When a member (beams, columns, shafts) is loaded, resisting forces are developed inside the material of the member. (Explanation: External force acting on the cross-section of a member develops pressure on the member. A resisting internal pressure (i.e. opposing to the external pressure) is developed in the material of the member. This resisting force per unit area is termed as STRESS.

Stress at a point is defined as, "The internally resisting force developed per unit area". Stress is represented by the symbol *(SIGMA)* σ.

Mathematically stress,

$$\sigma = \frac{F}{A}$$

Where, F = internal resisting force in Newton, N.

A = Cross-sectional area in mm^2.

Unit of stress: Stress is expressed in N/m^2.

Note: 1 bar = 10^5 Pa (Pressure is measured in bars and Pa stands for Pascal).

1Pa = $1N/m^2$.

1MPa = $1N/mm^2$.

1GPa = $1GN/m^2 = 1x10^3\ N/mm^2$.

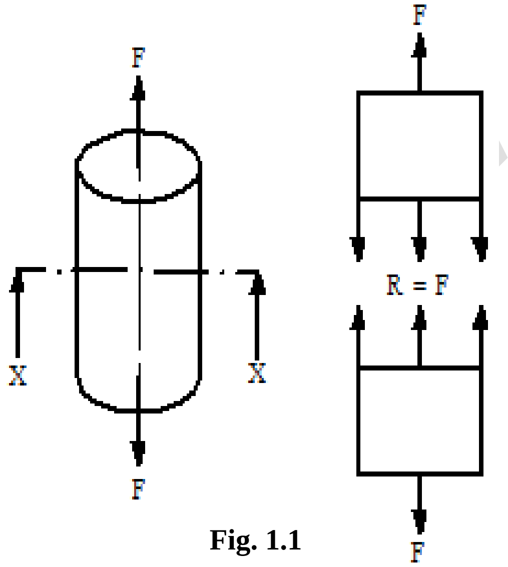


Fig. 1.1

Simple stress: A member subjected to only one type of stress (tensile stress or compressive stress or shear stress) is said to be under simple stress condition. The stresses are assumed to be uniform over the cross-section.

Normal Stress: Internally developed axial load per unit area acting normal (perpendicular) to the cross-section is called normal stress

Normal stress under which the member elongates, such a stress is called normal tensile stress (Fig. 1.2).

Normal stress under which the member compresses, such a stress is called normal compressive stress (Fig. 1.3).

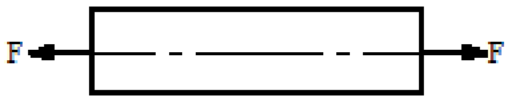


Fig. 1.2

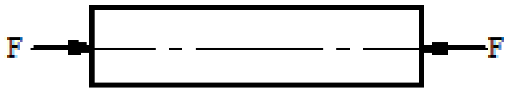


Fig. 1.3

Shear Stress: Internally developed force per unit area acting parallel to the cross-section is called shearing stress (Fig. 1.4). It is represented by the symbol (Tau) τ. Its unit is N/m^2.

$$\tau = \frac{F}{A}$$

Where, F = internal resisting force parallel to the cross section area A, in Newton, N

A = Cross-sectional area in mm^2.

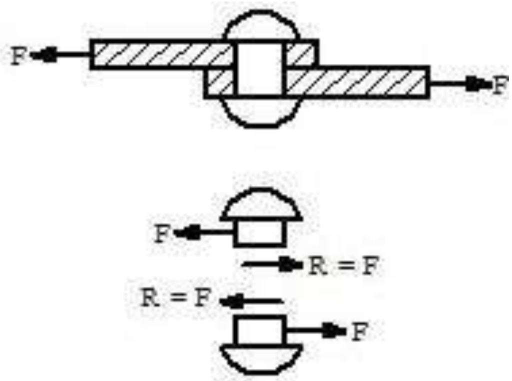


Fig. 1.4

Linear Strain: When a member is loaded it develops stress. The stressed member will undergo a change in its linear dimensions (Fig. 1.5). This change in dimension per unit is called strain. Strain is represented by the symbol ε (epsilon). Strain does not have a unit.

Linear strain is defined as the ratio of change in length to original length.

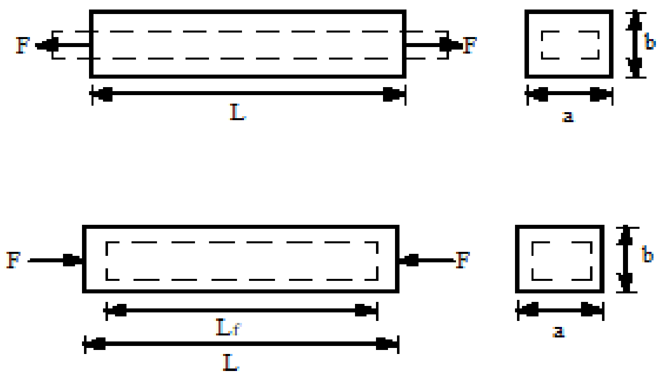


Fig. 1.5

Mathematically, Strain, $\varepsilon = \frac{Change\ in\ length}{Original\ length} = \frac{(Final\ length - Initial\ length)}{Initial\ length} = \frac{\Delta L}{L}$

Assumptions made in the study of mechanics of materials:

1) Material obeys Hooke's Law, i.e. stress is directly proportional to strain within elastic limit.
2) Material is homogeneous, i.e. same properties at all points in the material.
3) Material is isotropic, i.e. same properties in all directions from a point in the material.
4) No residual stresses.

Hooke's Law: Stress is directly proportional to strain within elastic limit.

Mathematically, $\sigma \propto \varepsilon$

$\sigma = E\varepsilon$ -------------------------------- (1.1)

Where, σ = Stress in N/mm^2

ε = Strain

E = Young's modulus or modulus of elasticity in N/mm^2.

Stress-strain diagram for mild steel (ductile material)

A mild steel specimen as per technical standards (as shown in Fig. 1.6) is tested in a universal testing machine by applying tensile force. From the experimental details a graph is plotted called the stress-strain diagram (Fig. 1.7).

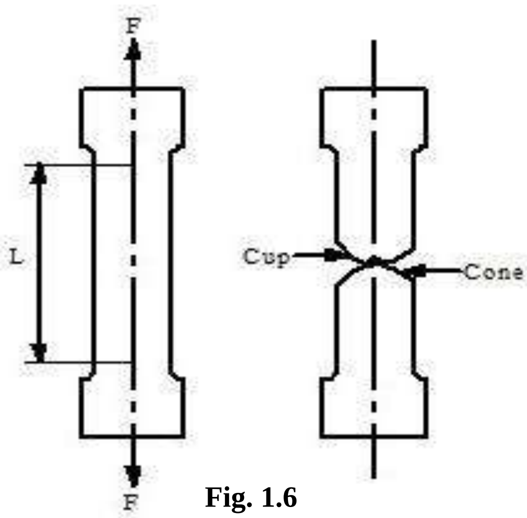


Fig. 1.6

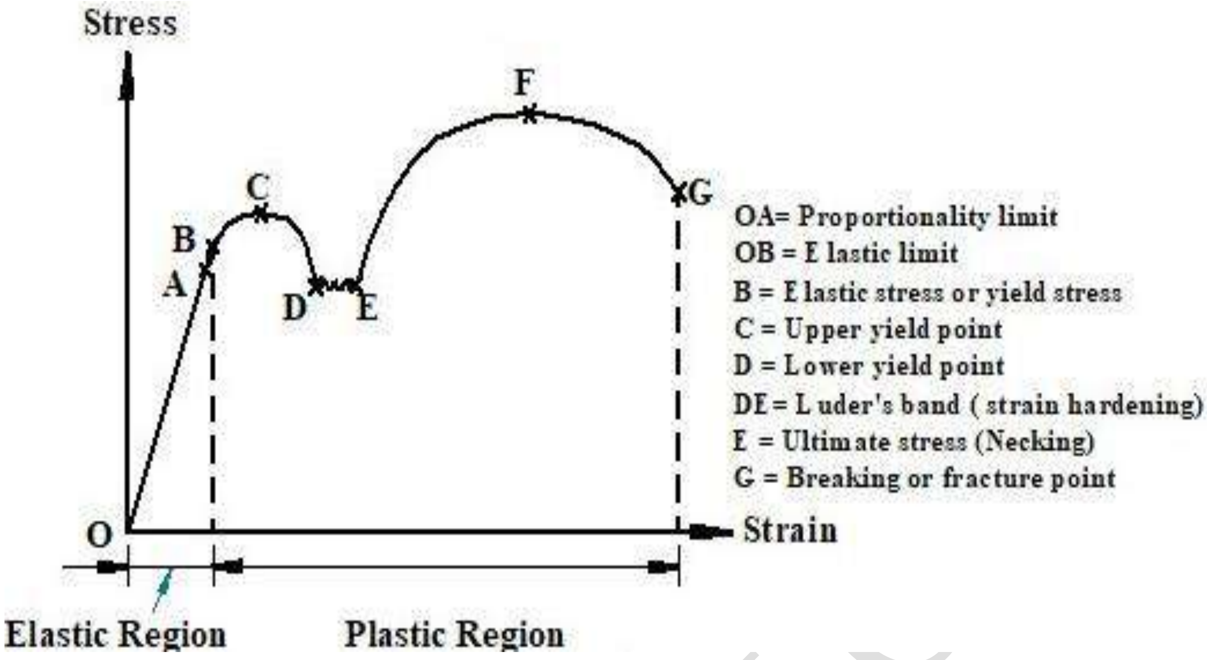


Fig. 1.7

1) OA (proportionality limit) = Linear straight line. Stress is proportional to strain.
2) A (proportionality limit) = Stress is directly proportional to strain till this point.
3) B (elastic limit) = Material behaves elastically i.e. when load is removed material regains its original shape. The stress at this point is called elastic stress. This point is the start of plasticity.
4) C (upper yield point) = Yielding stress beyond which load reduces but elongation increases.
   Note: When material enters plastic region, the changes in dimension become permanent. Even if the load is removed material will not regain its original shape. This is yielding.
5) DE (lower yield point) = At this point elongation takes place at constant load. Luder's band are formed which show stretch marks on the material surface. Strain hardening takes place and material becomes stronger.
6) F (ultimate load point) = this is the maximum load the material can withstand without breaking. Necking takes place at this point. The stress at this point is called ultimate stress.
   Note: Necking is reduction of diameter (i.e. area of cross section) of specimen.
7) G (fracture point) = the material breaks because of rapid cross-sectional area reduction. A cup and cone formation results in the fractured specimen.

