Module – 2

Analysis of Beam subjected to Flexure, shear and torsion

1.1 Introduction to Failure modes of beams

Failure Modes due to Shear

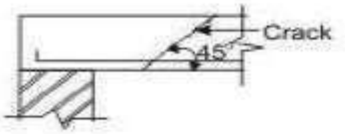

Figure 1.10 (a) Web shear progress along dotted lines

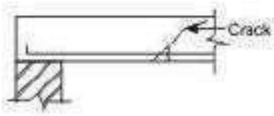

Figure 1.10 (b) Flexural tension

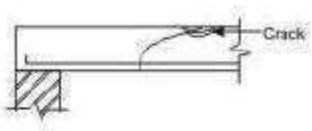

Figure 1.10 (b) Flexural compression

Bending in reinforced concrete beams is usually accompanied by shear, the exact analysis of which is very complex. However, experimental studies confirmed the following three different modes of failure due to possible combinations of shear force and bending moment at a given section:

(i) Web shear (Fig. 1.10a)

(ii) Flexural tension shear (Fig. 1.10b)

(iii) Flexural compression shear (Fig. 1.10c)

Web shear causes cracks which progress along the dotted line shown in Fig. 1.10a. Steel yields in flexural tension shear as shown in Fig. 1.10b, while concrete crushes in compression due to flexural compression shear as shown in Fig. 1.10c. An in-depth presentation of the three types of failure modes is beyond the scope here.

1.2 Objectives

1. To analyze the RCC beam as singly or doubly

1.3 Shear Stress

The distribution of shear stress in reinforced concrete rectangular, *T* and *L*-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):

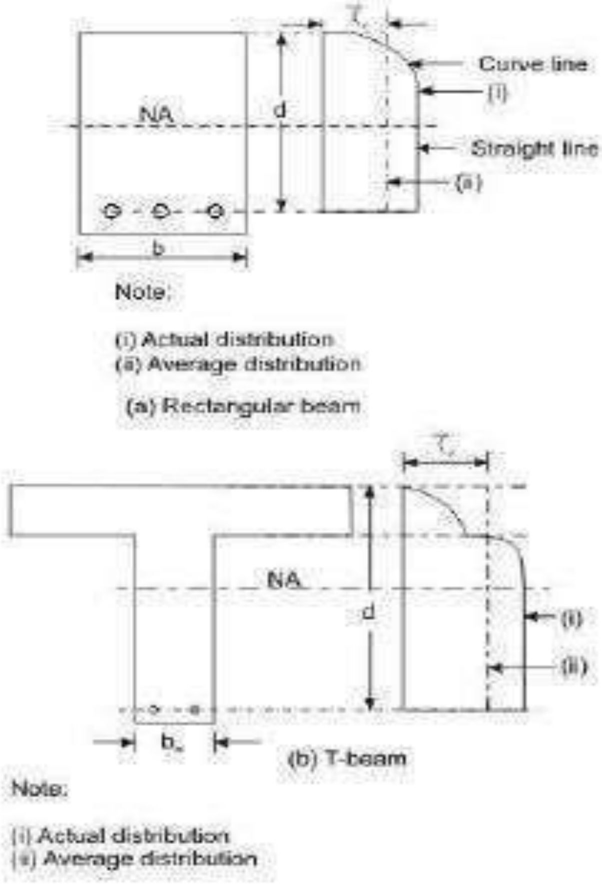

Figure 1.11: Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs. 1.11a and b):

$$\tau_v = \frac{V_u}{bd}$$

where V_u = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

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$$\tau_v = \frac{V_u}{bd}$$

where V_u = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

d = effective depth.

(ii) In beams of varying depth:

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd}$$

where τ_v, Vu, b or b_w and d are the same as in (i),

M_u = bending moment at the section, and

β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment M_u decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment M_u increases numerically in the same direction as the effective depth increases.

1.4 Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value τ_{cmax} depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table 3 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y} \quad (15)$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam b_w, and

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2.

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.

The minimum shear reinforcement is provided for the following:

(i) Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

(ii) Brittle shear failure is arrested which would have occurred without shear reinforcement.

(iii) Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.

(iv) To hold the reinforcement in place when concrete is poured.

(v) Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 d for vertical stirrups and d for inclined stirrups at 45^o, where d is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

1.5 Design of Shear Reinforcement (cl. 40.4 of IS 456)

When τ_v is more than τ_c given in Table 6.1, shear reinforcement shall be provided in any of the three following forms:

(a) Vertical stirrups,

(b) Bent-up bars along with stirrups, and

(c) Inclined stirrups

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement.

The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to

$$V_{us} = V_u - \tau_c bd \qquad (16)$$

where b is the breadth of rectangular beams or b_w in the case of flanged beams.

