] - 10x = 0$

$21.21x - 21.21 - 10x = 0$

$\therefore \boxed{x = 1.89\,m}$ from point D.

(9)



Soln:

(a) Resolving forces

$50\sin 50 = 38.3$ KN
$50\cos 50 = 32.13$ KN
$40\sin 70 = 37.58$ KN
$40\cos 70 = 13.68$ KN
$60\sin 40 = 38.56$ KN
$60\cos 40 = 45.96$ KN

(b) Reaction forces.

(1) Horizontal forces : $\Sigma F_x = 0$

$R_{AH} + 13.68 = 50\cos 50 + 60\cos 40$

$\therefore \boxed{R_{AH} = 64.41 \text{ KN}}$ — (1)

(2) Vertical forces : $\Sigma F_y = 0$

$R_{AV} + R_E = 38.3 + 37.58 + 38.56$

$\therefore \boxed{R_{AV} + R_E = 114.44}$ — (2)

(3) $\Sigma M_A = 0$

$-(38.3\times2) - (37.58\times5) - (38.56\times7) + R_E\times8 = 0$

$\therefore \boxed{R_E = 66.8 \text{ KN}}$

$\boxed{R_{AY} = 47.64 \text{ KN}}$

(c) Shear forces.

$F_A = R_{AY} = 47.64$ KN

(left) $F_B = 47.64$ KN

(Right) $F_B = 37.58 + 38.56 - 66.8 = 9.34$ KN

(left) $F_C = 47.64 - 38.3 = 9.34$ KN

(Right) $F_C = 38.56 - 66.8 = -28.24$ KN

(left) $F_D = 47.64 - 38.3 - 37.58 = -28.24$ KN

(Right) $F_D = -66.8$ KN,

(left) $F_E = 47.64 - 38.3 - 37.58 - 38.56 = -66.8$ KN

(d) Bending Moment

$M_A = -[38.3\times2] - [37.58\times5] - [38.56\times7] + [66.8\times8] = 0$

$M_B = -[37.58\times3] - [38.56\times5] + [66.8\times6] = 95.26$ KNm.

$M_C = -[38.56\times2] + [66.8\times3] = 123.28$ KNm

$M_D = 66.8\times1 = 66.8$ KNm

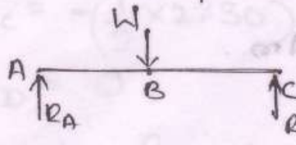
$M_E = 0$

Problems on simply supported beams.

① Calculate Reaction forces using equilibrium eq^n.

(a) $\Sigma F_Y = 0$ and (b) $\Sigma M = 0$ [moment about any point]

Ex:



To find two reaction forces.

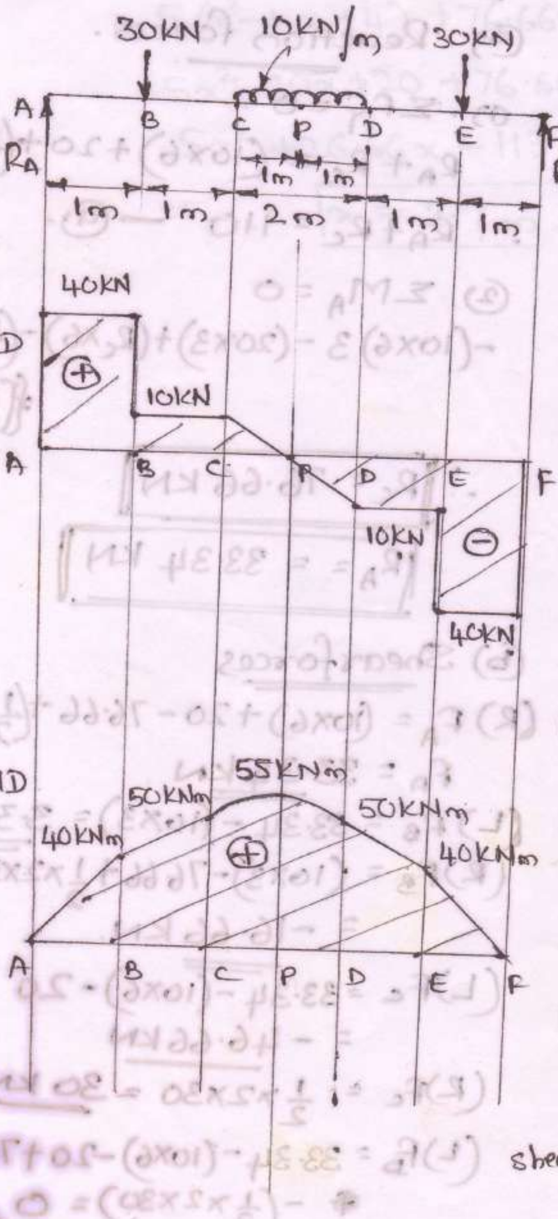
(a) $\Sigma F_Y = 0$, and (b) $\Sigma M_A = 0$.