Note:

1) Ductility is a property of material by which it can be drawn in thin wires.

Percentage elongation = $\frac{(Final\ length - Initial\ length)}{Initial\ length} X100 = \frac{(L_f - L)}{L} X100$

Percentage reduction in area = $\frac{(Initial\ area - Final\ area)}{Initial\ length} X100 = \frac{(A - A_f)}{A} X100$

Stress-strain diagram for aluminium and high strength steel

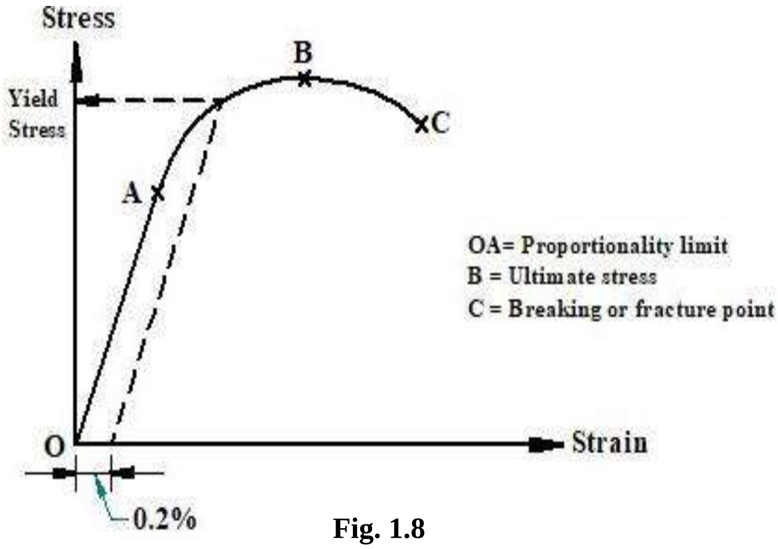


Fig. 1.8

In materials like aluminium and high strength steel the yielding is very less and not well-defined (i.e. start of plastic phase is not clear). The material yield stress is calculated as 0.2% strain (Fig. 1.8). This stress is called as **proof stress**.

If a member is unloaded at proof stress there will be a permanent strain on 0.2%.

Stress-strain diagram for brittle material

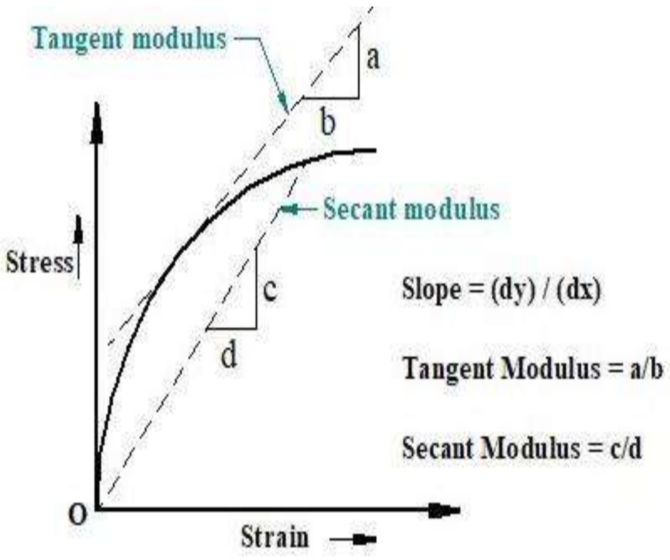


Fig. 1.9

Brittle materials when subjected to tensile force undergo very less elongation. The plastic phase is less for brittle materials. There is no yielding and no necking. Hence at ultimate load brittle materials break.

The modulus of elasticity in brittle materials is determined by (Fig. 1.9),

a) Tangent modulus: The slope of line tangent to the curve at any point.

$$Tangent\ modulus = \frac{dy}{dx} = \frac{a}{b}$$

b) Secant modulus: The slope of line joining the origin to any point on the curve.

$$Secant\ modulus = \frac{dy}{dx} = \frac{c}{d}$$

Nominal stress and True stress

The stress calculated by considering original cross-sectional area is called nominal stress

$$Nominal\ stress = \frac{Load}{Original\ cross - section\ area}$$

The stress calculated by considering instantaneous (actual) cross-sectional area is called true stress,

$$True\ stress = \frac{Load}{Actual\ cross - section\ area}$$

True stress-true strain diagram

True stress and true strain diagram is plotted by considering the actual cross-sectional area of the specimen at an instant corresponding to the applied load. Thus in the plastic region this curve deviates from the engineering stress-strain curve as shown in Fig. 1.10.

Both the curves are same till the elastic limit. Since stresses in a material must be within elastic limit, stress calculation is done by considering original cross-section area. This makes calculations simple.

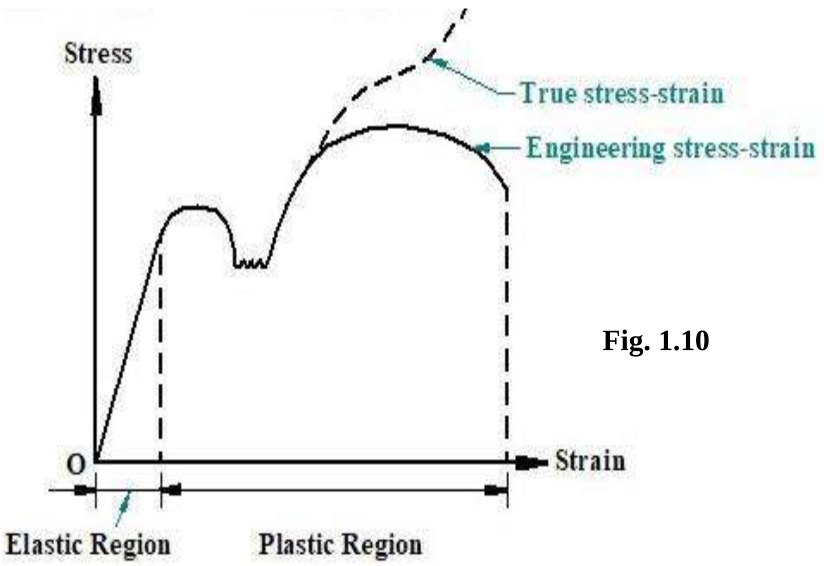


Fig. 1.10

Factor of safety

The material must be loaded within elastic limit to keep it safe and prevent plastic deformation. If the load on the material slightly increases beyond the elastic limit, then yielding starts. Thus the stress in working conditions must be less than the maximum stress that can be taken by the material.

Factor of safety for brittle material is defined as, "The ratio of ultimate stress to working stress".

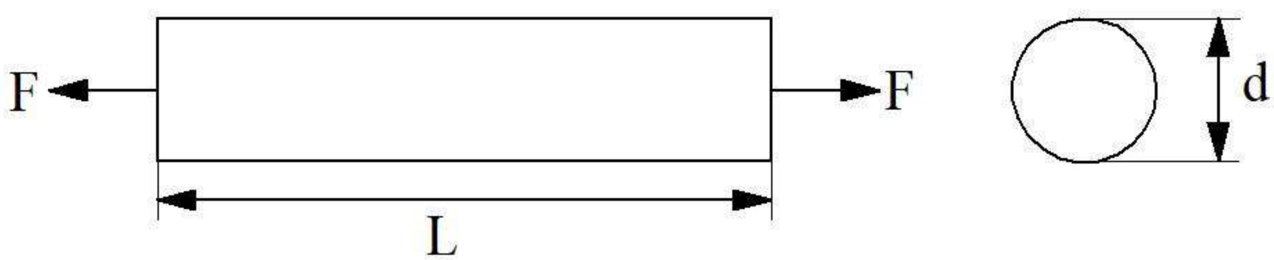
$$Factor\ of\ safety\ (FOS) = \frac{Ultimate\ stress}{Working\ stress}$$

Factor of safety for ductile material is defined as, "The ratio of yield stress to working stress".

$$Factor\ of\ safety\ (FOS) = \frac{Yield\ stress}{Working\ stress}$$

Derivations:

1) **Elongation in a bar of uniform cross-section**



Consider a bar of uniform cross-section subjected to an axial tensile force, "F".

Let, L = length of the bar
d = diameter of the bar
E = Young's modulus of elasticity (material constant)

According to Hooke's law, $\sigma \propto \epsilon$.

$$\therefore \sigma = E\,\epsilon$$

$$\frac{F}{A} = E\,\frac{\Delta L}{L}$$

Therefore elongation or change in length,

$$\boxed{\Delta L = \frac{FL}{AE}} \longrightarrow \textbf{Eq. 1.2}$$

.

Where, A = area of cross section in mm^2.

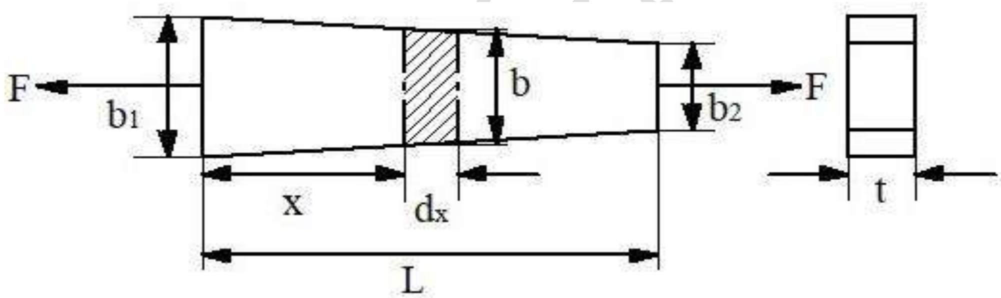
For circular cross-section, $A = \frac{\pi}{4} d^2$

For rectangular cross-section, A = width X thickness = b X t.

