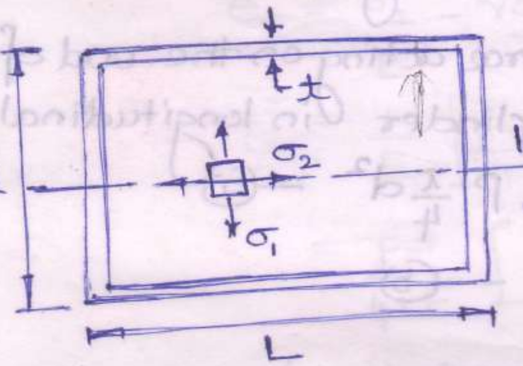
THIN AND THICK CYLINDERS

Cylinders and spheres are industrial components subjected to internal and external pressures. Cylinder/spheres with thickness less than $1/10^{th}$ to $1/15^{th}$ of the radius are called thin cylinders. The radial pressure in thin cylinders is negligible, but in thick cylinders it should be considered.

Stresses in thin cylinders.



Assumptions:-

(1) Radial stress is small and hence negligible.

(2) The hoop stress and longitudinal stress is uniformly distributed.

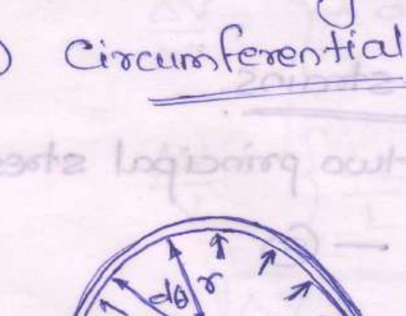
Consider a thin cylinder as shown, having both ends closed.

Let, d = diameter,
t = thickness
L = Length
p_1 = internal pressure.
σ_1 = circumferential stress
σ_2 = longitudinal stress

(a) Circumferential stress (Hoop stress) "σ_1"

Force on elemental length due to internal pressure = $p r d\theta L$ —(a).

Bursting pressure force = $p r L \cos\theta \, d\theta$ —



Total bursting force $= 2\int_0^{\pi/2} p r L \cos\theta \, d\theta$

$= 2 p r L \left[\sin\theta\right]_0^{\pi/2}$

Total bursting force $= pdL$ —— ① $\{\because 2r = d\}$

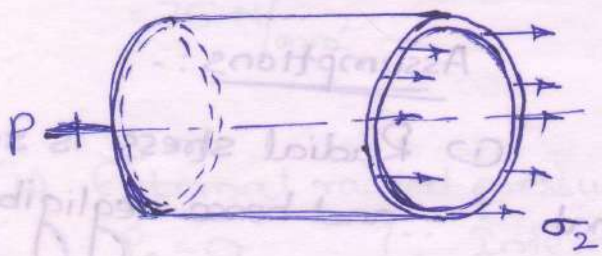
Resisting force $= 2\sigma_1 tL$ —— (2) It resists the bursting force

$\therefore$ Equating eq^n (1) and (2)

$$2\sigma_1 tL = pdL$$

$\therefore$ Hoop stress, $\boxed{\sigma_1 = \frac{pd}{2t}}$ —— (i)

(b) Longitudinal stress "σ_2"



Consider transverse section of the cylinder.

Force acting on the end of cylinder in longitudinal dir^n

$$= p\frac{\pi}{4}d^2 \quad \text{—— (a)}$$

Resisting force $= \sigma_2 \pi dt$ —— (b)

Equating (a) and (b).

$$\sigma_2 \pi dt = p\frac{\pi}{4}d^2$$

Longitudinal stress, $\boxed{\sigma_2 = \frac{pd}{4t}}$ —— (ii)

From (i) and (ii)

Longitudinal stress $= \frac{1}{2}$ Hoop stress

Circumferential and longitudinal strains.

(a) At any point in cylinder, the two principal stresses are

hoop stress, $\sigma_1 = \frac{pd}{2t}$ —— (a)

longitudinal stress, $\sigma_2 = \frac{pd}{4t}$ —— (b)

Let "γ" = Poisson's ratio.