② Moment at simple support point (i.e A & C in example above) is "zero" i.e $M_A = M_C = 0$.

③

Problems

(10)



Sol^n:-

(a) Reaction forces.

(i) $\Sigma F_Y = 0$

$R_A + R_F = 30 + (10 \times 2) + 30 = 80KN$ —(b)

Since beam is symmetrically loaded

$\boxed{R_A = R_F}$ $\therefore R_A + R_A = 80KN$

$\boxed{\begin{array}{l} R_A = 40KN \\ R_F = 40KN \end{array}}$ —(a)

(b) Shear forces.

$F_A = R_A = 40KN$.

(left) $F_B = 40KN$

(Right) $F_B = (10 \times 2) + 30 - 40 = 10KN$.

(left) $F_C = 40 - 30 = 10KN$.

(Right) $F_C = (10 \times 2) + 30 - 40 = 10KN$.

(left) $F_D = 40 - 30 - (10 \times 2) = -10KN$

(Right) $F_D = 30 - 40 = -10KN$.

(left) $F_E = 40 - 30 - (10 \times 2) = -10KN$

(Right) $F_E = -40KN$.

(left) $F_F = 40 - 30 - (10 \times 2) - 30 = -40KN$.

(left) $F_P = 40 - 30 - (10 \times 1) = 0$

(Right) $F_P = (10 \times 1) + 30 - 40 = 0$

Note :- Shear force a "P" is calculated to shearforce at mid-span of UDL-section "CD".