2) Elongation in a tapered bar of rectangular cross-section

Question: A bar of uniform thickness 't' tapers uniformly from a width of b_1 at one end to b_2 at other end in a length 'L'. Find the expression for the change in length of the bar when subjected to an axial force F.

Solution:



Consider an element of length 'dx' at a distance 'x' from the bigger end. Let the element width be 'b'.

a) Slope of the tapered bar,

$$\frac{(b_1 - b_2)}{2L} = \frac{(b_1 - b)}{2x}$$

$$b = b_1 - \frac{(b_1 - b_2)x}{L}$$

$$\therefore b = b_1 - kx \qquad where, k = \frac{(b_1 - b_2)}{L}$$

b) The element is so small in length that the width 'b' is considered to be uniform.
Elongation of the element (considering uniform bar of width 'b'),

$$\boxed{\Delta L_x = \frac{F\,dx}{A\,E}} \longrightarrow \text{Integrating within limits 0 to L}$$

$$\Delta L = \int_0^L \frac{Fdx}{AE} = \int_0^L \frac{Fdx}{b\,t\,E} = \frac{F}{tE}\int_0^L \frac{1}{b}dx = \frac{F}{tE}\int_0^L \frac{1}{(b_1 - kx)}dx$$

Note: Formula

$$\left\{\int \frac{1}{(ax+b)}dx = \frac{1}{a}\log(ax+b)\right\} \qquad (\text{where, a} = (-\text{k}) \text{ and b} = b_1)$$

$$\therefore \Delta L = \frac{F}{tE}\left(\frac{1}{-k}\right)\log[(-kx)+b_1]_0^L = \frac{F}{tE}\left(\frac{L}{-(b_1-b_2)}\right)\log\left[\left(-\frac{(b_1-b_2)}{L}x\right)+b_1\right]_0^L$$

Solving,

$$\therefore \Delta L = \frac{F}{tE}\left(\frac{L}{-(b_1-b_2)}\right)\{\log\{[-b_1+b_2+b_1]\} - \log b_1\}$$

$$\therefore \Delta L = \frac{F}{tE}\left(\frac{L}{-(b_1-b_2)}\right)\{\log\{b_2\} - \log b_1\}$$

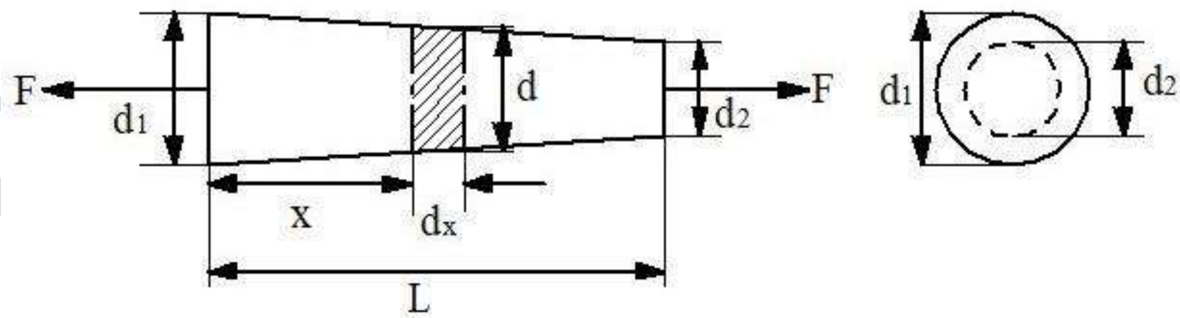
$$\therefore \Delta L = \frac{F}{tE}\left(\frac{L}{(b_1-b_2)}\right)\{\log\{b_1\} - \log b_2\}$$

$$\therefore \Delta L = \frac{FL}{tE(b_1-b_2)}\log\frac{b_1}{b_2} \longrightarrow \textbf{Eq. 1.3}$$

3) **Elongation in a tapered bar of circular cross-section**

Question: A tapering rod has diameter d_1 at one end and it tapers uniformly to a diameter d_2 at the other end in a length L. If the modulus of elasticity of the material is E, find the change in length when subjected to an axial force F.

Solution:



Consider an element of length 'dx' at a distance 'x' from the bigger end. The diameter of the element be'd'.

a) Slope of the tapered rod

$$\frac{(d_1 - d_2)}{2L} = \frac{(d_1 - d)}{2x}$$

$$d = d_1 - \frac{(d_1 - d_2)x}{L}$$

$$\therefore d = d_1 - kx \qquad where, k = \frac{(d_1 - d_2)}{L}$$

b) The element is so small in length that the diameter'd' is considered to be uniform. Elongation of the element (considering rod of uniform diameter'd'),

$$\boxed{\Delta L_x = \frac{F\,dx}{A\,E}} \longrightarrow \text{Integrating within limits 0 to L}$$

$$\therefore \Delta L = \int_0^L \frac{Fdx}{AE} = \int_0^L \frac{Fdx}{\left(\frac{\pi d^2}{4}\right)E} = \frac{4F}{\pi E}\int_0^L \frac{1}{d^2}dx = \frac{4F}{\pi E}\int_0^L \frac{1}{(d_1 - kx)^2}dx$$

$$\therefore \Delta L = \frac{4F}{\pi E}\int_0^L (d_1 - kx)^{-2}\,dx$$

Note: Formula

$$\left\{\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{(n+1)}}{(n+1)}\right\} \qquad (\text{where}, a = (-k), b = d_1 \text{ and } n = -2)$$

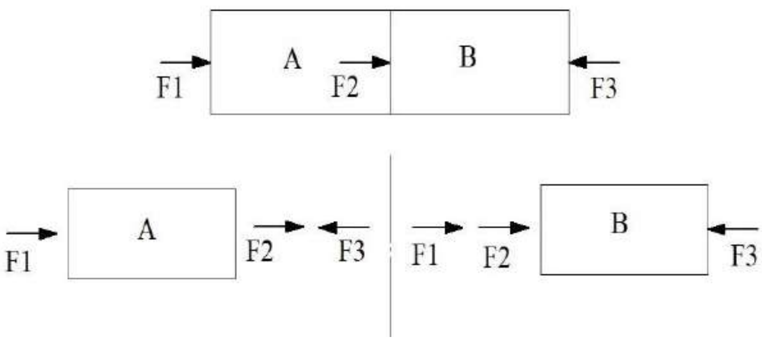
$$\therefore \Delta L = \frac{4F}{\pi E}\left(\frac{1}{-k}\right)\left[\frac{(-kx + d_1)^{(-2+1)}}{(-2+1)}\right]_0^L = \frac{4F}{\pi E}\left(\frac{L}{-(d_1 - d_2)}\right)\left[\frac{(-kx + d_1)^{(-1)}}{(-1)}\right]_0^L$$

$$\therefore \Delta L = \frac{4F}{\pi E}\left(\frac{L}{(d_1 - d_2)}\right)\left[\frac{1}{\left(-\left(\frac{d_1 - d_2}{L}\right)x\right) + d_1}\right]_0^L = \frac{4F}{\pi E}\left(\frac{L}{(d_1 - d_2)}\right)\left(\frac{1}{d_2} - \frac{1}{d_1}\right)$$

$$\boxed{\therefore \Delta L = \frac{4FL}{\pi E d_1 d_2}} \longrightarrow \textbf{Eq. 1.4}$$

Law of Superposition: "The total effect of a set of loads is same as the sum of the effects of individual loads".

The net load acting on a portion of the member is equal of sum of forces acting either on Left Hand Side or Right Hand Side of the member. Tensile force is taken positive and compressive force negative.



The net force on member A, LHS = RHS

$(-F_1) = (F_2 – F_3)$

Therefore, $F_a =$ $(-F_1) = (F_2 - F_3)$

Similarly, $F_b= (-F_1-F_2) = (-F_3)$.

Problems:

I. Type one based on stress-strain diagram data

1) A surveyor's steel tape 30m long has a cross-section of 15mm x 0.75mm. With this, line AB is measured as 150m. If the force applied during measurement is 100N more than the force applied at the time of calibration, what is the actual length of the line? Take modulus of elasticity for steel as 200kN/mm^2.

