Module – 3

Limit State Design of Beams

1.1 Introduction

Beam is a structural member which is normally placed horizontally. It provides resistance to bending when loads are applied on it. Most commonly used material for beam is RCC (Reinforced Cement Concrete). RCC beam can be various types depending on different criteria.

RCC beam can be various types depending on different criteria. Such as depending on shape, beam can be rectangular, T-beam etc. Depending on reinforcement placement, beam can be double reinforced beam, single reinforced beam, etc.

1.2 Objective

1. To design singly and doubly reinforced beam
2. To design flanged beams for combined bending, shear and torsion as per IS 456

1.3 Types of RCC Beams

RCC beams are 4 types depending on their supporting systems.

1. Simply supported beam
2. Semi-continuous beam
3. Continuous beam, and
4. Cantilever beam.

Simply Supported Beam

This type of beam has a single span. It is supported by two supports at both ends. This beam is also called simple beam.

Semi-Continuous Beam

This beam doesn’t have more than two spans. And supports are not more than three.
Technically this beam is a continuous beam.

Continuous Beam

This type of beam has more than two spans and has more than three supports along its length.
The supports are in one straight line thus the spans are also in a straight line.

Cantilever Beam

It has only one support in one end, another end is open.

There is another type of beam we can see in the civil engineering world which is called over-hanging beam. This beam extends beyond its supports. Actually, this beam is a combination of simply supported and cantilever beam. In this chapter, it is intended to learn the method of designing the beams using the principles developed in previous chapters. Design consists of selecting proper materials, shape and size of the structural member keeping in view the economy, stability and aesthetics.

1.4 Design procedure

The procedure for the design of beam may be summarized as follows:

1. Estimation of loads
2. Analysis
3. Design

1. Estimation of loads

The loads that get realized on the beams consist of the following:

a. Self weight of the beam.

b. Weight of the wall constructed on the beam

c. The portion of the slab loads which gets transferred to the beams. These slab loads are due to live loads that are acting on the slab dead loads such as self weight of the slab, floor finishes, partitions, false ceiling and some special fixed loads. The economy and safety of the beams achieved depends on the accuracy with which the loads are estimated.

The dead loads are calculated based on the density whereas the live loads are taken from IS: 875 depending on the functional use of the building.

2. Analysis

For the loads that are acting on the beams, the analysis is done by any standard method to obtain the shear forces and bending moments.

3. Design

a. Selection of width and depth of the beam.

The width of the beam selected shall satisfy the slenderness limits specified in IS 456 : 2000 clause 23.3 to ensure the lateral stability.

b. Calculation of effective span (le) (Refer clause 22.2, IS 456:2000)

c. Calculation of loads (w)

d. Calculation of critical moments and shears.

The moment and shear that exists at the critical sections are considered for the design. Critical sections are the sections where the values are maximum. Critical section for the moment in a simply supported beam is at the point where the shear force is zero. For continuous beams the critical section for the +ve bending moment is in the span and –ve bending moment is at the support. The critical section for the shear is at the support.

e. Find the factored shear (Vu) and factored moment (Mu)

f. Check for the depth based on maximum bending moment.

Considering the section to be nearly balanced section and using the equation

Annexure G, IS 456-2000 obtain the value of the required depth required. If the assumed depth "d" is greater than the "required", it satisfies the depth criteria based on flexure. If the assumed section is less than the" required", revise the section.

g. Calculation of steel.

As the section is under reinforced, use the equation G.1.1.(b) to obtain the steel.

h. Check for shear.

i. Check for developmental length.

j. Check for deflection.

k. Check for Ast min, Ast max and distance between the two bars.

Anchorage of bars or check for development length

In accordance with clause 26.2 IS 456: 2000, the bars shall be extended (or anchored) for a certain distance on either side of the point of maximum bending moment where there is maximum stress (Tension or Compression). This distance is known as the development length and is required in order to prevent the bar from pulling out under tension or pushing in under compression. The development length (Ld) is given by

$$L_d = \frac{\varnothing\ \sigma_s}{4\ Z_{bd}}$$

where, $\varnothing$ = Nominal diameter of the bar

σ_s – Stress in bar at the section considered at design load

Z_{bd} – Design bond stress given in table 26.2.1.1 (IS 456 : 2000)

Table 26.2.1.1: Design bond stress in limit state method for plain bars in tension shall be as below:

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design bond stress τ_{bd} N/mm^2	1.2	1.4	1.5	1.7	1.9

Note: Due to the above requirement it can be concluded that no bar can be bent up or curtailed upto a distance of development length from the point of maximum moment. Due to practical difficulties if it is not possible to provide the required embedment or development length, bends hooks and mechanical anchorages are used.