Pg 7/5

(c) Bending moment

$M_A = 0$

$M_B = -(10\times2)2 - (30\times4) + (40\times5) = 40\,kNm$

$M_C = -(10\times2)1 - (30\times3) + (40\times4) = 50\,kNm$

$M_D = -(30\times1) + (40\times2) = 50\,kNm$

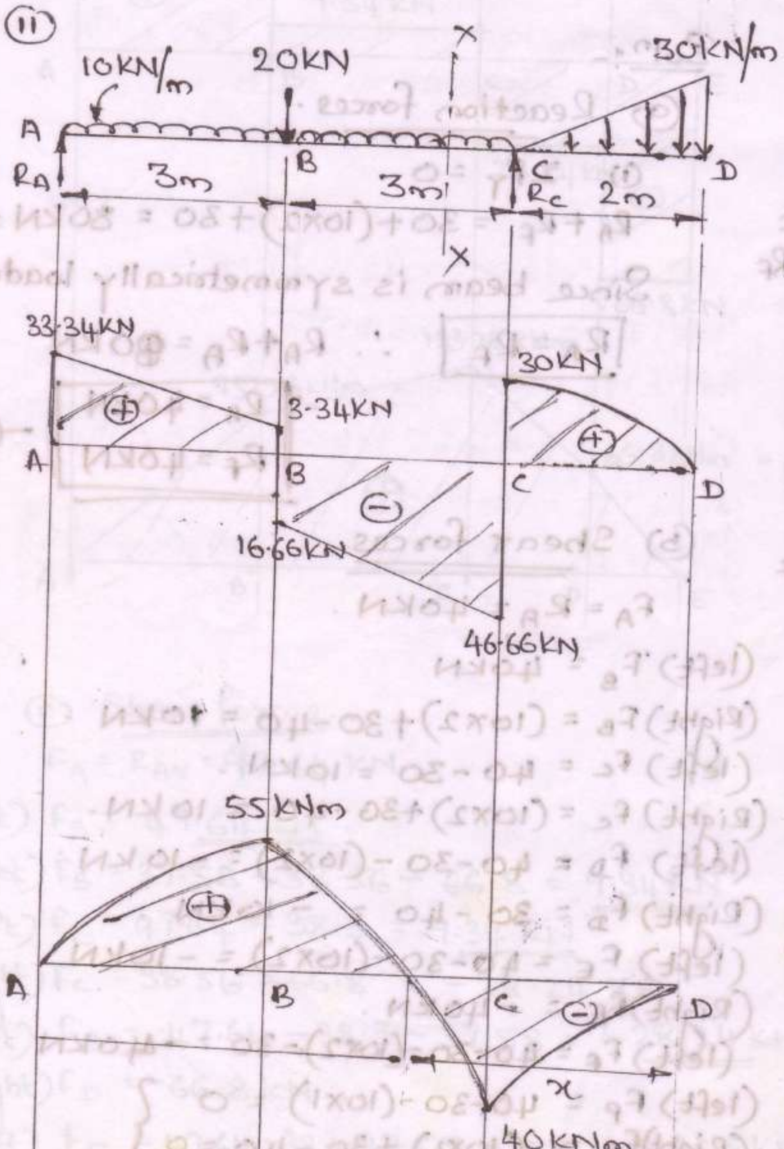
$M_E = 40\times1 = 40\,kNm$

$M_F = 0$

$M_P = -(10\times1)\frac{1}{2} - (30\times2) + (40\times3) = 55\,kNm$

Note :- To find the maximum BM on UDL section, we will calculate moment about "P"

(11)



Soln:

(a) Reaction forces.

(1) $\Sigma F_Y = 0$

$R_A + R_C = (10\times6) + 20 + (\frac{1}{2}\times2\times30)$

$\therefore R_A + R_C = 110$ — (a).

(2) $\Sigma M_A = 0$

$-(10\times6)3 - (20\times3) + (R_C\times6) - (\frac{1}{2}\times2\times30)\{[\frac{2}{3}\times2] + 6\}$

$\therefore \boxed{R_C = 76.66\,KN}$

$\boxed{R_A = 33.34\,KN}$

(b) Shearforces

(R) $F_A = (10\times6) + 20 - 76.66 + (\frac{1}{2}\times2\times30)$

$F_A = 33.34\,kN$

(L) $F_B = 33.34 - (10\times3) = 3.34\,KN$

(R) $F_B = (10\times3) - 76.66 + \frac{1}{2}\times2\times30$

$= -16.66\,KN$

(L) $F_C = 33.34 - (10\times6) - 20$

$= -46.66\,kN$

(R) $F_C = \frac{1}{2}\times2\times30 = 30\,KN$

(L) $F_D = 33.34 - (10\times6) - 20 + 76.66 - (\frac{1}{2}\times2\times30) = 0$

(c) Bending moment

$M_A = 0$.

$M_B = -(10\times3)1.5 + (76.66\times3) - \left(\frac{1}{2}\times2\times30\right)\left[3+\left(\frac{2}{3}\times2\right)\right] = \underline{\underline{55\ \text{KNm}}}$

$M_C = -\left(\frac{1}{2}\times2\times30\right)\left(\frac{2}{3}\times2\right) = -40\,\text{KNm}$

$M_D = 0$

(d) Point of contraflexure

$\Sigma M_{x-x} = 0$

$$-\left\{\left[10(x-2)\right]\left(\frac{x-2}{2}\right)\right\} + \left(76.66\times(x-2)\right) - \left(\frac{1}{2}\times2\times30\right)\left[(x-2)+\left(\frac{2}{3}\times2\right)\right]$$

$$5(x^2-4x+4) + 76.66x - 153.32 - 30\left[x-2+\frac{4}{3}\right] = 0$$

$$5x^2-20x+20+76.66x-153.32-30x+60-40 = 0$$

$$5x^2+26.66x-113.32 = 0$$

$\therefore \boxed{x = 2.7\,m}$ from point "D".