Data Given:
L = 30m = 30 X 10^3 mm
b X t = 15mm X 0.75mm
AB = 150m = 150 X 10^{3}mm
F = 100 N
E = 200 kN/mm^2

Solution:

a) **Total elongation of pipe**

$$\Delta L = \frac{FL}{AE}$$

$$\Delta L = \frac{100 \times 30 \times 10^3}{15 \times 0.75 \times 200 \times 10^3}$$

$$\boxed{\boldsymbol{\Delta L = 1.333mm}}$$

b) **Actual length measured using tape,**

$$L_1 = L + \Delta L = (30 \times 10^3) + 1.333 = \mathbf{30001.333\ mm}$$

c) **Actual length of line i.e. (AB)$_1$**

Tape length	Length of line AB
30 X 10^3 mm	150 X 10^3 mm
30001.333 mm	?

$$(AB)_1 = \frac{30001.333 \times 150 \times 10^3}{30 \times 10^3} = 150006.665\text{mm}$$

$$\boxed{(AB)_1 = 150.006665\text{m}}$$

2) The tensile test was conducted on a mild steel bar. The following data was obtained from the test. Diameter of steel bar =16mm, load at proportional limit =72kN, load at failure = 80kN, diameter of the rod at failure =12mm, gauge length =80mm, extension at a load of 60kN =0.115mm, final length =104mm. Determine: (i) Young's modulus (ii) Proportionality limit stress (iii) True breaking stress (iv) % elongation in length (v) % reduction in area. **(June/July 2018, Dec2014 /Jan 2015)**

Data Given:

$D = 16$ mm

$F = 72kN = 72 \times 10^3$ N

$F_f = 80kN = 80 \times 10^3$ N

$D_f = 12$ mm

$L_1 = 80$ mm

ΔL at 60kN = 0.115 mm

$L_2 = 104$ mm

Solution:

a) **Young's modulus**

By Hooke's Law, $\sigma = E\,\varepsilon$

$$\therefore E = \frac{\sigma}{\varepsilon} = \frac{F_e/A}{\Delta L/L} = \frac{F_e\Big/\left(\frac{\pi D^2}{4}\right)}{\Delta L/L} = \frac{(60 \times 10^3)\Big/\left(\frac{\pi 16^2}{4}\right)}{0.115/80} = \mathbf{207.593}\ \frac{\boldsymbol{kN}}{\boldsymbol{mm^2}}$$

b) **Proportionality limit stress**

$$\sigma = \frac{F}{A} = \frac{72 \times 10^3}{\left(\frac{\pi 16^2}{4}\right)} = \mathbf{358.098}\ \frac{\boldsymbol{N}}{\boldsymbol{mm^2}}$$

c) **True breaking stress**

$$\sigma_f = \frac{F_f}{A_f} = \frac{F_f}{\left(\frac{\pi D_f^2}{4}\right)} = \frac{80 \times 10^3}{\left(\frac{\pi \times 12^2}{4}\right)} = \mathbf{707.355}\ \frac{\boldsymbol{N}}{\boldsymbol{mm^2}}$$

d) **% elongation**

$$\left(\frac{L_2 - L_1}{L_1}\right) \times 100 = \left(\frac{104 - 80}{80}\right) \times 100 = \mathbf{30}\%$$

e) **% reduction in area**

$$\left(\frac{\frac{\pi D^2}{4} - \frac{\pi D_f^2}{4}}{\frac{\pi D^2}{4}}\right) \times 100 = \left(\frac{D^2 - D_f^2}{D^2}\right) \times 100 = \left(\frac{16^2 - 12^2}{16^2}\right) \times 100 = \mathbf{43.75}\%$$

3) Tension test was conducted on a specimen and the following readings recorded.

Diameter = 22mm

Gauge length of extensometer = 200mm

Least count of extensometer = 0.001mm

At a load of 22kN, extensometer reading = 60

At a load of 36kN, extensometer reading = 94

Yield load = 95kN
Maximum Load = 157kN
Diameter at neck = 15mm
Final length over 110mm original length = 132mm
Find young's modulus, yield stress, ultimate stress, % elongation and % reduction in area.

Data Given:

D_1 = 22 mm
L_E = 200 mm
L.C = 0.001 mm
At 22kN, L_{E1} = 60
At 36kN, L_{E2} = 94
F_y = 95kN = 95 X 103 N
F_{max} = 157kN =157 X 103 N
$D_2 = D_f$ = 15 mm
L_1 = 110 mm
$L_2 = L_f$ = 132 mm

Solution: "Each division of extensometer is equal to, Least Count = 0.001mm. L_{E1} = 60 and L_{E2} = 94 are the number of divisions reading for loads 22kN and 36kN respectively. Since two readings are given a average value of extensometer readings and average of corresponding loads will be considered for calculation".

a) Young's modulus

By Hooke's Law, σ = E ε

$$\boldsymbol{Load, F = \frac{(22+36)}{2} = 29kN\ and}$$

$$\boldsymbol{Extensometer\ reading, \Delta L = \left(\frac{60+94}{2}\right) \times L.C = 77 \times 0.001}$$

$$\therefore E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L} = \frac{(29\times 10^3)\Big/\left(\frac{\pi 22^2}{4}\right)}{(77\times 0.001)/200} = \mathbf{198.15}\ \frac{\boldsymbol{kN}}{\boldsymbol{mm^2}}$$

b) Yield stress

$$\sigma_y = \frac{F_y}{A} = \frac{95\times 10^3}{\left(\frac{\pi 22^2}{4}\right)} = \mathbf{249.91}\,\frac{\boldsymbol{N}}{\boldsymbol{mm^2}}$$

c) Ultimate stress

$$\sigma_{ut} = \frac{F_{max}}{A} = \frac{157\times 10^3}{\left(\frac{\pi 22^2}{4}\right)} = \mathbf{413.013}\,\frac{\boldsymbol{N}}{\boldsymbol{mm^2}}$$

d) % elongation

$$\left(\frac{L_2 - L_1}{L_1}\right)\times 100 = \left(\frac{132-110}{110}\right)\times 100 = \mathbf{20}\%$$

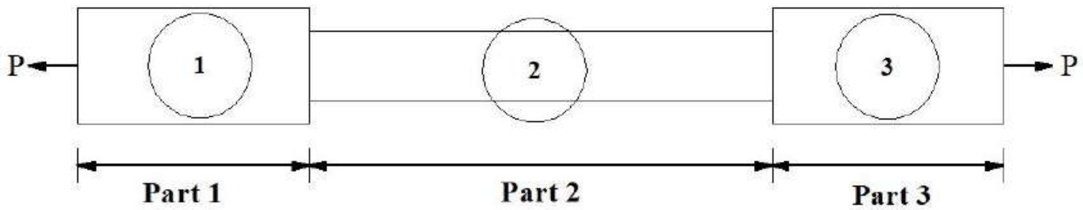
e) % reduction in area

$$\left(\frac{\frac{\pi D_1^2}{4}-\frac{\pi D_2^2}{4}}{\frac{\pi D_1^2}{4}}\right)\times 100=\left(\frac{D_1^2-D_2^2}{D_1^2}\right)\times 100=\left(\frac{22^2-15^2}{22^2}\right)\times 100=\mathbf{53.5}\%$$

II. **<u>Type two stepped bar (bars arranged in series i.e. coaxial)</u>**

<u>Procedure:</u>

a) The bar is divided into separate parts or sections based on diameter, material and forces acting at sections (as shown in figure below).

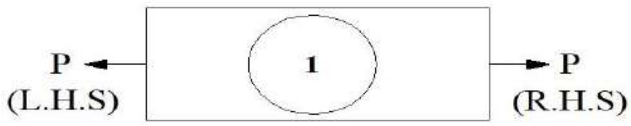


b) Net deformation (i.e. elongation) in the bar is equal to sum of individual part deformation.

$$\Delta L=\Delta L_1+\Delta L_2+\Delta L_3+\cdots+\Delta L_x$$

$$\therefore \Delta L=\left(\frac{FL}{AE}\right)_1+\left(\frac{FL}{AE}\right)_2+\left(\frac{FL}{AE}\right)_3+\cdots+\left(\frac{FL}{AE}\right)_x$$

c) Force on an individual part (portion or step) of bar is calculated using superposition principle.



Consider a part 1of the stepped bar, (Tensile forces are taken as positive and compressive forces are taken as negative)

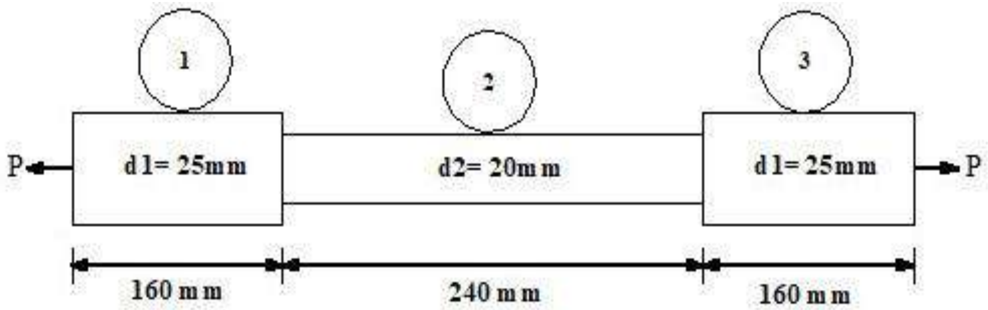
Force on part 1, F_1 = Summation of forces on the L.H.S of part 1 = Summation of forces on the R.H.S of part 1.

$\therefore F_1 = P_{L.H.S} = P_{R.H.S}$

d) The force value of each part of the bar calculated as shown in step (c) is substituted in step (a) to calculate the net elongation.

e) Using force values from step (c), calculate the stresses in each portion.

4) The bar shown in figure is tested in universal testing machine. It is observed that at a load of 40kN the total extension of the bar is 0.285mm. Determine the young's modulus of the material.