Flexural reinforcement shall not be terminated in a tension zone unless any one of the following condition is satisfied:

a. The shear at the cut-off points does not exceed two-thirds that permitted, including the shear strength of web reinforcement provided.

b. Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the cut-off point equal to three-fourths the effective depth of the member. The excess stirrup area shall be not less than $0.4bs/f_y$, where b is the breadth of the beam, s is the spacing and f_y is the characteristic strength of reinforcement in N/mm2. The resulting spacing shall not exceed d/8 where is the ratio of the area of bars cut-off to the total area of bars at the section, and d is the effective depth.

c. For 36 mm and smaller bars, the continuing bars provide double the area required for flexure at the cut-off point and the shear does not exceed three-fourths that permitted.

Positive moment reinforcement:

a. At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of the member into the support, to a length equal to $L_d/3$.

b. When a flexural member is part of the primary lateral load resisting system, the positive reinforcement required to be extended into the support as described in (a) shall be anchored to develop its design stress in tension at the face of the support.

c. At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that L_d computed for f_d by 26.2.1
IS 456:2000 does not exceed.

$$\frac{M_1}{V} + L_0$$

Where, M_1 = moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d;

$f_d = 0.87f_y$ in the case of limit state design and the permissible stress in the case of working stress design;

V = shear force at the section due to the design loads;

L_0 = sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support; and at a point of inflection, L_0 is limited to the effective depth of the members or 12 , whichever is greater; and

ϕ= diameter of bar.

The value of M_1/V in the above expression may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction.

Negative moment reinforcement:

At least one third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflection for a distance not less than the effective depth of the member of 12 or one-sixteenth of the clear span whichever is greater.

Anchorage of bars

Anchoring of bars is done to provide the development length and maintain the integrity of the structure.

Anchoring bars in tension:

a. Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should normally be provided for plain bars in tension.

b. Bends and hooks – shall conform to IS 2502

1. Bends – The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 450 bend subject to a maximum of 16 times the diameter of the bar.

2. Hooks – The anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

Anchoring bars in compression:

The anchorage length of straight bar in compression shall be equal to the development length of bars in compression as specified in clause 26.2.1 of IS 456:2000. The projected length of hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall only be considered for development length.

Mechanical devices for anchorage:

Any mechanical or other device capable of developing the strength of the bar without damage to concrete may be used as anchorage with the approval of the engineer-in-charge.

Anchoring shear reinforcement:

a. Inclined bars – The development length shall be as for bars in tension; this length shall be measured as under:

1. In tension zone, from the end of the sloping or inclined portion of the bar, and

2. In the compression zone, from the mid depth of the beam.

b. Stirrups – Not withstanding any of the provisions of this standard, in case of secondary reinforcement, such as stirrups and transverse ties, complete development lengths and anchorages shall be deemed to have been provided when the bar is bent through an angle of at least 900 round a bar of at least its own diameter and is continued beyond the end of the curve for a length of at least eight diameters, or when the bar is bent through an angle of 1350 and is continued beyond the end of the curve for a length of at least six bar diameters or when the bar is bent through an angle of 1800 and is continued beyond the end of the curve for a length of at least four bar diameters.

Reinforcement requirements

1. Minimum reinforcement:

The minimum area of tension reinforcement shall be not less than that given by the following:

Where,

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

Where, A_s = minimum area of tension reinforcement.

b = breadth of beam or the breadth of the web of T-beam, d =

effective depth, and

f_y = characteristic strength of reinforcement in N/mm2

2. Maximum reinforcement – The maximum area of tension reinforcement shall not exceed 0.04bD

Compression reinforcement:

The maximum area of compression reinforcement shall not exceed 0.04bD.

Compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint.

Pitch and diameter of lateral ties:

The pitch of shear reinforcement shall be not more than the least of the following distances:

1. The least lateral dimension of the compression members;

2. Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and 3.

300 mm.

The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 16 mm.

Slenderness limits of beams to ensure lateral stability

A beam is usually a vertical load carrying member. However, if the length of the beam is very large it may bend laterally. To ensure lateral stability of a beam the following specifications have been given in the code.

A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed 60b or whichever is less, where d is the effective depth of the beam and b the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25b or whichever is less.

T–beams and L–beams

Beams having effectively T-sections and L-sections (called *T*-beams and *L*-beams) are commonly encountered in beam-supported slab floor systems [Figs]. In such situations, a portion of the slab acts integrally with the beam and bends in the longitudinal direction of the beam. This slab portion is called the *flange* of the T- or L-beam. The beam portion below the flange is often termed the *web*, although, technically, the web is the full rectangular portion of the beam other than the overhanging parts of the flange. Indeed, in shear calculations, the web is interpreted in this manner.

When the flange is relatively wide, the flexural compressive stress is not uniform over its width. The stress varies from a maximum in the web region to progressively lower values at points farther away from the web. In order to operate within the framework of the theory of flexure, which assumes a uniform stress distribution across the width of the section, it is necessary to define a reduced effective flange.

The effective width of flange 'may be defined as the width of a hypothetical flange that resists in-plane compressive stresses of uniform magnitude equal to the peak stress in the original wide flange, such that the value of the resultant longitudinal compressive force is the same (Fig).