(i) Then strains due to stress in circumference

$$\epsilon_1 = \frac{\sigma_1}{E} - \gamma\frac{\sigma_2}{E}$$

$$= \frac{1}{E}\left[\frac{pd}{2t} - \gamma\frac{pd}{4t}\right]$$

$$\epsilon_1 = \frac{pd}{4tE}\left[2-\gamma\right] \quad —— ①$$

Also, ϵ_1 = diametrical strain = $\frac{\Delta d}{d} = \frac{pd}{4tE}\left[2-\gamma\right]$ — ⓒ.

(ii) Strain due to longitudinal stress.

$$\epsilon_2 = \frac{\sigma_2}{E} - \frac{\gamma\sigma_1}{E}$$

$$= \frac{1}{E}\left[\frac{pd}{4t} - \gamma\frac{pd}{2t}\right]$$

$$= \frac{pd}{4tE}\left[1-2\gamma\right] \quad —— ②$$

Also, ϵ_2 = longitudinal strain = $\frac{\Delta L}{L} = \frac{pd}{4tE}(1-2\gamma)$ — ⓓ.

(iii) Volumetric strain = $\frac{\Delta V}{V}$ — ⓔ

Volume, $V = \frac{\pi}{4}d^2L$ —(f) is a function of "d" and "L"

Differentiating (f), ∴ $\Delta V = dV = \frac{\pi}{4}d^2\Delta L + 2\frac{\pi}{4}d\Delta d L$ —(g)

Substituting (f) and (g) in ⓔ.

$$\frac{\Delta V}{V} = \left\{\frac{\pi}{4}d^2\Delta L + \frac{\pi}{2}dL\Delta d\right\} \Big/ \frac{\pi}{4}d^2L$$

$$\frac{\Delta V}{V} = \left\{\frac{\Delta L}{L} + 2\frac{\Delta d}{d}\right\}$$

$$\therefore \boxed{\frac{\Delta V}{V} = 2\epsilon_1 + \epsilon_2} \quad —— ③$$ Substituting from ① and ② in ③

2/2

$$\frac{\Delta V}{V} = 2\left\{\frac{pd}{4tE}\left[2-\nu\right]\right\} + \left\{\frac{pd}{4tE}\left[1-2\nu\right]\right\}$$

$$= \frac{pd}{4tE}\left\{4-2\nu+1-2\nu\right\}$$

$$\therefore \boxed{\frac{\Delta V}{V} = \frac{pd}{4tE}\left[5-4\nu\right]} \quad \text{—} (4)$$

Problems :-

(1) A cylindrical shell is 3m long and is having 1m internal diameter and 15mm thickness. Calculate the maximum intensity of shear stress induced and also the changes in the dimensions of the shell, if it is subjected to an internal fluid pressure of 1.5 N/mm^2. Take, $E = 2\times10^5\ N/mm^2$ and $\nu = 0.3$.

Soln : Data Given

$L = 3m = 3\times10^3\ mm$

$d = 1m = 1\times10^3\ mm$

$t = 15\ mm$

$p = 1.5\ N/mm^2$

$E = 2\times10^5\ N/mm^2$

$\nu = 0.3$

Soln :-

(a) Shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$ — (1)

$\sigma_1 = \frac{pd}{2t} = \frac{1.5\times1\times10^3}{2\times15} = 50\ N/mm^2$

$\sigma_2 = \frac{pd}{4t} = \frac{1.5\times1\times10^3}{4\times15} = 25\ N/mm^2$

$\therefore \tau_{max} = \frac{50-25}{2} = 12.5\ N/mm^2$

(b) $\epsilon_1 = \frac{\Delta d}{d} = \frac{pd}{4tE}\left[2-\nu\right] = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E}$

$$\therefore \Delta d = \frac{1\times10^3}{2\times10^5}\left[50-(0.3\times25)\right]$$

$$\therefore \boxed{\Delta d = 0.2125}\ mm \quad \text{—} (a)$$

(c) $\epsilon_2 = \frac{\Delta L}{L} = \frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E}$