Data given: (Note: The stepped bar has three distinguished parts. Hence the subscripts 1, 2 and 3 are used)

$P = 40kN = 40 \times 10^3$ N

$\Delta L = 0.285$ mm

$D_1 = 25$ mm

$D_2 = 20$ mm

$D_3 = 25$ mm

$L_1 = 160$ mm

$L_2 = 240$ mm

$L_3 = 160$ mm

Solution:

a) Net deformation (i.e. elongation) in the bar

$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$$

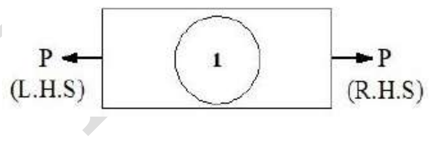
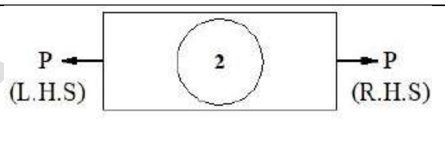
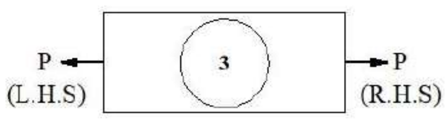
$$\therefore \Delta L = \left(\frac{FL}{AE}\right)_1 + \left(\frac{FL}{AE}\right)_2 + \left(\frac{FL}{AE}\right)_3 = \left(\frac{F_1 L_1}{\left(\frac{\pi D_1^2}{4}\right) E_1}\right) + \left(\frac{F_2 L_2}{\left(\frac{\pi D_2^2}{4}\right) E_2}\right) + \left(\frac{F_3 L_3}{\left(\frac{\pi D_3^2}{4}\right) E_3}\right)$$

$$\therefore 0.285 = \left(\frac{F_1 \times 160}{\left(\frac{\pi 25^2}{4}\right) E_1}\right) + \left(\frac{F_2 \times 240}{\left(\frac{\pi 20^2}{4}\right) E_2}\right) + \left(\frac{F_3 \times 160}{\left(\frac{\pi 25^2}{4}\right) E_3}\right)$$

Since the bar is made of same material, $E_1 = E_2 = E_3 = E$

$$\therefore 0.285 = \left(\frac{0.325\ F_1}{E}\right) + \left(\frac{0.76\ F_2}{E}\right) + \left(\frac{0.325\ F_3}{E}\right) - - - - - - (1)$$

b) Force in each part of the bar (Applying superposition principle on each part)

<table>
<tr><td>(1)</td><td>
</td><td>Forces on L.H.S = Forces on R.H.S
$\therefore P = P$
Hence, $F_1 = P$</td></tr>
<tr><td>(2)</td><td>
</td><td>Forces on L.H.S = Forces on R.H.S
$\therefore P = P$
Hence, $F_2 = P$</td></tr>
<tr><td>(3)</td><td>
</td><td>Forces on L.H.S = Forces on R.H.S
$\therefore P = P$
Hence, $F_3 = P$</td></tr>
<tr><td colspan="3">$\therefore$ **$F_1 = F_2 = F_3 = P$ - - - - - (2)**</td></tr>
</table>

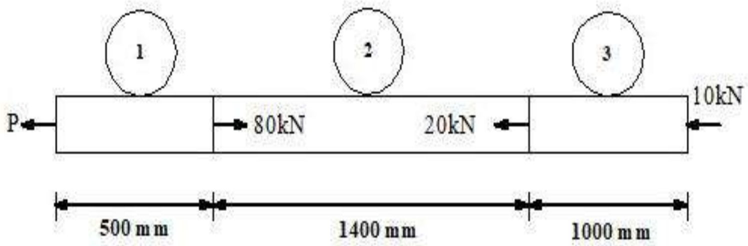
c) Substituting equation (2) in equation (1)

$$\therefore 0.285 = \left(\frac{0.325 \times P}{E}\right) + \left(\frac{0.76 \times P}{E}\right) + \left(\frac{0.325 \times P}{E}\right)$$

$$\therefore 0.285 = \frac{(0.325 \times 40 \times 10^3) + (0.76 \times 40 \times 10^3) + (0.325 \times 40 \times 10^3)}{E} = \frac{56400}{E}$$

$$\therefore \boldsymbol{E = 197.894 \times 10^3 = 197.894 \frac{kN}{mm^2}} - - - - - (\mathbf{3})$$

5) A brass bar having uniform cross-section area of 300mm² is subjected to a load as shown in figure. Find the total elongation of bar and the magnitude of load 'P' if young's modulus is 84GPa.



Data given: (Since the bar is made of same material i.e. brass)

$E_1 = E_2 = E_3 = E = 84GPa = 84\times10^3$ MPa $= 84\times10^3$ N/mm²

$A_1 = A_2 = A_3 = A = 300mm^2$ (Since bar is of uniform cross-sectional area)

$\therefore D_1 = D_2 = D_3 = D$

$L_1 = 500$ mm

$L_2 = 1400$ mm

$L_3 = 1000$ mm

Solution:

a) Net deformation (i.e. elongation) in the bar

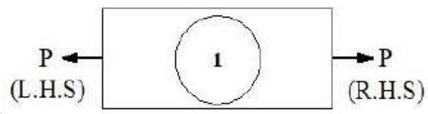
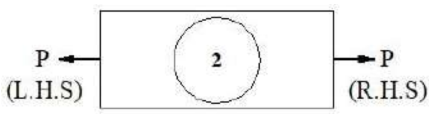
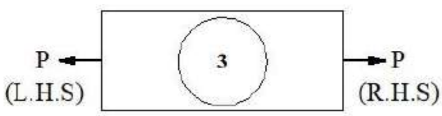
$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$$

$$\therefore \Delta L = \left(\frac{FL}{AE}\right)_1 + \left(\frac{FL}{AE}\right)_2 + \left(\frac{FL}{AE}\right)_3$$

$$\therefore \Delta L = \left(\frac{F_1 \times 500}{300 \times 84 \times 10^3}\right) + \left(\frac{F_2 \times 1400}{300 \times 84 \times 10^3}\right) + \left(\frac{F_3 \times 1000}{300 \times 84 \times 10^3}\right)$$

$$\therefore \Delta L = (1.984 \times 10^{-5} \times F_1) + (5.555 \times 10^{-5} \times F_2) + (3.968 \times 10^{-5} \times F_3) - -(1)$$

b) Force in each part of the bar (Applying superposition principle on each part)

(1)		Forces on L.H.S = Forces on R.H.S ∴ P = 80-20-10 Hence, **F_1 = P = 50 kN - - -(2a)** **(Magnitude of force, P = 50 kN)**
(2)		Forces on L.H.S = Forces on R.H.S ∴ P-80 = -20-10 ∴50-80 = -20-10 ∴ (-30) = (-30) Hence, **F_2 = -30 kN - - -(2b)**
(3)		Forces on L.H.S = Forces on R.H.S ∴ P-80+20 = (-10) ∴50-80+20 = (-10) ∴ (-10) = (-10) Hence, **F_3 = -10 kN - - -(2c)**

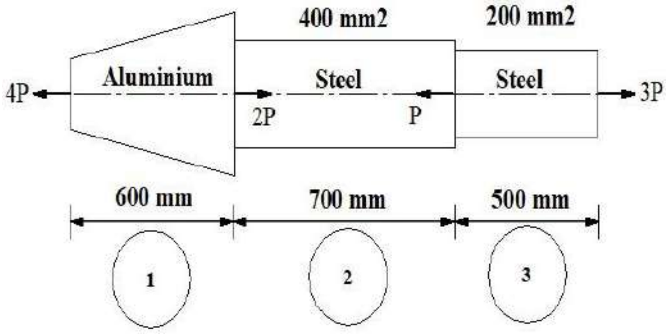
c) Substituting equation (2a), (2b) and (2c) in equation (1)

$$\therefore \Delta L = (1.984 \times 10^{-5} \times 50 \times 10^3) + (5.555 \times 10^{-5} \times (-30) \times 10^3) + (3.968 \times 10^{-5} \times (-10) \times 10^3)$$

$$\therefore \boldsymbol{\Delta L = 0.992 - 1.6665 - 0.3968 = -1.0713\, mm - - - (3)}$$

(NOTE: Negative sign indicates reduction in the total length)

6) A round bar with stepped portion is subjected to the forces as shown in figure. Determine the magnitude of force P, such that net deformation in the bar does not exceed 1mm. E for steel is 200GPa and aluminium is 70GPa. Big end diameter and small end diameter of the tapering bar are 40mm and 12.5mm respectively.



Data given:

$\Delta L \leq 1$mm

$E_1 = E_s = 200$ GPa $= 200\times10^3$ N/mm^2

$E_2 = E_{Al} = 70$ GPa $= 70\times10^3$ N/mm^2

$D_1 = 40$ mm

$D_2 = 12.5$ mm

$A_2 = 400$mm^2

$A_3 = 200$mm^2

$L_1 = 600$ mm

$L_2 = 700$ mm

$L_3 = 500$ mm

Solution:

a) Net deformation (i.e. elongation) in the bar

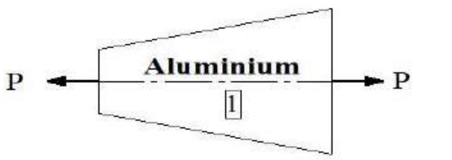
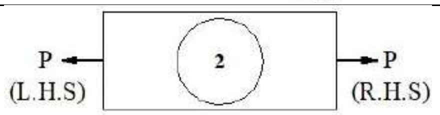
$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$$

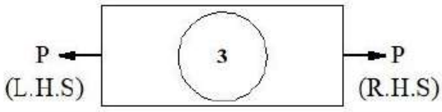
$$\therefore \Delta L = \left(\frac{4FL}{\pi E D_1 D_2}\right)_1 + \left(\frac{FL}{AE}\right)_2 + \left(\frac{FL}{AE}\right)_3$$

$$\therefore 1 = \left(\frac{4 \times F_1 \times 600}{\pi \times 70 \times 10^3 \times 40 \times 12.5}\right)_1 + \left(\frac{F_2 \times 700}{400 \times 200 \times 10^3}\right)_2 + \left(\frac{F_3 \times 500}{200 \times 200 \times 10^3}\right)_3$$

$$\therefore 1 = (2.182 \times 10^{-5} \times F_1) + (0.875 \times 10^{-5} \times F_2) + (1.25 \times 10^{-5} \times F_3) - - - (1)$$

b) Force in each part of the bar (Applying superposition principle on each part)

(1)		Forces on L.H.S = Forces on R.H.S ∴ 4P = 2P-P+3P ∴4P = 4P Hence, **F_1 = 4P - - -(2a)**
(2)		Forces on L.H.S = Forces on R.H.S ∴ 4P-2P = -P+3P ∴2P = 2P Hence, **F_2 = 2P - - -(2b)**

(3)		Forces on L.H.S = Forces on R.H.S ∴ 4P-2P+P = 3P ∴3P = 3P Hence, **F_3 = 3P - - -(2c)**

c) Substituting equation (2a), (2b) and (2c) in equation (1)

$$\therefore 1 = (2.182 \times 10^{-5} \times 4P) + (0.875 \times 10^{-5} \times 2P) + (1.25 \times 10^{-5} \times 3P)$$

$$\therefore 1 = 14.228 \times 10^{-5} \times P$$

$$\therefore \boldsymbol{P = 7.028 \times 10^{3} - - - (3)}$$

UNIT 2 : STRESS IN COMPOSITE SECTION

Elastic Constants

(1) Modulus of rigidity "G" :- The ratio of shear stress and shear strain within the elastic limit is known as modulus of rigidity or shear modulus of elasticity.

i.e $G = \frac{\tau}{\gamma}$ — (1). Larger the modulus of rigidity, lesser is the distortion when a body is subjected of shear stress.