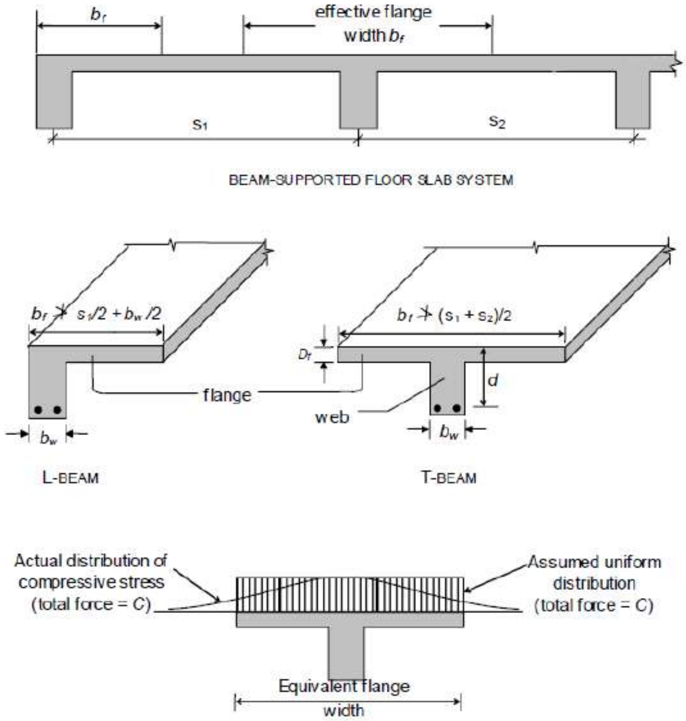


Figure. T-beams and L-beams in beam-supported floor slab systems

The effective flange width is found to increase with increased span, increased web width and increased flange thickness. It also depends on the type of loading (concentrated, distributed, etc.) and the support conditions (simply supported, continuous, etc.). Approximate formulae for estimating the 'effective width of flange' b_f (Cl. 23.1.2 of Code) are given as follows:

$$b_f = \begin{cases} l_0/6 + b_w + 6D_f & \text{for } T-Beam \\ l_0/12 + b_w + 3D_f & \text{for } L-Beam \end{cases} \quad (12)$$

where b_w is the breadth of the web, D_f is the thickness of the flange [Fig 2.8], and l_0 is the "distance between points of zero moments in the beam" (which may be assumed as 0.7 times the effective span in continuous beams and frames). Obviously, b_f cannot extend beyond the slab portion tributary to a beam, i.e., the actual width of slab available. Hence, the calculated b_f should be restricted to a value that does not exceed $(s_1+s_2)/2$ in the case of T–beams, and $s_1/2 + b_w/2$ in the case of L–beams, where the spans s_1 and s_2 of the slab are as marked in Fig.

$$b_f = \begin{cases} \dfrac{l_0}{l_0/b+4} + b_w & \text{for isolated } T-Beams \\ \dfrac{0.5\, l_0}{l_0/b+4} + b_w & \text{for isolated } L-Beam \end{cases} \quad (13)$$

where b denotes the *actual* width of flange; evidently, the calculated value of b_f should not exceed b.

1.5 Problems:

1. Design a singly reinforced SSB of clear span 5m to support a working live load of 25 kN/m run. Use Fe 415 steel and M 20 grad concrete. Assume the support thickness as 230 mm.

Step 1 (a): Fixing up the depth of the section.

Taking $\frac{L}{d} = 20$, for SSB [Refer 23.2.1, pg 37]

$$d = \frac{L}{20} = \frac{5}{20} = 0.25 \text{ m} = 250 \text{ mm}$$

Providing a cover of 25 mm, overall depth D = 250 + 25 = 275 mm

Dimensions of the section.

Width b = 230 mm

depth d = 250 mm

Step 1 (b): Check for lateral stability/lateral buckling

Refer page 39, clause 23.3

Allowable l = 60b or $\frac{250\, b^2}{d}$

Allowable l = 60b = 13800 mm = 13.8 m

Or $\frac{250\, b^2}{d}$ = 52900 mm = 52.9 m

Allowable l = Lesser of the two values

= 13.8 m

Actual l of the beam (5m) < Allowable value of l. Hence ok

Or $\frac{100\, b^2}{d}$ = 11750 mm = 11.75 m

Allowable l = Lesser of the two values

= 5.75 m

Actual l of the beam (5m) < Allowable value of l.

Hence ok

Step 2: Effective span

Referring class 22.2 page 34,

Effective span l_e = clear span + d

Or l_e = clear span + $\frac{1}{2}$ support thickness

= clear span + $\frac{l_s}{2}$

Whichever is lesser.