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) Vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} \qquad (17)$$

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} (\sin\alpha + \cos\alpha) \qquad (18)$$

(c) For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87 f_y A_{sv} d \sin\alpha \qquad (19)$$

where A_{sv} = total cross-sectional area of stirrup legs or bent-up bars within a distance s_v,

s_v = spacing of stirrups or bent-up bars along the length of the member,

τ_v = nominal shear stress,

τ_c = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web b_w,

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm^2,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45^o, and

d = effective depth.

The following two points are to be noted:

(i) The total shear resistance shall be computed as the sum of the resistance for the various types separately where more than one type of shear reinforcement is used.

(ii) The area of stirrups shall not be less than the minimum specified in cl. 26.5.1.6.

Numerical Problem

Find the moment of resistance of a singly reinforced concrete beam of 200 mm width 400mm effective depth, reinforced with 3-16 mm diameter bars of Fe 415 steel. Take M20 grade of concrete.

Solution

$$A_{st} = 3x\frac{\pi}{4}(16)^2 = 603.19\,mm^2$$

$$\%\,p_t = 100x\frac{603.19}{200x400} = 0.754\%$$

$$\frac{x_u}{d} = 2.417\,p_t\frac{f_y}{f_{ck}} = 2.417x\frac{0.754}{100}x\frac{415}{20} = 0.378$$

Now for Fe 415 grade of steel, $\frac{x_{u,\max}}{d} = 0.479$

Hence the beam is under-reinforced.

The moment of resistance is given by

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$= 0.87x415x603.19x400\left(1 - \frac{415x603.19}{20x200x400}\right)$$

=73.48 KN-m.

1.6 Bond

The bond between steel and concrete is very important and essential so that they can act together without any slip in a loaded structure. With the perfect bond between them, the plane section of a beam remains plane even after bending. The length of a member required to develop the full bond is called the anchorage length. The bond is measured by bond stress.

The local bond stress varies along a member with the variation of bending moment.

Thus, a tensile member has to be anchored properly by providing additional length on either side of the point of maximum tension, which is known as Development length in tension‘.

Similarly, for compression members also, we have Development length *Ld* in compression‘.

Accordingly, IS 456, cl. 26.2 stipulates the requirements of proper anchorage of reinforcement in terms of development length *Ld* only employing design bond stress η*bd*

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Design bond stress – values

The average bond stress is still used in the working stress method and IS 456 has mentioned about it in cl. B-2.1.2. However, in the limit state method of design, the average bond stress has been designated as design bond stress η*bd* and the values are given in cl. 26.2.1.1

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design Bond Stress τ_{bd} in N/mm2	1.2	1.4	1.5	1.7	1.9

For deformed bars conforming to IS 1786, these values shall be increased by 60 per cent. For bars in compression, the values of bond stress in tension shall be increased by 25 per cent.

1.7 Development Length

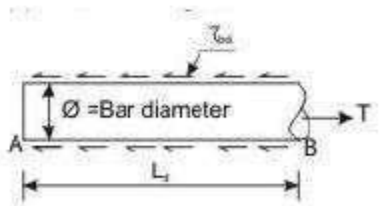

Figure 1.13 Development length of bar

Figure 1.13 shows the free body diagram of the segment AB of the bar. At B, the tensile force *T* trying to pull out the bar is of the value *T* = (π θ2 ζs /4), where θ is the nominal diameter of the bar and ζs is the tensile stress in bar at the section considered at design loads. It is necessary to have the resistance force to be developed by η*bd* for the length *Ld* to overcome the tensile force. The resistance force = π θ (*Ld*) (η*bd*). Equating the two, we get π θ (*Ld*) (η*bd*) = (π θ2 ζs/4) (19)

Equation (19), thus gives

$$L_d = \frac{\phi \sigma_s}{4\tau_{bd}} \quad (20)$$

The above equation is given in cl. 26.2.1 of IS 456 to determine the development length of bars.

The example taken above considers round bar in tension. Similarly, other sections of the bar should have the required L_d as determined for such sections. For bars in compression, the development length is reduced by 25 per cent as the design bond stress in compression τ_{bd} is 25 per cent more than that in tension (see the last lines below Table 6.4). Following the same logic, the development length of deformed bars is reduced by 60 per cent of that needed for the plain round bars. Tables 64 to 66 of SP-16 present the development lengths of fully stressed plain and deformed bars (when $\sigma_s = 0.87 f_y$) both under tension and compression. It is to be noted that the consequence of stress concentration at the lugs of deformed bars has not been taken into consideration.

1.8 Outcome

1. Able to analyze singly and doubly reinforced beam
2. Able to know failure modes of beams
3. Able to know the shear behavior of beams

1.9 Assignment questions

1. What is the difference is between singly reinforced and doubly reinforced beam?
2. Explain different types of stirrups with a neat sketch.
3. Describe the failure modes of beams with a neat sketch.
4. What is development length?

1.10 Future Study

https://nptel.ac.in/courses/105105104/pdf/m5l11.pdf