(2) Volumetric strain :- Ratio between change in volume and original volume of a body.

$$\epsilon_v = \frac{dV}{V} \quad — (2)$$

(3) Bulk modulus :- A body subjected to three mutually ⊥ar stresses of same magnitude is said to be subjected to spherical stress or bulk stress.

The ratio between spherical stress and volumetric strain is known as Bulk modulus of elasticity "K".

$$\therefore K = \frac{\sigma}{\epsilon_v} \quad — (3)$$

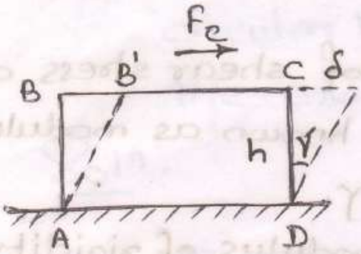
(4) Homogeneous body :- A body having uniform properties throughout is known as homogeneous body. Thus properties are not a function of position of a point in the body.

(5) Isotropic body :- A body possessing same properties in all the directions at a point is known as isotropic body. The properties are not a function of orientation at a point.

(6) Orthotropic body :- Material properties are different in three mutually ⊥ar directions at a point in an orthotropic body. Ex: Wood.

(7) Anisotropic body :- An anisotropic body has material properties different in all the directions at a point.

Shear Stress and Shear Strain



Consider a body subjected to shear stress (τ). The bottom face "AD" is fixed and a shear force acts on the top face "BC". Because of two equal and opposite tangential forces a shear deformation is produced in the body.

The face ABCD takes a new shape AB'C'D due to the shear deformation which is given by $CC' = \delta$.

"Shear strain is defined as change in right angle between two planes originally $\perp^{ar}$ to each other".

i.e $\tan\gamma = \frac{\delta}{h}$

$$\therefore \boxed{\gamma = \frac{\delta}{h}} \quad -(1)$$

Since, $\tan\gamma = \gamma$ very small angle.

Note:-

(1) Shear strain is expressed in radians

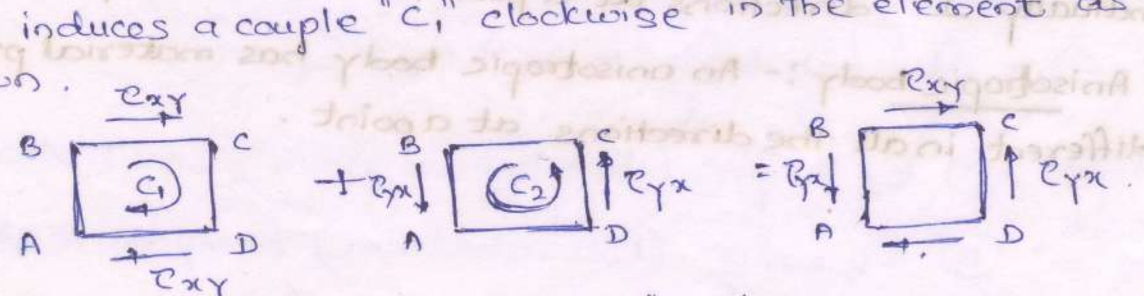
(2) Shear stress is directly proportional to shear strain within the elastic limit, $\tau = G\gamma$.

where, G = modulus of rigidity

Complementary Shear Stress Theory

"The shear stresses induced in a plane is accompanied by an equal shear stress in a plane perpendicular to the former".

Proof :- Consider an element ABCD of unit thickness. Let a shear force "F_{τ_1}" act on faces BC and AD. This shear force induces a couple "C_1" clockwise in the element as shown.



For equilibrium, let shear force "F_{τ_2}" act on AB and CD. This induces a couple "C_2" anti-clockwise in the element.

$F_{e_1} = \tau_{xy} \times \underbrace{BC \times 1}_{Area}$ — (a)

$\therefore C_1 = F_{e_1} \times AB = \tau_{xy} \times AB \times BC$ — (b)

$F_{e_2} = \tau_{yx} \times \underbrace{AB \times 1}_{Area}$ — (c)

$C_2 = F_{e_2} \times BC = \tau_{yx} \times AB \times BC$ — (d)

Couples "C_1 & C_2" are equal and opposite.

$\therefore C_1 = C_2$ — from (b) and (d)

$\tau_{xy} \times AB \times BC = \tau_{yx} \times AB \times BC$

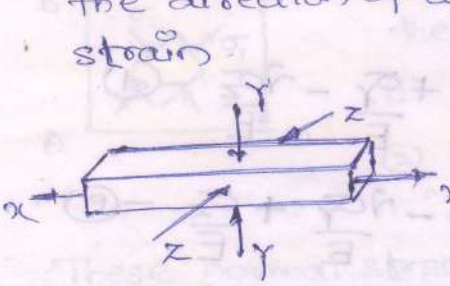
$\therefore \boxed{\tau_{xy} = \tau_{yx}}$ — (i) Hence proved.

Poisson's Ratio :- " The ratio between lateral strain and longitudinal strain is known as Poisson's ratio". It represents elastic property of the material and is indicated by "ν" (Nu) or $1/m$.

Explanation :-

(a) Strain induced along the direction of application of stress is known as longitudinal strain.

(b) Strain induced along the direction perpendicular to the direction of application of stress is known as lateral strain.

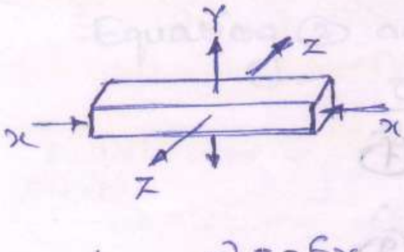


(i) Consider prismatic bar subjected to tensile force in x-direction.

$\therefore$ x-dir^n tensile strain ϵ_x

y-dir^n compressive strain $-\epsilon_y$

z-dir^n — " — $-\epsilon_z$



(ii) If compressive force is applied in x-direction.

$\therefore$ x-dir^n compressive strain $-\epsilon_x$.

y-dir^n tensile strain ϵ_y

z-dir^n tensile strain ϵ_z.

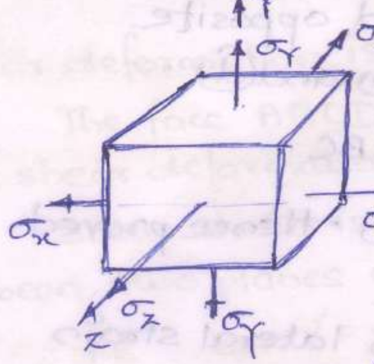
$\therefore \nu = -\dfrac{\epsilon_y}{\epsilon_x}$ and $\nu = -\dfrac{\epsilon_z}{\epsilon_x}$

Note :

① Longitudinal strain accompanied by the lateral strain of opposite nature is known as Poisson's Effect.

② Lateral strain induced due to Poisson's Effect is not accompanied by stress in same direction.

Generalised Hooke's Law



Consider a body subjected to stresses σ_x, σ_Y and σ_Z in x, Y and z-direction.

∴ Net strain in x-direction is,

ⓐ Longitudinal strain in x-dir^n because of stress in x-dir^n = ϵ_x

ⓑ Lateral strain in x-dir^n because of stress in Y-dir^n = $-\nu\epsilon_Y$

ⓒ Lateral strain in x-dir^n because of stress in z-dir^n = $-\nu\epsilon_Z$.

∴ Net strain in x-dir^n = $\epsilon_x - \nu\epsilon_Y - \nu\epsilon_Z$ — ⓐ

From Hooke's law, $\epsilon = \frac{\sigma}{E}$ — ⓑ applying in eq^n ⓐ

∴ Net strain in x-dir^n = $\frac{\sigma_x}{E} - \nu\frac{\sigma_Y}{E} - \nu\frac{\sigma_Z}{E}$ — ⓒ

Similarly,

Net strain in Y-dir^n = $-\nu\frac{\sigma_x}{E} + \frac{\sigma_Y}{E} - \nu\frac{\sigma_Z}{E}$ — ⓓ

Net strain in z-dir^n = $-\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_Y}{E} + \frac{\sigma_Z}{E}$ — ⓔ

Note: Eq^n ⓒ, ⓓ and ⓔ are for tensile stresses. If any stress in compressive, then substitute that stress with negative sign.

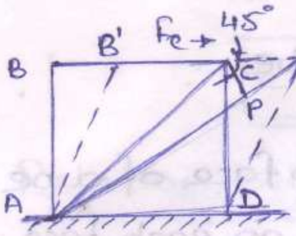
Ex: σ_Y compressive.

$$\therefore \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu(-\sigma_Y)}{E} - \frac{\nu\sigma_Z}{E} \quad \text{— (f)}$$

$$\epsilon_Y = \frac{-\sigma_Y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_Z}{E} \quad \text{— (g)}$$

$$\epsilon_Z = \frac{\sigma_Z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu(-\sigma_Y)}{E} \quad \text{— (h)}$$

Relation between Young's Modulus and Modulus of Rigidity



Consider an element ABCD of unit thickness.
(1) Face AD is fixed rigidly.
(2) Face BC is subjected to shear force "F_e"
The element deforms to new shape AB'C'D. Put a perpendicular CP on diagonal AC'.