L_e = 2 m + 450 mm = 2450 mm

Or $L_e = 2\text{ m} + \frac{230}{2} = 2115$ mm

Therefore l_e = 2115 mm

Step 3: Calculation of loads

Consider 1m length of the beam

a. Dead load = (0.23 x 0.475 x 1m x 25 kN/m^3)x1.5 = 4.096 ≈ 4.1 kN/m
b. Factored live load = 30 kN/m

Total Factored load W_u = 34.1 kN/m ≈ 35 kN/m

Factored moment $M_u = \frac{W_u \times l_e^2}{2} = \frac{35 \times 2.115^2}{2} = 78.28$ kN-m

Factored shear = 35 x 2.115 = 74.025 kN

Step 4: Check for depth based on flexure or bending moment consideration

Assuming the section to be nearly balanced, and equating M_u to M_{ulim},

$M_u = M_{ulim}$ = 78.28 kN-m

Using the equation G 1.1 (c)

$$M_{ulim} = 0.36\frac{x_{umax}}{d}\left(1 - 0.42\frac{x_{umax}}{d}\right)bd^2 f_{ck}$$

$$78.28 \times 10^6 = 0.36 \times 0.48\,(1 - 0.42 \times 0.48)230d^2 \times 20$$

d = 222 mm

$d_{assumed} > d_{required}$

Hence ok.

Step 5: Calculation of steel

Since the section is under reinforced we have,

Using equation G 1.1 (b)

$$M_u = 0.87 f_y A_{st}\, d\left(1 - \frac{A_{st} f_y}{bd\, f_{ck}}\right)$$

$$78.28 \times 10^6 = 0.87 \times 415 \times A_{st} \times 450\left(1 - \frac{A_{st} \times 415}{230 \times 450 \times 20}\right)$$

Solving the quadratic equation, A_{st} = 540.33 mm^2 ≈ 540 mm^2

Choosing 16 mm diameter bars,

$$\text{Area of 1 bar} = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$$

Therefore number of bars of 8mm required = 2.69= 3 bars

Distance between any two bars

Minimum distance between two bars is greater of the following:

a. Size of the aggregate + 5 mm
   20 mm + 5 mm
b. Size of the bar (whichever is greater)=16mm

Therefore minimum distance = 25 mm

$$\text{Distance between the bars} = \frac{230-2\times25-2\times16-2\times8}{2} = 58\text{mm}$$

Distance provided = 58mm > Minimum distance 25mm

Hence ok.

Check for $A_{st\ min}$

$$A_{st\ min} = \frac{0.85bd}{0.87f_y}$$

$$A_{st\ min} = \frac{0.85 \times 230 \times 450}{0.87 \times 415} = 243.66 \text{ mm}^2$$

A_{st} provided = 3 x $\frac{\pi}{4}$ x 16^2 = 603.18 mm^2 > $A_{st\ min}$

Hence ok.

Check for $A_{st\ max}$

$A_{st\ max}$ = 0.04 x b x D = 4370 mm^2

A_{st} provided = 603.18 mm^2

$A_{st\ min} < A_{st} < A_{st\ max}$

Hence ok.

Check for shear

V_u = 74.025 kN

$$\tau_v = \frac{V_u}{bd} = 0.715 \text{ N/mm}^2$$

$$P_t = \frac{100A_{st}}{bd} = \frac{100\times603.18}{230\times450} = 0.58$$

From table 19,

τ_c = 0.51 N/mm^2

From table 20,

$\tau_{c\ max}$ = 2.8 N/mm^2

$\tau_c < \tau_v < \tau_{c\ max}$

Hence design of shear reinforcement is required

Selecting 2 leg vertical stirrups of 8 mm diameter, Fe 415 steel,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100 \text{ mm}^2$$

V_c = Shear force taken up by the concrete

$= \frac{\tau_c bd}{1000} = \frac{0.51 \times 230 \times 450}{1000} = 52.78$ kN

$V_u = 74.025$ kN

$V_{us} = V_u - V_c$

$= 74.025 - 52.785 = 21.24$ kN

$V_{us} = \frac{0.87 \times f_y \times A_{sv} \times d}{S_v}$

$21.24 \times 10^3 = \frac{0.87 \times 415 \times 100 \times 450}{S_v}$

$S_v = 764$ mm

Check for maximum spacing

Maximum spacing = 0.75d or 300mm whichever is lesser

Maximum spacing = 337.5 or 300mm

Therefore maximum spacing allowed = 300mm

Let us provide 8 mm dia 2-leg vertical stirrups at a spacing of 300 mm.

Check for $A_{sv\ min}$:

A_{sv} provided = 100 mm^2

$A_{sv\ min} = \frac{0.4 b S_v}{0.87 f_y} = 76.44$ mm^2

A_{sv} provided > $A_{sv\ min}$

Hence ok.

Check for deflection:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f

Basic $\frac{l}{d} = 7$ as the beam is cantilever

From fig 4, $M_t = 1.2$
From fig 5, $M_c = 1$

From fig 6, $\frac{b_w}{b_f} = 1$ [Since it is rectangular section $b_w = b_f$]

Therefore allowable l/d = 7 x 1.2 x 1 x 1 = 8.4

Actual l/d = $\frac{2115}{450} = 4.7$ < Allowable l/d. Hence ok.