$\therefore \Delta L = \frac{3\times10^3}{2\times10^5}\left[25-(0.3\times50)\right]$

$\therefore \boxed{\Delta L = 0.15}$ mm — (b)

(d) $\frac{\Delta V}{V} = 2\epsilon_1 + \epsilon_2$

$\therefore \Delta V = \left\{\frac{\pi}{4}\times(1\times10^3)^2\times3\times10^3\right\}\left[(2\times2.125\times10^{-4})+(5\times10^{-5})\right]$

$\therefore \boxed{\Delta V = 1119.1923\times10^3}$ mm³ — (c)

(2) A thin cylindrical shell, 2 m long has 200 mm diameter and thickness of metal 10 mm. It is filled completely with a fluid at atmospheric pressure. If an additional 25000 mm³ fluid is pumped in, find the pressure developed and hoop stress developed. Find also the changes in diameter and length. Take, $E = 2\times10^5$ N/mm² and $\nu = 0.3$.

Soln:- Data:-

$L = 2m = 2\times10^3$ mm

$d = 200$ mm

$t = 10$ mm

$P_1 = P_{atm} = 1 bar = 10^5 Pa$.

$V_2 - V_1 = \Delta V = 25000$ mm³

$E = 2\times10^5$ N/mm²

$\nu = 0.3$

Soln:-

(a) Hoop stress, $\sigma_1 = \frac{pd}{2t}$ —(a)

$\therefore \sigma_1 = \frac{200p}{2\times10} = 10p$ —(a)

(b) $\epsilon_1 = \frac{\Delta d}{d} = \frac{pd}{4tE}\left[2-\nu\right]$

$\therefore \epsilon_1 = \frac{200p}{4\times10\times2\times10^5}\left[2-0.3\right]$

$\therefore \Delta d = 4.25\times10^{-5}\, p\times200$

$\therefore \Delta d = 8.5\times10^{-3}\, p$ —(b)

(c) $\epsilon_2 = \frac{\Delta L}{L} = \frac{pd}{4tE}\left[1-2\nu\right]$

$\therefore \Delta L = \frac{2\times10^3 \times 200\, p}{4\times10\times2\times10^5}\left[1-(2\times0.3)\right]$

$\therefore \Delta L = 0.02\, p$ — (c)

(d) $\frac{\Delta V}{V} = 2\epsilon_1 + \epsilon_2$

$\frac{25000}{\frac{\pi}{4}d^2 L} = \left[(2\times4.25)+1\right]\times10^{-5} p$

from (b) & (c)
$\epsilon_1 = 4.25\times10^{-5} p$
$\epsilon_2 = 1\times10^{-5} p$

$\therefore p = \frac{3.978\times10^{-4}}{9.5\times10^{-5}}$

$\therefore \boxed{p = 4.187}\ N/mm^2$ — (1)

(e) Hoop stress, $\sigma_1 = 10\times p = 41.87\ N/mm^2$ — (2)

longitudinal stress, $\sigma_2 = \frac{pd}{4t} = \frac{\sigma_1}{2} = 20.935\ N/mm^2$ — (3)

$\Delta d = 8.5\times10^{-3} p = 0.0355$ mm — (4)

$\Delta L = 0.02 p = 0.0837$ mm — (5)

June/July'13 [10 marks]

(3) A thin cylindrical shell 1m diameter and 3m long has a metal thickness of 10mm. It is subjected to an internal fluid pressure of 3MPa. Determine the change in length, diameter and volume. Also find the maximum shearing stress in the shell. Assume Poisson's ratio is 0.3 and E = 210 GPa.