$\therefore AC = AP$ and $\angle PCC' = 45°$ — (1)

Diagonal "AC" is subjected to normal strain ϵ_{AC},

$$\epsilon_{AC} = \frac{dL}{L} = \frac{AC' - AP}{AC} = \frac{PC'}{AC} \quad \text{— (a)}$$

Now, $PC' = CC' \sin 45 = \frac{CC'}{\sqrt{2}}$ — (b)

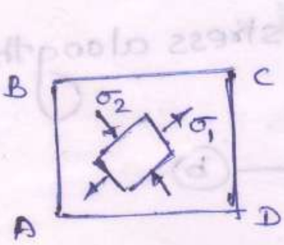
$$AC = \sqrt{AD^2 + AD^2} = \sqrt{2}\, CD \quad \text{— (c)} \quad (\because AD = CD).$$

Put (b) and (c) in (a) we get.

$$\epsilon_{AC} = \frac{CC'}{2CD} = \frac{\gamma}{2} \quad \text{— (d)} \qquad \gamma = \text{Shear Strain}$$

Since $G = \tau/\gamma$ then eq^n (d) becomes.

$$\boxed{\epsilon_{AC} = \frac{\tau}{2G}} \quad \text{— (2)}$$



If an element is considered oriented along the diagonals, then
(1) Along diagonal "AC" tensile stress "σ_1" acts
(2) Along diagonal "BD" compressive stress "σ_2" acts

These normal stresses, $\sigma_1 = \sigma_2 = \tau$. Therefore net strain

$$\epsilon_{AC} = \frac{\sigma_1}{E} + \frac{\gamma \sigma_2}{E} = \frac{\tau}{E}(1+\gamma) \quad \text{— (3)}$$

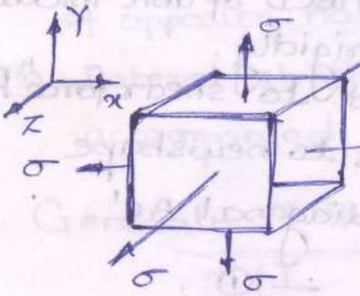
{ σ_2 is having -ve sign. compressive

Equating (2) and (3),

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\gamma)$$

$$\therefore \boxed{E = 2G(1+\gamma)} \quad \text{— (i)}.$$

Volumetric Strain



Consider a cube.

Let,

a = side of each face of cube

σ = stress acting on each face.

da = deformation in each side.

∴ Initial volume of cube $= a^3$ — (1)

Final volume of cube $= (a+da)^3$ — (2)

Change in volume $= (a+da)^3 - a^3$

$= a^3 + 3a^2 da + 3da^2 a + da^3 - a^3$

Neglecting higher order terms

$dV = 3a^2 da$ — (3)

Volumetric strain, $\epsilon_v = \frac{dV}{V} = \frac{3a^2 da}{a^3} = 3\frac{da}{a}$

but $\frac{da}{a} = \epsilon$, linear strain

$$\therefore \boxed{\epsilon_v = 3\epsilon} \quad \text{— (a)}$$

∴ Volumetric strain is three times the linear strain when a body is subjected to same stress along three mutually $\perp^{ar}$ directions.

In general, $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$ — (b)

Relation between Young's Modulus and Bulk Modulus.

Consider a cube of side "a", subjected to stress "σ" along three mutually $\perp^{ar}$ directions.

From Hooke's generalised law, $\epsilon_x = \frac{\sigma}{E} - \frac{\nu\sigma}{E} - \frac{\nu\sigma}{E}$

$\therefore \epsilon_x = \frac{\sigma}{E}(1-2\nu)$ — (1)

Volumetric strain, $\epsilon_v = 3\epsilon$ $\{\epsilon = \epsilon_x$ = linear strain

$\therefore \epsilon_v = 3\frac{\sigma}{E}(1-2\nu)$ — (2)

Bulk modulus, $K = \frac{\sigma}{\epsilon_v}$ — (3)

Substituting eq^n (2) in (3), we get.

$$\therefore \frac{\sigma}{K} = \frac{3\sigma}{E}(1-2\nu)$$

$$\therefore \boxed{E = 3K(1-2\nu)} \quad \text{—} (4)$$

Relation between E, K and G

$$\left.\begin{array}{l} E = 2G(1+\nu) \quad \text{—} (1) \\ E = 3K(1-2\nu) \quad \text{—} (2) \end{array}\right\} \text{Solving}$$

$$2G(1+\nu) = 3K(1-2\nu)$$

$$\therefore 2G + 2\nu G = 3K - 6\nu K$$

$$-2G + 3K = \nu(2G + 6K)$$

$$\therefore \nu = \frac{3K-2G}{(2G+6K)} \quad \text{—} (3) \text{ Substitute in eq}^n (1)$$

$$\therefore E = 2G\left[1 + \frac{3K-2G}{2G+6K}\right]$$

$$= 2G\left[\frac{2G+6K+3K-2G}{2G+6K}\right]$$

$$= \frac{18GK}{2G+6K}$$

$$\therefore \boxed{E = \frac{9GK}{G+3K}} \quad \text{—} (4)$$

Problems:

Dec'11 (8 mks)

1. When a bar of 25 mm diameter is subjected to a pull of 61 kN, the extension on a 50 mm gauge length is 0.1 mm and decrease in diameter is 0.013 mm. Calculate the value of elastic constants E, G, K and μ

Sol^n: Data Given:-

$d = 25\ mm$

$P = 61\ kN = 61 \times 10^3\ N$

$L = 50\ mm$

$\Delta L = 0.1\ mm$

$\Delta d = 0.013\ mm$

(1) Young's modulus.

Deformation, $\Delta L = \frac{PL}{AE}$ — (a)

$\therefore E = \frac{PL}{\frac{\pi}{4} d^2 \Delta L} = \frac{61 \times 10^3 \times 50}{\frac{\pi}{4} \times 25^2 \times 0.1}$

$\therefore E = 62.13\ KN/mm^2$ — (1)

(2) Poisson's Ratio : $\mu = \nu = \frac{\epsilon_{lat}}{\epsilon_{long}}$ — (b)

$\therefore \nu = \frac{(\Delta d/d)}{(\Delta L/L)} = \left(\frac{0.013}{25}\right) / \left(\frac{0.1}{50}\right)$

$\therefore \boxed{\nu = 0.26}$ — (2)

(3) Modulus of Rigidity :

$E = 2G(1+\nu)$ — (c)

$\therefore G = \frac{62.13 \times 10^3}{2(1+0.26)} = 24.65\ KN/mm^2$ — (3)

(4) Bulk modulus

$E = 3K(1-2\nu)$ — (d)

$\therefore K = \frac{62.13 \times 10^3}{3[1-2(0.26)]} = 43.14\ KN/mm^2$ — (4)

June'12 (06 mks).

(2) The modulus of rigidity of a material is $0.8 \times 10^5\ N/mm^2$ when a 6mm × 6mm rod of this material was subjected to an axial pull of 3600N, it was found that the lateral dimension of the rod changed to 5.9991 mm × 5.9991 mm. Find the Poisson's ratio and the modulus of elasticity.

Soln: Given Data

$G = 0.8 \times 10^5\ N/mm^2$

$A = b \times t = 6 \times 6\ mm^2$

$P = 3600 N$

$A_f = 5.9991 \times 5.9991\ mm^2$

Young's modulus, $E = 3G(1-2\nu)$ — ① *wrong formula. [corrected in red: $2G(1+\nu)$]

$\therefore E = 3 \times 0.8 \times 10^5 [1-2\nu]$

$\therefore E = 240 \times 10^3 (1-2\nu)$ — ⓐ

Poisson's Ratio, $\nu = \dfrac{\epsilon_{lat}}{\epsilon_{long}}$ — ②

$\epsilon_{lat} = \dfrac{A - A_f}{A} = \dfrac{(6\times6) - (5.9991 \times 5.9991)}{(6\times6)} = 2.99\times10^{-4}$ — ⓑ

$\therefore \epsilon_{long} = \dfrac{\Delta L}{L} = \dfrac{\epsilon_{lat}}{\nu} = \dfrac{2.99\times10^{-4}}{\nu}$ — ⓒ

By Hooke's law, $\epsilon = \dfrac{\sigma}{E}$

$\therefore E = \dfrac{\sigma}{\epsilon} = \dfrac{P/A}{\epsilon_{long}}$ $\{\epsilon = \epsilon_{long}\}$

$\therefore E = \left(\dfrac{3600}{6\times6}\right) \Big/ \dfrac{2.99\times10^{-4}}{\nu} = 334.448\times10^3 \nu$ — ⓓ

Substitute ⓓ in ⓐ.

$\therefore 334.448\times10^3 \nu = 240\times10^3(1-2\nu)$

$1.393\nu = 1-2\nu$

$\therefore \nu = \dfrac{1}{3.393}$

$\therefore \boxed{\nu = 0.29}$ — ③ $\nu = 0.86$

$\therefore$ Young's modulus, $E = \boxed{100.8\times10^3}$ N/mm² — ④. 298.5×10^3 N/mm²

Dec'12 (10mks)

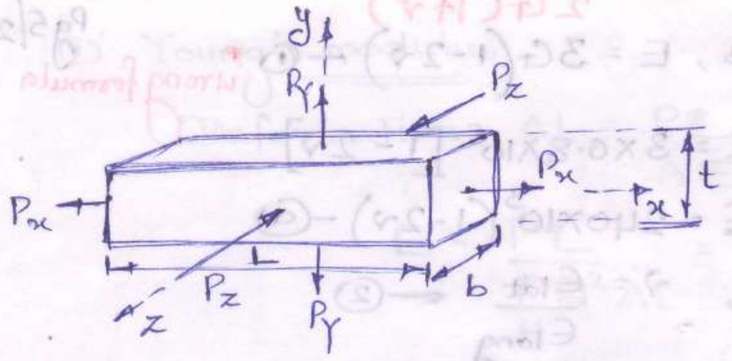
③ A C.I flat, 300 mm long, 50 mm wide and 30 mm thick is acted upon by the following forces, 25 kN tensile in the dirⁿ of length, 350 kN compressive, in the direction of width and 200 kN tensile, in the direction of thickness. Determine: ⓐ Change in volume of flat ⓑ Modulus of rigidity ⓒ Bulk modulus. Take, E = 140 GN/m² and 1/m = 0.25.