3. Design a reinforced concrete beam of rectangular section using the following data:

Effective span	= 5 m
Width of beam	= 250 mm
Overall depth	= 500 mm
Service load (DL+LL)	= 40 kN/m
Effective cover	= 50 mm
Materials	: M20 grade concrete and Fe 415 steel

a. Data

$b = 250$ mm $\quad f_{ck} = 20$ N/mm^2

$D = 500$ mm $\quad f_y = 415$ N/mm^2

$d = 450$ mm $\quad E_s = 2 \times 10^5$ N/mm^2

$d' = 50$ mm

$l_e = 5$ m

$w = 40$ kN/m and $W_u = 40 \times 1.5 = 60$ kN/m

b. Ultimate moments and shear forces

$$M_u = \frac{W_u \times l_e^2}{8} = \frac{60 \times 5^2}{8} = 187.5 \text{ kN-m}$$

$$V_u = \text{Factored shear} = \frac{W_u \times l_e}{2} = 150 \text{ kN}$$

c. Determination of M_{ulim} and f_{sc}

$$M_{ulim} = 0.36\frac{x_{umax}}{d}\left(1 - 0.42\frac{x_{umax}}{d}\right)bd^2 f_{ck}$$

$$M_{ulim} = 0.36 \times 0.48\,(1 - 0.42 \times 0.48)250 \times 450^2 \times 20$$

$= 140$ kN.m

Since $M_u > M_{u\,lim}$, design a doubly reinforced section

$(M_u - M_{u\,lim}) = 187.5 - 140 = 47.5$ kN.m

$$f_{sc} = \epsilon_{sc} \times E_s$$

Where, $\epsilon_{sc} = \left\{\frac{0.0035(x_{u\,max} - d')}{x_{u\,max}}\right\}$

$$f_{sc} = \left\{\frac{0.0035(x_{u\,max} - d')}{x_{u\,max}}\right\} E_s$$

$$= \left\{\frac{0.0035[(0.48 \times 450) - 50]}{0.48 \times 450}\right\} 2 \times 10^5$$

$= 538$ N/mm^2

But $f_{sc} \not> 0.87 f_y = (0.87 \times 415) = 361$ N/mm^2

Therefore $f_{sc} = 361$ N/mm^2

steel $A_{sc} = \left[\frac{(M_u - M_{u\,lim})}{f_{sc}(d - d')}\right]$

$$= \left[\frac{(47.5 \times 10^6)}{361 \times 400}\right] = 329 \text{ mm}^2$$

Provide 2 bars of 16mm diameter ($A_{sc} = 402$ mm^2)

$$A_{st2} = \left(\frac{A_{sc} f_{sc}}{0.87 f_y}\right) = \left(\frac{329 \times 361}{0.87 \times 415}\right) = 329 \text{ mm}^2$$

$$A_{st1} = \left[\frac{0.36 f_{ck} b x_{u\,lim}}{0.87 f_y}\right]$$

$$= \left[\frac{0.36 \times 20 \times 250 \times 0.48 \times 450}{0.87 \times 415}\right] = 1077 \text{ mm}^2$$

Total tension reinforcement $= A_{st} = (A_{st1} + A_{st2})$

$= (1077 + 329)$

$= 1406 \text{ mm}^2$

Provide 3 bars of 25mm diameter ($A_{st} = 1473 \text{ mm}^2$)

d. Shear reinforcements

$\tau_v = (V_u/bd) = (150 \times 10^3) / (250 \times 450) = 1.33 \text{ N/mm}^2$

$$P_t = \frac{(100A_s)}{bd} = \frac{100 \times 1473}{250 \times 450} = 1.3$$

Referring table 19 of IS : 456 – 2000 ,

$\tau_c = 0.68 \text{ N/mm}^2$

$\tau_{cmax} = 2.8 \text{ N/mm}^2$ for M20 concrete from table 20 of IS 456-2000

Since $\tau_c < \tau_v < \tau_{cmax}$, shear reinforcements are required.

$V_{us} = [V_u - (\tau_c bd)]$

$= [150-(0.68 \times 250 \times 450)10^{-3}] = 73.5 \text{ kN}$

Using 8 mm diameter 2 legged stirrups,

$$S_v = \frac{0.87 \times f_y \times A_{sv} \times d}{V_{us}} = \frac{0.87 \times 415 \times 2 \times 50 \times 450}{73.5 \times 10^3} = 221\text{mm}$$

Maximum spacing is 0.75d or 300 mm whichever is less

$S_v > 0.75d = (0.75 \times 450) = 337.5 \text{ mm}$

Adopt a spacing of 200 mm near supports gradually increasing to 300 mm towards the centre of the span.