Soln: Data:

$d = 1m = 1\times10^3$ mm

$L = 3m = 3\times10^3$ mm

$t = 10$ mm

$p = 3$ MPa

$\nu = 0.3$

$E = 210$ GPa $= 210\times10^3$ MPa

Sol^n :- (a) Hoop's stress, $\sigma_1 = \frac{pd}{2t}$

$$= \frac{3 \times 1 \times 10^3}{2 \times 10}$$

$$\therefore \boxed{\sigma_1 = 150} \ N/mm^2 \quad \text{—(a)}$$

(b) Longitudinal stress, $\sigma_2 = \frac{\sigma_1}{2} = \frac{pd}{4t}$

$$\sigma_2 = \frac{150}{2}$$

$$\therefore \boxed{\sigma_2 = 75} \ N/mm^2 \quad \text{—(b)}$$

(c) Circumferential strain, $\epsilon_1 = \frac{\Delta d}{d} = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E}$

$$\therefore \Delta d = \left(\frac{1 \times 10^3}{210 \times 10^3}\right)\left[150 - (0.3 \times 75)\right]$$

$$\therefore \boxed{\Delta d = 0.607} \ mm \quad \text{—(c)}$$

(d) Longitudinal strain, $\epsilon_2 = \frac{\Delta L}{L} = \frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E}$

$$\therefore \Delta L = \frac{3 \times 10^3}{210 \times 10^3}\left[75 - (0.3 \times 150)\right]$$

$$\boxed{\Delta L = 0.428} \ mm \quad \text{—(d)}$$

(e) Volumetric change, $\frac{\Delta V}{V} = 2\epsilon_1 + \epsilon_2$

$$\therefore \Delta V = \frac{\pi}{4} d^2 L (2\epsilon_1 + \epsilon_2)$$

$$= \frac{\pi}{4} \times (1 \times 10^3)^2 \times (3 \times 10^3) \left[(2 \times 6.076 \times 10^{-4}) + (1.426 \times 10^{-4})\right]$$

$$\therefore \boxed{\Delta V = 3.197 \times 10^6} \ mm^3 \quad \text{—(e)}$$

(f) Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{150 - 75}{2}$

$$\therefore \boxed{\tau_{max} = 37.5} \ N/mm^2 \quad \text{—(f)}$$

Dec'11 (12 marks)

(4) A thin cylinder of 75 mm internal diameter and 250 mm long has 2.5 mm thick walls. The cylinder is subjected to an internal pressure of 7 MN/m². Determine the change in internal diameter and change in the length of the cylinder. Also, compute, the Hoop stress and Longitudinal stress in the cylinder. Take $E = 2\times10^5$ N/mm² and $\nu = 0.3$

Soln:- Data:-

$d = 75$ mm

$L = 250$ mm

$t = 2.5$ mm

$p = 7\ MN/m^2 = \boxed{7\ N/mm^2}$

$E = 2\times10^5\ N/mm^2$

$\nu = 0.3$

Soln:-

(a) Hoop's stress

$$\sigma_1 = \frac{pd}{2t} = \frac{7\times75}{2\times2.5}$$

$$\therefore \boxed{\sigma_1 = 105}\ N/mm^2$$

(b) Longitudinal Stress,

$$\sigma_2 = \frac{pd}{4t} = \frac{7\times75}{4\times2.5}$$

$$\therefore \boxed{\sigma_2 = 52.5}\ N/mm^2$$

(c) $\epsilon_1 = \frac{\Delta d}{d} = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E}$

$$\therefore \frac{\Delta d}{75} = \frac{105-(0.3\times52.5)}{2\times10^5}$$

$$\epsilon_1 = \frac{\Delta d}{d} = 4.4625\times10^{-4}$$

$$\therefore \boxed{\Delta d = 0.0334}\ mm$$

(d) $\epsilon_2 = \frac{\Delta L}{L} = \frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E}$

$$\epsilon_2 = \frac{52.5-(0.3\times105)}{2\times10^5}$$

$$\epsilon_2 = \frac{\Delta L}{L} = 1.05\times10^{-4}$$

$$\therefore \boxed{\Delta L = 0.02625}\ mm$$

(5) The diameter of a city water supply pipe is 750 mm. It has to withstand a water head of 60 m. Find the thickness of the seamless pipe, if the permissible stress is 20 N/mm². Take unit weight of water as 9810 N/m³.