Solⁿ: Data Given

$L = 300$ mm $\quad P_x = 25$ kN $\quad E = 140$ GN/m² $= 140$ kN/mm²

$b = 50$ mm $\quad P_y = -350$ kN $\quad 1/m = \nu = 0.25$

$t = 30$ mm $\quad P_z = 200$ kN



(i) Stresses:

(a) x-direction, $\sigma_x = \frac{P_x}{A_x} = \frac{25\times10^3}{50\times30}$ $\because$ $A_x = b\times t$

$\therefore \sigma_x = 16.66 \ N/mm^2$ — (a)

(b) Y-direction, $\sigma_Y = \frac{P_Y}{A_Y} = \frac{P_Y}{L\times b} = \frac{+200\times10^3}{300\times50}$

$\therefore \sigma_Y = +13.33 \ N/mm^2$ — (b) {+ve sign indicates tensile stress}

(c) z-direction, $\sigma_z = \frac{P_z}{A_z} = \frac{P_z}{L\times t} = \frac{-350\times10^3}{300\times30}$

$\therefore \sigma_z = -38.88 \ N/mm^2$ — (c) {-ve sign for compressive stress}

(2) Net strains in all directions

Substitute, $\sigma_z = -38.88 N/mm^2$

(1) $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_Y}{E} - \nu\left(\frac{+\sigma_z}{E}\right)$

$= \frac{1}{140\times10^3}\left[16.66 - 0.25\left[13.33 - 38.88\right]\right]$

$\therefore \epsilon_x = 1.646\times10^{-4}$ — (d)

(2) $\epsilon_Y = \frac{\sigma_Y}{E} - \frac{\nu\sigma_x}{E} - \nu\left(\frac{+\sigma_z}{E}\right)$

$= \frac{1}{140\times10^3}\left[13.33 - 0.25(16.66 - 38.88)\right]$

$\therefore \epsilon_Y = 1.348\times10^{-4}$ — (e)

(3) $\epsilon_z = +\frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_Y}{E}$

$= \frac{1}{140\times10^3}\left[-38.88 - 0.25\left[16.66 + 13.33\right]\right]$

$\epsilon_z = -3.312\times10^{-4}$ — (f)

Volumetric strain:

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z \quad \text{— (g)}$$

$$\frac{dV}{V} = (1.646 + 1.348 - 3.312) \times 10^{-4}$$

$\therefore$ Change in volume, $dV = (-0.318 \times 10^{-4}) \times [300 \times 50 \times 30]$

$$\therefore \boxed{dV = -14.31} \text{ mm}^3 \quad \text{— (1)}$$

(4) Modulus of rigidity

$$E = 2G(1+\nu) \quad \text{— (h)}$$

$$\therefore G = \frac{140 \times 10^3}{2(1+0.25)} = 56 \times 10^3 \text{ N/mm}^2 \quad \text{— (2)}$$

(5) Bulk modulus:

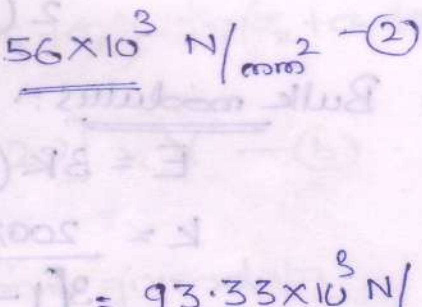
$$E = 3K(1-2\nu)$$

$$\therefore K = \frac{140 \times 10^3}{3[1-(2 \times 0.25)]} = 93.33 \times 10^3 \text{ N/mm}^2 \quad \text{— (3)}$$

June/July '13 (08 mks)

(4) A bar of 20 mm diameter is tested in tension. It is observed that when a load of 37.7 KN is applied, the extension measured over a gauge length of 200 mm is 0.12 mm and contraction in diameter is 0.0036 mm. Find Poisson's ratio and elastic constants E, G, K.

Soln: Data Given



$d = 20$ mm

$P = 37.7$ KN $= 37.7 \times 10^3$ N

$\Delta L = 0.12$ mm

$L = 200$ mm

$\Delta d = 0.0036$ mm.

(i) Young's modulus

$$E = \sigma / \epsilon \quad \text{— (a)}$$

$$\sigma = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d^2} = \frac{37.7 \times 10^3}{\frac{\pi}{4} \times 20^2}$$

$$\therefore \sigma = 120 \text{ N/mm}^2$$

$$\epsilon = \frac{\Delta L}{L} = \frac{0.12}{200} = 6 \times 10^{-4}$$

from (a).

$$\therefore E = \frac{120}{6 \times 10^{-4}}$$

$$\therefore E = 200 \times 10^3 \text{ N/mm}^2 \quad \text{— (1)}$$

(2) Poisson's Ratio

$$\nu = \frac{\epsilon_{lat}}{\epsilon_{long}} = \frac{\Delta d/d}{\Delta L/L} = \frac{0.0036/20}{0.12/200}$$

$$\therefore \boxed{\nu = 0.3} \quad — (2)$$

(3) Modulus of Rigidity

$$E = 2G(1+\nu)$$

$$\therefore G = \frac{200\times10^3}{2(1+0.3)} = \underline{76.923\times10^3 N/mm^2} \quad —(3)$$

(4) Bulk modulus :-

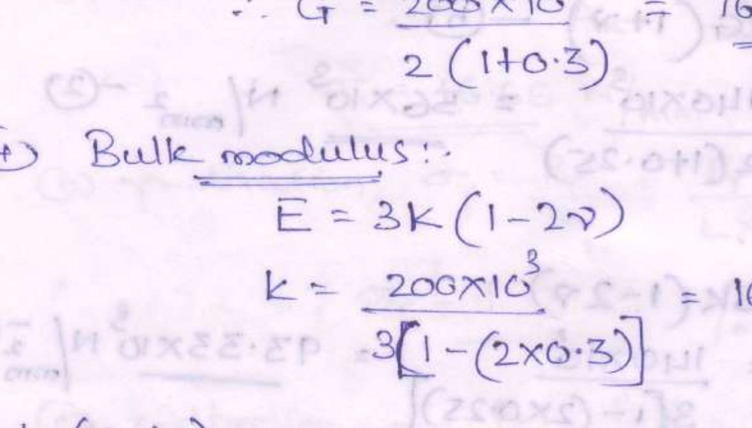
$$E = 3K(1-2\nu)$$

$$K = \frac{200\times10^3}{3[1-(2\times0.3)]} = \underline{166.66\times10^3 N/mm^2} \quad —(4)$$

Jan'14 (09 mks)

(5) A bar of rectangular c/s is subjected to stresses σ_x, σ_Y and σ_Z in x, Y and Z directions respectively. Show that if sum of these stresses is zero, there is no change in volume of the bar.

Sol^n :-



Data Given : If, $\sigma_x + \sigma_Y + \sigma_Z = 0$, Prove that, $dV = 0$

(1) Stresses :

$$\left.\begin{aligned} x\text{-direction} &= \sigma_x \\ Y\text{-direction} &= \sigma_Y \\ Z\text{-direction} &= \sigma_Z \end{aligned}\right\} — (1)$$

(2) Net strain in all directions.

(a) $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_Y}{E} - \frac{\nu\sigma_Z}{E}$

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y - \nu\sigma_z\right] \quad \text{— (2)}$$

(b) $$\epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E} = \frac{1}{E}\left[\sigma_y - \nu\sigma_x - \nu\sigma_z\right] \quad \text{— (3)}$$

(c) $$\epsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{1}{E}\left[\sigma_z - \nu\sigma_x - \nu\sigma_y\right] \quad \text{— (4)}$$

(3) Volumetric strain :-

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z \quad \text{— (a)}$$

Substituting from (2), (3) & (4) in (a)

$$\therefore \epsilon_v = \frac{dV}{V} = \frac{1}{E}\left[(\sigma_x + \sigma_y + \sigma_z) - 2\nu(\sigma_x + \sigma_y + \sigma_z)\right]$$

$$\therefore dV = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}\left[1 - 2\nu\right] \times V \quad \text{— (b)}$$

Since, $\sigma_x + \sigma_y + \sigma_z = 0$ from given data.

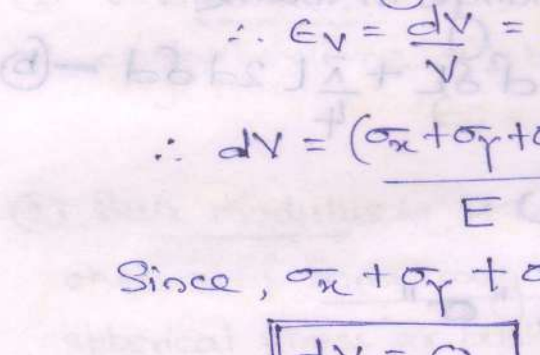
$$\boxed{dV = 0} \quad \text{— (i)}$$

~~Not a proper solution~~

(6) Derive expression for volumetric strain of rectangular bar subjected to normal stress "σ" along its axis.

Soln

Volumetric strain, $\epsilon_v = \frac{dV}{V}$ — (1)



Volume, $V = l.b.t$ — (a)

Change in volume,

$dV = bt\,\delta L + lt\,\delta b + lb\,\delta t$ — (b)

Substituting (a) and (b) in (1).

$$\epsilon_v = \frac{\delta L}{L} + \frac{\delta b}{b} + \frac{\delta t}{t} = \epsilon_L + \epsilon_b + \epsilon_t \quad \text{— (c)}$$

Strains due to stress "σ"

(a) $\epsilon_l = \frac{\sigma}{E}$ (longitudinal strain) (tensile strain)

(b) $\epsilon_b = -\nu\frac{\sigma}{E}$ (lateral strain) (compressive strain)

(c) $\epsilon_t = -\nu\frac{\sigma}{E}$ (lateral strain) (— " —)

$$\therefore \epsilon_v = \frac{\sigma}{E}(1 - 2\nu) \quad \text{— (2)} \;//$$