e. Check for deflection control

$(l/d)_{actual} = (5000/450) = 11.1$

$(l/d)_{allowable} = [(l/d)_{basic} \times M_t \times M_c \times M_f]$

$P_t = 1.3$ and $P_c = [(100 \times 402) / (250 \times 450)] = 0.35$

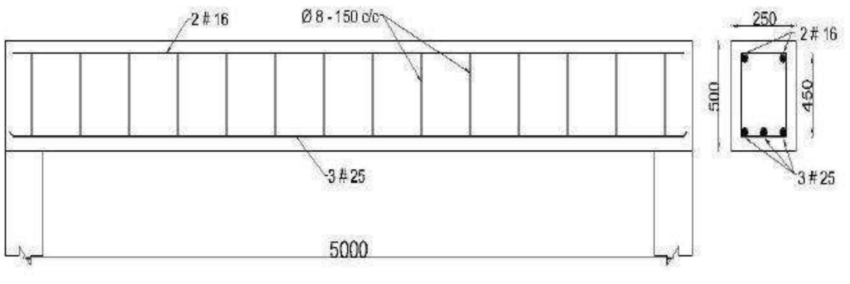
Refer Fig 4, $M_t = 0.93$
Fig 5, $M_c = 1.10$
Fig 6, $M_f = 1.0$

$(l/d)_{allowable} = [(20 \times 0.93 \times 1.10 \times 1] = 20.46$

$(l/d)_{actual} < (l/d)_{allowable}$

Hence deflection control is satisfied.

f. Reinforcement details

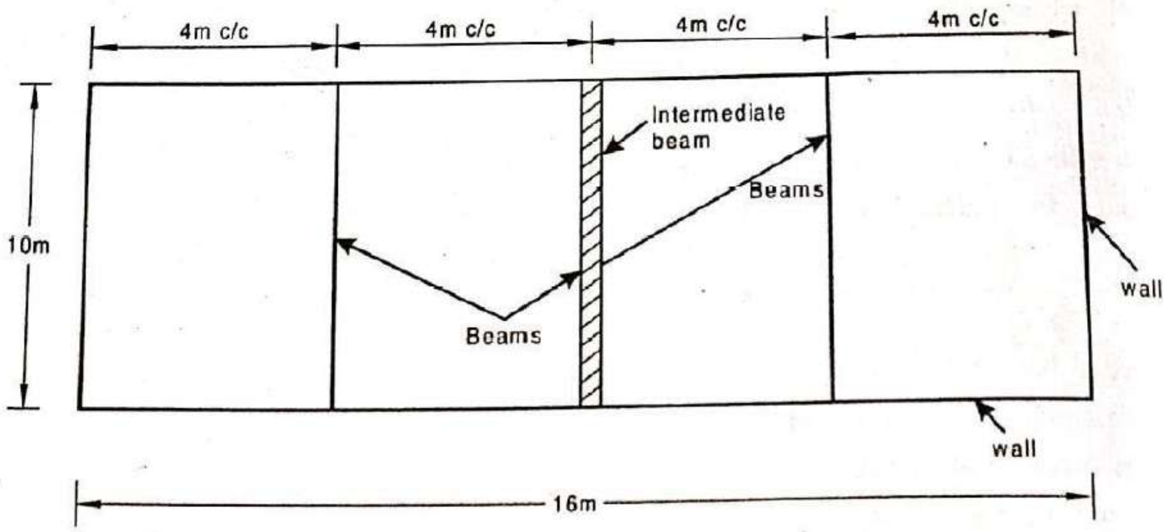


A floor of a hall measures 16m × 10m to the faces of the supporting walls. The floor consists of three beams spaced at 4m c/c and the slab thickness is 150 mm. The floor carries a live lead of 4 kN/m². Design the intermediate T - beam. Use M20 concrete and *Fe* 415 steel. The support width may be assumed as 300mm.

Given : l = 10m, support width = 300mm, LL = 4kN/m^2

Slab depth = D_f = 150mm, Beam spaced = 4m c/c, f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2

Hall measures = 16m × 6m



Step : 1 Selection of cross-sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{span length}}{15} = \frac{10000}{15} = 666.67, \text{ Say } 700\text{mm}$$

(b) Overall depth of beam (D)

$$D = d + d' = 700 + 50 = 750\text{mm (Assume } d' = 50\text{mm)}$$

(c) Width of web (b_w) : Assume b_w = 300mm

(d) Flange thickness (D_f) : D_f = 150mm (Given)

(e) Effective width of flange (b_f)

Effective span (l_0) : should be least of the following two

(i) l_0= Clear span + bearing = 10000 + 300 = 10,300mm or 10.3m

(ii) l_0 = clear span + Effective depth (d) = 1000 + 700 = 10,700mm or 10.7m

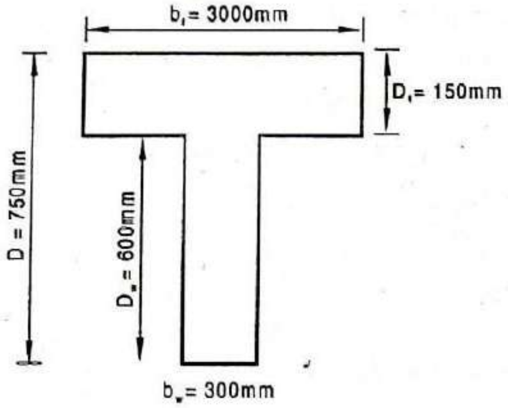
$$\text{Say } l_0 = 10.3 \text{ m}$$

Refer IS : 456-2000, Clause 23.1.2 (a)