Soln:- Data :-

$d = 750\,mm$

$H = 60\,m$

$\sigma_{all} = 20\,N/mm^2$

$w = 9810\,N/m^3$

Soln :-

(a) Pressure head, $P = \rho g h = w h$ — (a)

$\left\{ \begin{array}{ll} \rho = \text{mass density} & kg/m^3 \\ w = \text{weight density} & N/m^3 \end{array} \right\}$

$P = 9810 \times 60$

$\therefore$ Pressure, $P = 0.5886 \times 10^6\,N/m^2$

$$\therefore \boxed{p = 0.5886}\ N/mm^2 \quad — (i)$$

(b) Hoop's stress, $\sigma_1 = \frac{pd}{2t}$ — (b)

Assuming $\sigma_1 = \sigma_{all} = 20\,N/mm^2$, put in (b)

$$\therefore t = \frac{pd}{2\sigma_1} = \frac{0.5886 \times 750}{2 \times 20}$$

$$\therefore \boxed{t = 11.036}\ mm$$

THICK CYLINDERS

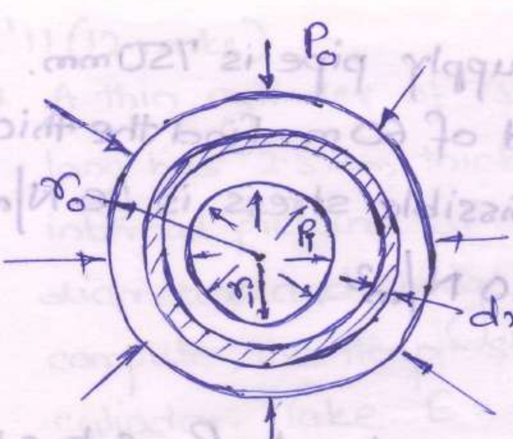
Cylinders with thickness more than $1/10^{th}$ of the diameter are called thick cylinders. The radial ~~pressure~~ stress and hoop stress variation across the section are to be considered.

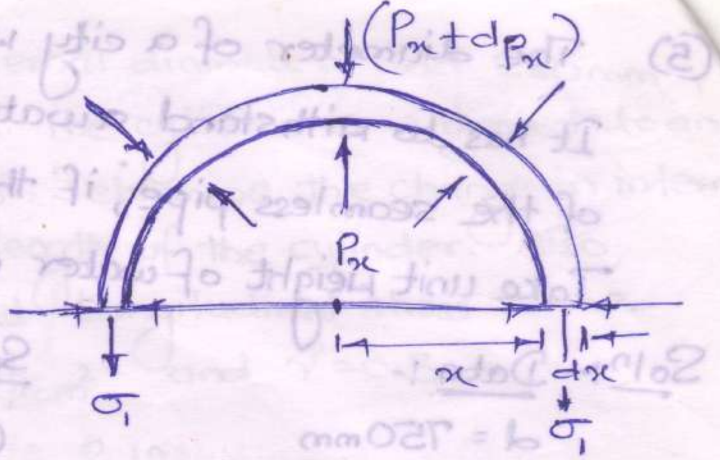
Assumption

(1) Material is homogeneous and isotropic

(2) Limit of proportionality is not exceeded.

(3) Longitudinal strain is uniform across the c/s of cylinder [Lame's the





Let, r_i = inner radius
r_o = outer radius
P_i = internal radial pressure at, $r = r_i$
P_o = external radial pressure at, $r = r_o$

Now, consider a elemental ring of,
x = radius
dx = thickness
L = length of elemental ring.