For T - beam,

$$b_f = \left(\frac{l_0}{6}\right) + b_w + 6D_f = \left(\frac{10300}{6}\right) + 300 + 6 \times 150 = 2916.66 \text{ mm}$$

$$\therefore \text{Say } b_f = 3000\text{mm or } 3\text{m}$$



Step : 2 Load Calculation

1. Self weight of beam $= [(b_f \times D_f) + (b_w \times D_w)] \times$RCC density
   $= (3 \times 0.15 + 0.3 \times 0.6) \times 25$
   $= 15.75$ kN/m
2. Live load (Given) $= 4 \times 1 = 4$ kN/m

   Total load, W $= 19.75$ kN/m

   $\therefore$ Factored load, $W_u = 1.5 \times W = 1.5 \times 19.75$

   $W_u = 29.62$ kN/m

Live load on slab = 4 kN/m^2
Consider
Per meter width
$\therefore$ LL $= 4 \times 1 = 4 \text{kN/m}^2$

Step : 3 Calculation of Maximum BM

$$M_u = \frac{W_u l_0^2}{8} = \frac{29.62 \times 10.3^2}{8} = 392.79 \text{ kN.m}$$

Step : 4 Calculation of moment capacity of flange

$$M_{uf} = 0.36 f_{ck} b_f x_u (d - 0.42\, x_u)$$

Assuming N – A (Neutral Axis) lies with in the flange

$$\therefore x_u \le D_f$$

$$\text{Consider } x_u = D_f$$

$$\text{Here } D_f = 150\text{mm}$$

$$\therefore M_{uf} = 0.36 f_{ck} b_f D_f (d - 0.42\, D_f)$$

$$= 0.36 \times 20 \times 3000 \times 150\,(700 - 0.42 \times 150)$$

$$M_{uf} = 2063.88 \text{ kN.m}$$

$$\text{Since } M_{uf} > M_u$$

$$\Rightarrow D_f > x_u$$

Hence Neutral axis lies with in the flange. In this case the beam can be treated as a normal rectangular beam of width $b_f = b$ and depth d

Hence singly reinforced section is to be designed

Step : 5 Calculation of Main reinforcement

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b_f}} = \sqrt{\frac{392.79 \times 10^6}{0.138 \times 20 \times 3000}} = 217.80\text{mm} < d_{provided}\ (700\text{mm})$$

Hence design is safe

$$M_u = 0.87 f_y A_{st}\, d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} b_f d}\right)\right]$$

$$392.79 \times 10^6 = 0.87 \times 415 \times 700\, A_{st} \left[1 - \left(\frac{415 A_{st}}{20 \times 3000 \times 700}\right)\right]$$

$$\therefore 2.49 A_{st}^2 - 252735\, A_{st} + 392.79 \times 10^6 = 0$$

$$\therefore A_{st} = 1578.71 \text{ mm}^2$$

Assume 25mm ϕ bars

$$\therefore \text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{1578.71}{\frac{\pi}{4} \times 25^2} = 3.21, \text{ say } 4$$

$$A_{st_{provided}} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.49\text{mm}^2$$

∴ Provide 4-25 mm ϕ bars at tension side and 2-12mm ϕ bars at compression side (Anch bars) - [Assume]

Step : 6 Design of Shear reinforcement

- Design of shear force, $V_u = \frac{W_u l_o}{2} = \frac{29.62 \times 10.3}{2} = 152.54$ kN
- Nominal shear stress, $\tau_v = \frac{V_u}{b_w d} = \frac{152.54 \times 10^3}{300 \times 700} = 0.72$ N/mm^2
- Percentage of tension steel reinforcement, $p_t = \frac{100 A_{st_{provided}}}{b_w d} = \frac{100 \times 1963.49}{300 \times 700} = 0.93\%$

Calculation of τ_c, Refer table 19 of IS : 456-2000
for $p_t = 0.93\%$ and $f_{ck} = 20$ N/mm^2
∴ $\tau_c = 0.60$ N/mm^2 (By interpolation)

- Design shear strength, $\tau_{c_{max}} = 2.8$ N/mm^2 (From table 20)
- Comparisons

$\tau_v > \tau_c$ – Hence shear reinforcement is required

(i) Calculate shear carried by concrete

$V_{uc} = \tau_c b_w d = 0.6 \times 300 \times 700 = 126$ kN

(ii) Calculate shear carried by stirrups

$V_{us} = V_u - V_{uc} = 152.54 - 126 = 26.54$ kN

Spacing of stirrups should be least of the following three

1. $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$

$A_{sv} = 100.53$ mm^2 for 2L – 8mm ϕ vertical stirrups

$\therefore S_v = \frac{0.87 \times 415 \times 100.53 \times 700}{26.54 \times 10^3} = 957.32$mm

2. $S_v \not> 0.75d$, $\therefore S_v = 0.75 \times 700 = 525$mm
3. $S_v \not> 300$mm, $\therefore S_v = 300$mm

Say, $S_v = 300$mm

Hence provide 2L – 8mm ϕ bars @ 300 mm c/c

Step : 7 Check for deflection control

(a) Note down the percentage of steel provided

$$p_t = 0.93\%$$

(b) Stress in steel, Refer IS: 456-2000, Fig. 4

$$f_s = 0.58 f_y \left[\frac{A_{st_{req}}}{A_{st_{pro}}}\right] = 0.58 \times 415 \times \frac{1578.71}{1963.49} = 193.53 \text{ N/mm}^2$$

read out the modification factor (k_t) using p_t and f_s

$$\therefore k_t = 1.22$$

Now, ratio $\frac{b_w}{b_f} = \frac{300}{3000} = 0.1$

read out the reduction factor (k_f) using $\frac{b_w}{b_f}$ referring IS : 456–2000, Fig. 6.

$$\therefore k_f = 0.8$$

(1) $\left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times k_c \times k_f$ $\quad k_c = 1 \because$ No compression reinforcement

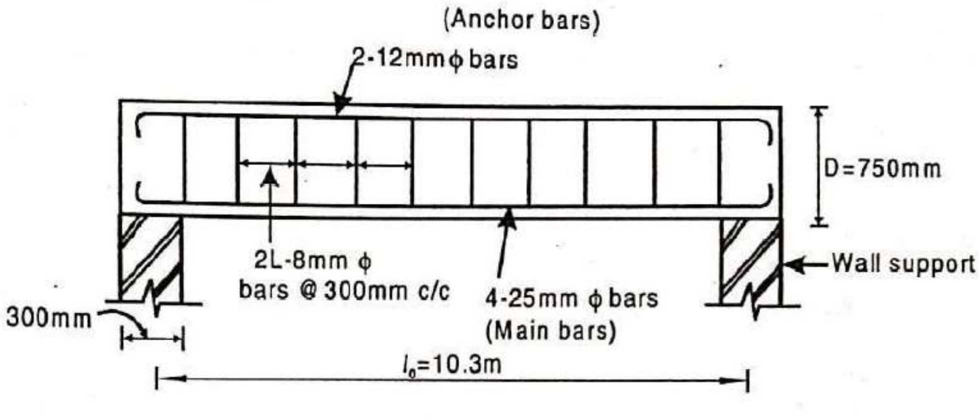
$$= 20 \times 1.22 \times 1 \times 0.8 = 19.52$$

(2) $\left(\frac{L}{d}\right)_{provided} = \frac{10300}{700} = 14.71$

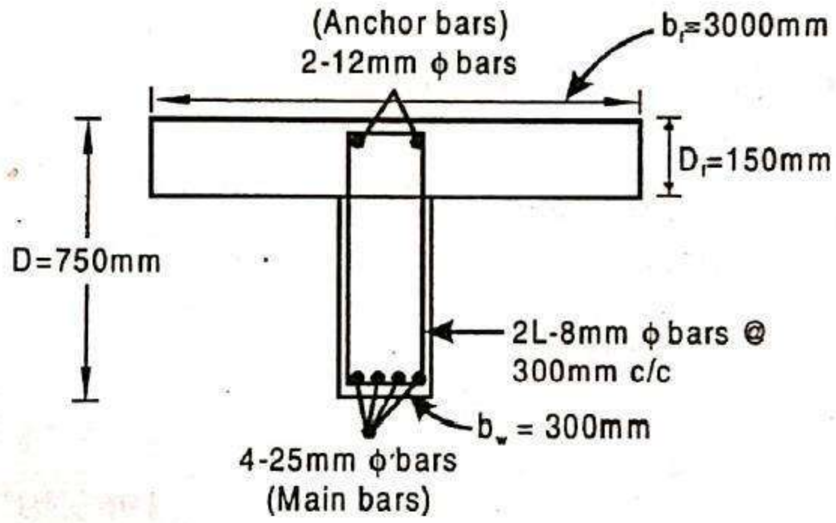
$$\therefore \left(\frac{L}{d}\right)_{max.} > \left(\frac{L}{d}\right)_{provided}$$

Hence deflection control is satisfied

Step : 8 Reinforcement details



L/S of T-beam



C/S of T-beam

1.6 Outcome

1. Able to design singly, doubly reinforced beam and T-beam

1.7 Assignment questions

A tee beam slab floor of an office comprises of a slab 150 mm thick spanning between ribs spaced at 3m centres. The effective span of the beam is 8 m. Live load on floor is 4 kN/m^2. Using M-20 grade concrete and Fe-415 HYSD bars, design one of the intermediate tee beam.

1.8 Future Study

https://nptel.ac.in/courses/105105105/11

2.1 Outcome

1. Able to analyze singly and doubly reinforced beam
2. Able to know failure modes of beams
3. Able to know the shear behavior of beams

2.2 Assignment questions

1. What is the difference is between singly reinforced and doubly reinforced beam?
2. Explain different types of stirrups with a neat sketch.
3. Describe the failure modes of beams with a neat sketch.
4. What is development length?

2.3 Future Study

https://nptel.ac.in/courses/105105104/pdf/m5l11.pdf