(a) Bursting force, $= P_x 2xL - (P_x + dP_x)\,2(x+dx)\,L$

$= \cancel{P_x(2x)L} - \cancel{P_x(2x)L} - P_x(2dx)L - dP_x(2x)L - dP_x(2dx)L$

Bursting force $= -P_x\,2dx\,L - dP_x\,2x.L$ — (a) {Neglecting higher order terms

(b) Hoop's Stress, "σ_1"
∴ Resisting force $= \sigma_1\,2dx\,L$ — (b)
Equating equations (a) and (b).

$$\sigma_1\,2dx\,L = -2P_x\,dx\,L - 2\,dP_x.x.L$$

$$\sigma_1\,dx = -P_x\,dx - x\,dP_x \quad \to \text{Divide by } dx.$$

$$\boxed{\sigma_1 + P_x + x\frac{dP_x}{dx} = 0} \quad \text{— (1)}$$

(c) Longitudinal stress "σ_2", then strain in longitudinal direction

$$\epsilon_2 = \frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E} + \nu\frac{P_x}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu}{E}\left[\sigma_1 - P_x\right] \quad \text{— (c)}$$

According to Lame's theory, "Longitudinal strain is constant".

In equation (c), ϵ_2 = constant,

ν, E and σ_2 are constants, Then $\sigma_1 - P_x$ = constant

∴ Assuming, $\sigma_1 - P_x = 2a$

$$\therefore \sigma_1 = P_x + 2a \quad \text{—(d)}$$ Put in eq^n (i).

$$P_x + 2a + P_x + x\frac{dP_x}{dx} = 0$$

$$x\frac{dP_x}{dx} = -2\left[P_x + a\right]$$

$$\frac{dP_x}{(P_x + a)} = -2\,\frac{dx}{x}$$ Integrating:

$$\log(P_x + a) = -2\log x + C \qquad \left\{ \text{Let } C = \log b \right.$$

$$\log(P_x + a) = -\log x^2 + \log b$$

$$= \log\left(\frac{b}{x^2}\right)$$

$$\therefore P_x + a = \frac{b}{x^2} \quad \text{—(e)}$$ Put in (d)

$$\therefore \boxed{\sigma_1 = \frac{b}{x^2} + a} \quad \text{—(i)}$$

$$\boxed{P_x = \frac{b}{x^2} - a} \quad \text{—(ii)}$$

Lame's equation for thick cylinders.

where, a and b = arbitrary constants.

x = distance from centre of element radially

i.e $x = r_i$ and $x = r_o$ to find P_i and P_o respectively.

Problems:

July'15 (10 marks)

(1) A C.I pipe has 200 mm internal diameter and 50 mm metal thickness and carries water under a pressure of $5\ N/mm^2$. Calculate the maximum and minimum intensities of circumferential stress and sketch the distribution of circumferential stress intensity and intensity of radial pressure across the section.

Soln: Data:

$d_i = 200\ mm$

$t = 50\ mm$

$P_i = 5\ N/mm^2$

$P_o = 0$

Soln:-

(a) Inner radius, $r_i = \frac{d_i}{2} = 100\ mm$

Outer radius, $r_o = \frac{d_i}{2} + t = 150\ mm$

Inner radial pressure, $P_i = 5\ N/mm^2$

Outer radial pressure, $P_o = 0$.

(b) From Lame's equation

$$P_x = \frac{b}{x^2} - a$$

at $x = r_i$, $P_i = \frac{b}{r_i^2} - a$

$$\therefore 5 = \frac{b}{(100)^2} - a$$

$$\therefore b = 100^2(5 + a) \quad -(a)$$

at $x = r_o$, $P_o = \frac{b}{r_o^2} - a$

$$\therefore 0 = \frac{b}{(150)^2} - a$$

$$\therefore b = 150^2 a \quad -(b)$$

Solving (a) and (b)

$$\left.\begin{array}{l} a = 4 \\ b = 90{,}000 \end{array}\right\} -(c)$$

(c)

(i) Radial pressure.

at $x = r_i = 100\ mm$, $P_i = 5\ N/mm^2$

at $x = r_2 = 125\ mm$, $P_2 = \frac{90000}{125^2} - 4$

$$\therefore P_2 = 1.76\ N/mm^2$$

at $x = r_o = 150\ mm$, $P_o = 0$

(ii) Hoop stress

at $x = r_i = 100\ mm$

$$\sigma_1 = \frac{b}{100^2} + a = 13\ N/mm^2$$

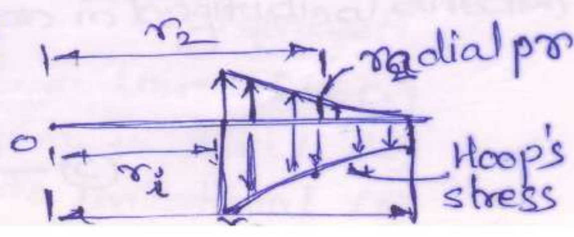
at $x = r_2 = 125\ mm$

$$\sigma_1 = \frac{90000}{125^2} + 4 = 9.76\ N/mm^2$$

at $x = r_o = 150\ mm$

$$\sigma_1 = 8\ N/mm^2$$

(d)



Jan'15 (10 marks)

(2) A thick cylinder with internal diameter 80 mm and external diameter 120 mm is subjected to an external pressure of 40 N/mm², when the internal pressure is 120 N/mm². Calculate circumferential stress at external and internal surfaces of the cylinder. Plot the variation of circumferential stress and radial pressure on the thickness of the cylinder.

Soln: Data:-

$d_i = 2r_i = 80\text{ mm} \quad \therefore r_i = 40\text{ mm}$

$d_o = 2r_o = 120\text{ mm} \quad \therefore r_o = 60\text{ mm}$

$P_o = 40\text{ N/mm}^2$

$P_i = 120\text{ N/mm}^2$

(a) According to Lame's equation

$$P_x = \frac{b}{x^2} - a \quad —(a)$$

at $x = r_i$, $P_i = \frac{b}{r_i^2} - a = \frac{b}{40^2} - a \quad \therefore b = 40^2 a + (120 \times 40^2) \quad —(i)$

at $x = r_o$, $P_o = 40 = \frac{b}{r_o^2} - a$

$\therefore 40 = \frac{b}{60^2} - a \quad \therefore b = 60^2 a + (40 \times 60^2) \quad —(ii)$

Solving (i) and (ii) simultaneously we get

$$\left.\begin{aligned} a &= 24 \\ b &= 230400 \end{aligned}\right\} —(b)$$

(b) Radial Pressures.

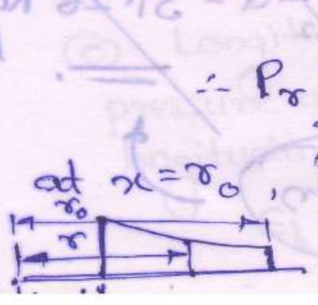
at $x = r_i$, $\underline{P_i = 120\text{ N/mm}^2}$

at $x = r = 50\text{ mm}$, $P_r = \frac{b}{r^2} - a$

$\therefore P_r = \frac{230400}{50^2} - 24$

$\therefore \underline{P_r = 68.16\text{ N/mm}^2}$

at $x = r_o$, $\underline{P_o = 40\text{ N/mm}^2}$

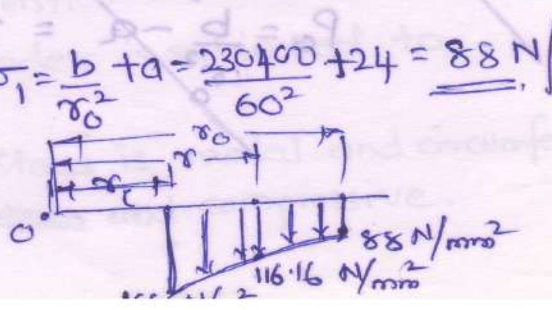


(c) Circumferential stresses.

$\sigma_1 = \frac{b}{r_i^2} + a = \frac{230400}{40^2} + 24 = \underline{168\text{ N/mm}^2}$

$\sigma_1 = \frac{b}{r^2} + a = \frac{230400}{50^2} + 24 = \underline{116.16\text{ N/mm}^2}$

$\sigma_1 = \frac{b}{r_o^2} + a = \frac{230400}{60^2} + 24 = \underline{88\text{ N/mm}^2}$



JUNE '12 (08 marks)

(4) A pressure vessel with outer and inner diameters of 400 mm and 320 mm respectively is subjected to an external pressure of 8 MPa. Determine the circumferential stress induced at the inner and outer surfaces. Prove that the longitudinal strain is constant through out the cylinder.

Soln:- Data:-

$d_o = 400$ mm

$d_i = 320$ mm

$P_o = 8$ MPa

(a) Radial Pressures.

By Lame's theorem.

(i) $P_i = \frac{b}{x^2} - a$ at $x = r_i = 160$ mm

$\therefore 0 = \frac{b}{160^2} - a$

$\therefore b - 160^2 a = 0$ — (a)

(ii) $P_o = \frac{b}{x^2} - a$ at $x = r_o = 200$ mm

$8 = \frac{b}{200^2} - a.$

$\therefore b - 200^2 a = 8 \times 200^2$ — (b)

Solving (a) and (b) we get

$$\left.\begin{aligned} b &= -568888.88 \\ a &= -22.2 \end{aligned}\right\} — (1).$$

(b) Circumferential stresses

$\sigma_i = \frac{b}{x^2} + a$ at $x = r_i$

$\therefore \sigma_i = \frac{-568888.88}{160^2} + (-22.2)$

$\therefore \boxed{\sigma_i = -44.44 \text{ MPa}}$

$\sigma_o = \frac{b}{x^2} + a$ at $x = r_o$.

$\sigma_o = \frac{-568888.88}{200^2} + (-22.2)$

$\therefore \boxed{\sigma_o = -36.4 \text{ MPa}}$

(c) Longitudinal strain :- Circumferential stress and radial pressure are compressive, hence cylinder is subjected to longitudinal tensile strain.

$\because$ Stress is radial and circumferential ~~hence~~ and compressive.

$\epsilon_i = \nu \frac{\sigma_i}{E} + \nu \frac{P_i}{E}$ — (6)

$$\boxed{\epsilon_i = 6.66\times10^{-5}} \quad -(2) \quad = \frac{\nu}{E}\,44.4 \;(\text{---}(2))$$

$$\epsilon_o = \frac{\nu\sigma_o}{E} + \frac{\nu P_o}{E} = \frac{\nu}{E}[36.4+8] = \frac{\nu}{E}\,44.4 \quad -(3)$$

Hence $\epsilon_i = \epsilon_o$.

(3) Soln :-

$r_i = 200\text{mm}$

$r_o = 250\text{ mm}$

$\sigma_{all} = 75\ MN/m^2$

$= 75\ N/mm^2$

(a) $\sigma_i = \sigma_{all} = 75\ N/mm^2$

$\therefore \sigma_i = \frac{b}{x^2} + a$ at $x = r_i$

$75 = \frac{b}{200^2} + a$

$\therefore b + 200^2 a = 75\times200^2 \quad -(a)$

(b) External radial pressure

$P_o = 0$ ($\because$ Internal max pressure, given in statement)

$P_o = \frac{b}{x^2} - a$ at $x = r_o$

$0 = \frac{b}{250^2} - a$

$\therefore b - 250^2 a = 0 \quad -(b)$ Solving (a) and (b)

$$\left.\begin{aligned} \therefore b &= 1829268.29 \\ a &= 29.26 \end{aligned}\right\} -(1)$$

(c) Max internal pressure.

$P_i = \frac{b}{x^2} - a$ at $x = r_i$.

$P_i = \frac{1829268.29}{200^2} - 29.26$

$$\boxed{P_i = 16.47}\ N/mm^2$$